ON EMITTANCE GROWTH IN SPACE-CHARGE DOMINATED BEAMS

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Abstract

An analytic model for emittance growth in the round beam transported in arbitrary electrostatic channel is presented to reproduce the main features of the beam dynamics in real, nonlinear fields. The model bases on the set of equations for adiabatic transport of single-fluid plasma which can be reduced to the equations with respect to the coefficients of aberration representation for the beam flow lines. The first equation closes to envelope equation for K-V beam. The second equation defines disturbance caused by field nonlinearity and the other effects in the beam interior. Simple combination of their solutions gives so-called fluid part of rms emittance responsible for beam phase-ellipse crooking. If the solutions are within some limits the beam distortion will stay reversible. Otherwise beam appears as a multi-stream or turbulence flow, entropy of which grows due to multivaluedness of stream velocity.

1 INTRODUCTION

A tendency to consider halo formation in Liouvillian beams as a randomization process or chaotic behavior of the single particles is observed, although emittance is statistical parameter. Furthermore these particles are considered as the test ones, and their migration does not change the beam field. Halo formation appears as escaping particles from the beam core so is transfer of mass and driven by the mass flow velocity. If beam collisionless is true, phase space projection area may grow only because of coupling between degrees of freedom. An affective area may rise due to filamentation in nonlinear fields.

Much used now the idea of relating emittance growth to an expected redistribution of space charge and to the beam field energy released in this process allows an asymptotic estimation only [1]. The reason is lack of an a priory information on the real beam profile at given section. In evolution process, a beam, like charged plasma, infinitely extends in free space and tends to an equilibrium configuration in confining fields. At the equilibrium configuration, the total electric field counterbalances exactly heat motion of beam particles and is entirely absent in the cold beam. Such equilibrium configurations can be computed [2]. In practice, the space charge distribution oscillates about equilibrium one due to excess of energy (free energy). A change in the confining field as well as the free energy conversation into emittance growth will modify the equilibrium configuration. Hence, actual emittance growth is determined by the details of the dynamical process, the simplest model for that is described below. The model employs the concept of beam flow lines. Tangent to the flow line specifies direction of the particle average velocity at the point of tangency. In ordinary case, the phase coordinates of the flow lines correspond to axis of the ellipse which presents the beam in the phase space (Fig.1).

2 ABERRATION MODEL FOR INTENSE BEAM

In the general case a beam of charged particles is characterized by the distribution function $f(\vec{r},\vec{v},t)$. Transfer of mass is handled with the local charge density $\rho(\vec{r},t) = qn(\vec{r},t)$ and local average velocity $\vec{V}$ referred also to mass-flow or directed one and defined as follows .

$$\vec{V}(\vec{r},t) = \int_{-\infty}^{+\infty} \vec{v}\, f(\vec{r},\vec{v},t)d\vec{v} \Big/ n(\vec{r},t),$$

$$n(\vec{r},t) = \int_{-\infty}^{+\infty} f(\vec{r},\vec{v},t)d\vec{v}.$$

A single particle velocity at the point $\vec{r}$ is assumed to be a random value with the expectation $\vec{V}$ and specified by the matrix of the second moments or covariance matrix

$$T_{ps} = \int_{-\infty}^{+\infty} (v_p - V_p)(v_s - V_s)\, f(\vec{r},\vec{v},t)d\vec{v} \Big/ n(\vec{r},t)$$

which is often referred as a temperature tensor too. Rela-

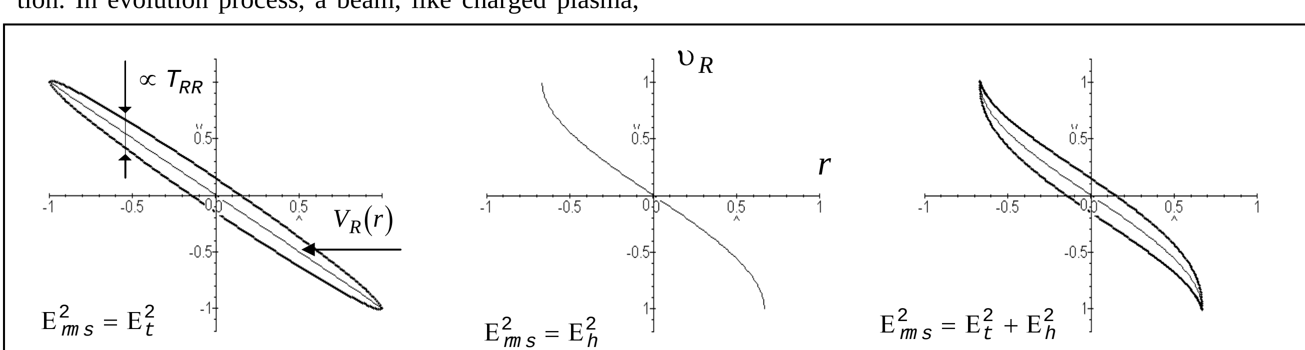

Fig.1

tionships between mass-flow velocity, charge density and temperature tensor are known as the equations for transfer of mass, momentum and heat [4,6], and derived by integrating the Vlasov equation over the velocity space. The equations for the temperature tensor involves the third moments of the distribution function, which in turn include the fourth ones, and so on indefinitely.

In the model, charge particle flow is considered neglecting the third moments and nondiagonal terms of temperature tensor. Physical sense of the limitation will discuss later on. For these assumptions the set of transport equations for axisymmetric beam with no magnetic field follows [4]:

$$\frac{dV_R}{dt} = \frac{q}{m}\frac{\partial \Phi}{\partial r} - \frac{1}{\rho}\frac{\partial(\rho T_{RR})}{\partial r} + \frac{T_{\theta\theta}}{r} - \frac{T_{RR}}{r},$$

$$\frac{dV_Z}{dt} = \frac{q}{m}\frac{\partial \Phi}{\partial z} - \frac{1}{\rho}\frac{\partial(\rho T_{ZZ})}{\partial z}, \qquad \frac{dT_{ZZ}}{dt} = -2T_{ZZ}\frac{\partial V_Z}{\partial z},$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) + \frac{\partial^2 \Phi}{\partial z^2} = \frac{\rho}{\varepsilon_0}, \qquad \frac{dT_{\theta\theta}}{dt} = -2T_{\theta\theta}\frac{V_R}{r},$$

$$\frac{dT_{RR}}{dt} = -2T_{RR}\frac{\partial V_R}{\partial r}, \qquad \frac{d\rho}{dt} + \rho \,\mathrm{div}\vec{V} = 0\cdot$$

The equations above give the well-known integrals along flow lines:

$$T_{\theta\theta} r^2 = const. \qquad T_{ZZ}V_{ZZ}^2 = const. \quad T_{RR}V_{RR}^2 = const.$$

$$\rho = \frac{const.}{rV_R V_Z} \qquad \frac{\rho}{\sqrt{T_{\theta\theta}T_{RR}T_{ZZ}}} = const \quad \frac{\rho V_Z}{\sqrt{T_{\theta\theta}T_{RR}}} = const.$$

The flow lines are governed by the equation $r' = V_R/V_Z$ and can be expanded in the terms of Lagrangian variable ξ marking initial location of the flow lines in the phase plane. Discarding the higher terms in the expansion, i.e., considering the lines of flow near axis, one may write

$$\begin{cases} r(z) = \xi R(z) + \xi^3\chi(z), \\ r'(z) = \xi R'(z) + \xi^3\chi'(z) \end{cases} \quad 0 \le \xi \le 1$$

The coefficients R and χ can be found from the set of equations disposed in frame at the bottom of the page, where I is the beam full current. In obtaining Eqs. (1)-(2) all longitudinal thermal effects have been ignored by putting $T_{ZZ} = 0$, and

$$V_Z^2(r,z) \approx \frac{2q}{m}\Phi_0(z)\left\{1 + r^2\left[\frac{J(0,z)}{4\varepsilon_0\sqrt{\Phi_0^3(z)\,2q/m}} - \frac{\Phi_0''(z)}{4\Phi_0(z)}\right]\right\}$$

been assumed (details see [9]). The most important macroscopic quantities can be expressed in the terms of R and χ as follows: the current density

$$J(r,z) \approx J(0,0)\frac{R^2(0)}{R^2(z)}\left\{1 + \frac{2r^2}{R^2(z)}\left[\frac{\mathrm{P}_2}{\mathrm{P}_0} - 2\frac{\chi(z)}{R(z)}\right]\right\},$$

the effective temperature

$$T_{RR}(r,z) \approx T_{RR}(0,0)\frac{R^2(0)}{R^2(z)}\left\{1 + \frac{r^2}{R^2(z)}\left[1 + 2\frac{\chi(z)}{R(z)}\right]\right\},$$

$$T_{\theta\theta}(r,z) \approx T_{\theta\theta}(0,0)\frac{R^2(0)}{R^2(z)}\left\{1 + \frac{r^2}{R^2(z)}\left[1 - 2\frac{\chi(z)}{R(z)}\right]\right\},$$

the fluid part of rms emittance (definition see [5], Fig.1)

$$\mathrm{E}_h^2 = \kappa\frac{(R\chi' - R'\chi)^2}{72}$$

(the factor κ closes to unity, being slowly dependent on the transverse beam profile). The consideration $\chi R' = \chi' R$ is obviously required for the phase ellipse to appear with no distortion. The ratio $R = -3\chi$ indicates origin of so-called fold with two-stream character of the local velocity distribution (Fig.2). "Bifurcation point" corresponds such ξ that $(dr/d\xi)/(dr'/d\xi) = 0$.The well-known causes of the phenomenon are: first, nonlinear total transverse force; second, nonlinear transverse pressure of particles; third, spread in the longitudinal average speed across the beam due to the field divergent property. They may be described as oscillation frequency dependence vs. amplitude and interpreted as resonance overlap mechanism [3,8]. Hence, the aberration model gives reliable criterion for the most fast stage of the halo formation, scenario of which sketched in Fig.2. For a particular case, the model allows to describe analytically evolution of the space-charge density shape and rms emittance. An optimal transport conditions are found to be in a special profile of

$$\begin{cases} R'' = -\dfrac{\Phi_0' R'}{2\Phi_0} - \dfrac{\Phi_0'' R}{4\Phi_0} + \dfrac{Q}{4\Phi_0^{3/2}R} + \dfrac{\mathrm{E}^2}{\Phi_0 R^3}\left(1 - \dfrac{\rho_2 R^2}{\rho_0} - \dfrac{4\chi}{R}\right), \\ \chi'' = \dfrac{\Theta_1}{u_0^2}\chi - \dfrac{\Theta_0}{u_0^2}\chi' + R^3\left(\dfrac{\Theta_3}{u_0^2} - \dfrac{2u_2}{u_0}\dfrac{\Theta_1}{u_0^2}\right) + R^2R'\left(\dfrac{2u_2}{u_0}\dfrac{\Theta_0}{u_0^2} - \dfrac{\Theta_2}{u_0^2}\right), \end{cases} \tag{1)-(2}$$

$$R(0) = R_b(0),\ R'(0) = R_b'(0),\ \chi(0) = \chi'(0) = 0$$

$$\frac{\Theta_0}{u_0^2} = \frac{\Phi_0'}{2\Phi_0}, \qquad \frac{\Theta_1}{u_0^2} = \frac{R''}{R} + \frac{1}{2}\frac{\Phi_0' R'}{\Phi_0 R}, \qquad \frac{\Theta_2}{u_0^2} = -\frac{1}{8}\left(\frac{\Phi_0'''}{\Phi_0} + \frac{1}{2}\frac{\Phi_0' Q}{\Phi_0^{5/2}R^2}\right),$$

$$\frac{\Theta_3}{u_0^2} = \frac{\Phi_0^{IV}}{32\Phi_0} + \frac{Q}{32\Phi_0^{3/2}R^2}\left(\frac{4\rho_2}{\rho_0} - \frac{\rho_0''}{\rho_0}\right) + \frac{\mathrm{E}^2}{\Phi_0 R^4}\frac{\rho_2}{\rho_0}\left(1 - \frac{2\chi}{R} + \frac{\rho_2 R^2}{\rho_0}\right),$$

$$\frac{u_2}{u_0} = \frac{1}{8}\left(\frac{Q}{\Phi_0^{3/2}R^2} - \frac{\Phi_0''}{\Phi_0}\right), \qquad \frac{\rho_2}{\rho_0} = \frac{1}{R^2}\left(\frac{2\mathrm{P}_2}{\mathrm{P}_0} - \frac{4\chi}{R}\right) - \frac{u_2}{u_0}, \qquad \frac{\rho_0''}{\rho_0} = \frac{3}{4}\left(\frac{\Phi_0'}{\Phi_0}\right)^2 - \frac{\Phi_0''}{2\Phi_0} - \frac{2R''}{R},$$

$$\Phi_0(z) = \Phi(0,z), \qquad Q = \frac{J(0,0)R^2(0)}{\varepsilon_0\sqrt{2q/m}}, \qquad \mathrm{E}^2 = \frac{T_{RR}(0,0)R^2(0)}{q/m}, \qquad \frac{\mathrm{P}_2}{\mathrm{P}_0} = \frac{I}{J(0,0)\pi R^2(0)} - 1\cdot$$

beam and a certain phase shift per cell [10]. The model is best suited to design optics with compensated aberration by beam transverse profile control.

3 DISCUSSION

The applied set of equations ignores dissipation process and corresponds to an adiabatic plasma transport with entropy conservation. Dissipation is always associated with irreversibility of motion and entropy growth. So-called viscosity process or internal friction, described with nondiagonal elements of temperature tensor occurs only when the substance motion proceeds along the flow lines with different velocities. In this case there are moving of the flow parts relative to each other and interchange energy of the directed motion by heat migration of beam particles [7]. In other words, viscosity takes effect of the transfer of the directed motion energy from place with higher flow velocity to that with lower one and dissipation of the energy into beam heat, i.e., increasing particle speed spread. Handling of the collisionless dissipation requires the equation for nondiagonal terms of temperature tensor,

$$\frac{dT_{ZR}}{dt} = -T_{ZR}\left(\frac{\partial V_R}{\partial r} + \frac{\partial V_Z}{\partial z}\right) - T_{ZZ}\frac{\partial V_R}{\partial z} - T_{RR}\frac{\partial V_Z}{\partial r}, \quad (3)$$

and appropriate additions,

$$\frac{dV_R}{dt} = \cdots - \frac{1}{\rho}\frac{\partial(\rho T_{ZR})}{\partial z}, \qquad \frac{dT_{RR}}{dt} = \cdots - 2T_{ZR}\frac{\partial V_R}{\partial z},$$

$$\frac{dV_Z}{dt} = \cdots - \frac{1}{\rho}\frac{\partial(\rho T_{ZR})}{\partial r} - \frac{T_{ZR}}{r}, \quad \frac{dT_{ZZ}}{dt} = \cdots - 2T_{ZR}\frac{\partial V_Z}{\partial r},$$

to be included in the model. The equation analysis shows that heating of the beam due to internal friction near the axis decreases with increasing speed of particles and is negligible in the quasi-parallel beams of low temperature. In particular, a K-V beam conserves transverse emittance in linear fields as long as all beam particles have the same longitudinal velocity. Otherwise, a change in beam parameter results in growth of transverse phase ellipse thickness. The same fundamental process can be described as the Arnold diffusion [3,8]. The two last terms of Eq.(3) determines the process speed.

The entropy conservation requires in addition to low viscosity some conditions for eliminating the heat flow. These conditions are associated with a certain symmetry in the distribution function or, more precisely, with zero third moment [4,6]. From this viewpoint the approximation used consists in replacing actual speed distribution by a symmetric one.

On the other hand, it is known that entropy growth in adiabatic flows can result from irreversible losses of the directed motion energy, when the losses are accompanied by forming surfaces of discontinuity (jump) in temperature, pressure or density. Such a surface with moment flow through it is usually referred as a shock wave [5,7]. Obviously, shock wave forming is followed by the fold with typical for it asymmetry in distribution function about local average velocity. For this reason and requirement to conserve a certain ratio between solution to the first approximation envelope equation (1) and its perturbation χ, application of the model is limited within the range $R \geq 3|\chi|$.

Forced elimination of the fold is made difficult by multivaluedness of the flow line phase curve, then by necessity for selected action on the group of particles located at the same point of the configuration space. In the absence of electric field an asymptotical straightening of the fold is possible always and associated with beam cooling on expanding, i.e., transformation of the particle heat energy into the work of pressure forces.

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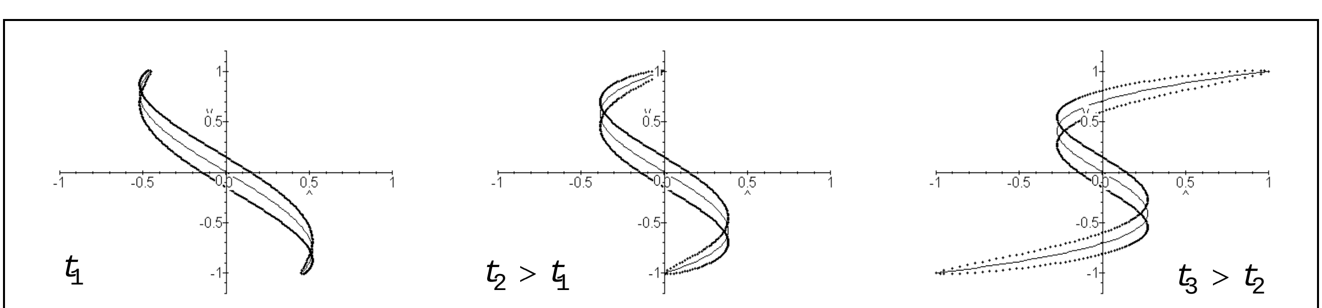

Fig.2