CALIBRATION OF QUADRUPOLE MAGNETS VIA RESPONSE MATRIX FITTING

L. Farvacque and R. Nagaoka, ESRF, Grenoble, France

Abstract

The paper describes an attempt made at the ESRF to calibrate the main storage ring quadrupoles through the response matrix fitting. The developed procedure as well as the outcomes are presented along with features that turned out to be noteworthy.

1 INTRODUCTION

The best knowledge of quadrupole calibration is essential in exploiting the full performance of the challenging optics of the new generation light sources. A calibration using the orbit response matrix had previously brought about a significant improvement in the optics modelling [1]. A more direct way of determining the effective quadrupole strengths and therefore the optics has recently been attempted via a least square fit of the measured response matrix.♦ Three features have been additionally taken into account. 1) To fit the measured displacement and deduce simultaneously the steerer calibration. 2) To remove the dispersive component from the measured displacement. 3) To acquire the response matrix and operate the machine over a wide range of quadrupole currents to study the global behaviour [3].

2 CALIBRATION MODEL

There are two basic sets of ingredients; $\{I,\ G\}_{Poisson}$ and $\{I,\ (Gl)\}_{meas}$ ($G \equiv \partial B_z/\partial x$). For a given quadrupole current I, its corresponding *default* strength k_0 and magnetic length L_0 are given by [1],

$$k_0 = \frac{(G)_{\rm int}}{B\rho}, \qquad L_0 = \frac{(Gl)_{\rm int}}{(G)_{\rm int}}, \tag{2.1}$$

where *int* signifies *interpolated*. Our goal is to find the *effective* strength k, or equivalently, *coef* defined by

$$coef \equiv \frac{k}{k_0} = k \cdot \frac{B\rho}{(G)_{\rm int}} = kL_0 \cdot \frac{B\rho}{(Gl)_{\rm int}}. \tag{2.2}$$

One could as well define the magnetic length L to experience the same degree of change as the field:

$$\sqrt{coef} \equiv \frac{k}{k_0} = \frac{L}{L_0} = L \cdot \frac{(G)_{\rm int}}{(Gl)_{\rm int}}. \tag{2.3}$$

♦ After this work had been completed, we came across a work made in a similar spirit [2].

The modified definition of *coef* allows the effective integrated strength to have the same expression in the two models. As different ways of distributing the integrated strength into k and L may influence the degree of subsequent optics fitting, some comparisons are made.

3 RESPONSE MATRIX FITTING

3.1 Least Square Fit

What is performed is simply the least square fit of

$$F \equiv \sum_{u=h,\,v} \sum_{i=1}^{M} \sum_{j=1}^{N_u} [A_{ij}^{(u)} - R_{ij}^{(u)}]^2 \tag{3.1}$$

where

M: Number of BPMs, N_u: Number of steerers in plane u,
$R_{ij}^{(u)}$, $A_{ij}^{(u)}$: Measured and fitted response matrices,

by varying the strength of all existing quadrupole families. In the ESRF machine, there are eight quadrupole families, $M = 224$ and $N_h = N_v = 96$. There are as many as $224 \times 96 = 21{,}504$ matrix elements. However, since our goal is to calibrate the quadrupoles by families, and since the designed optics in all cases possess a 16 fold symmetry, we shall average over every 16 elements of $R_{ij}^{(u)}$ that are ideally identical.

3.2 Steerer Calibration

The measured response matrices $R_{ij}^{(u)}$'s actually depend on the steerer calibration since,

$$R_{ij}^{(u)} = dUij \,/\, d\theta_j = dU_{ij}^{(u)} \,/\, (c_j \cdot dI_j)\,, \tag{3.2}$$

where

dU_{ij} : Displacement measured at i-th BPM,
$d\theta_j$: Kick angle given to j-th steerer,
dI_j : Current increment,
c_j : Steerer calibration coefficient.

Even though dU_{ij} could be measured with high accuracy, the same degree of accuracy may not be kept for $R_{ij}^{(u)}$ unless there is a good knowledge of c_j. We shall therefore work with the original displacement dU_{ij} and attempt to determine the plausible c_j by including into the fitting algorithm, a step to minimise the function

$$F_j^{(u)} \equiv \sum_{i=1}^{M} (A_{ij}^{(u)} \cdot c_j dI_j - dU_{ij})^2 \qquad (3.3)$$

whenever a new set of $A_{ij}^{(u)}$, namely, a new optics is computed. The above condition leads to

$$c_j = \frac{\sum_{i=1}^{M} A_{ij}^{(u)} \cdot dU_{ij}}{dI_j \cdot \sum_{i=1}^{M} A_{ij}^{(u)2}}. \qquad (3.4)$$

3.3 Path Length Effect

A kick $d\theta_j$ of j-th steerer at which there is a non-zero horizontal dispersion D_j creates a path length deformation of the closed orbit by $dL_C = -D_j \cdot d\theta_j$, shifting therefore the energy by

$$\frac{dp}{p} = -\frac{D_j \cdot d\theta_j}{\alpha L_C} \qquad (3.5)$$

Here, L_C is the machine circumference and α denotes the momentum compaction. This means that dU_{ij} contains an extra dispersive term besides the part we are interested in,

$$dU_{ij} = R_{ij}^{(u)} \cdot d\theta_j - \frac{D_i D_j}{\alpha L_C} \cdot d\theta_j. \qquad (3.6)$$

We shall utilise the measured dispersion $(D_i)_{meas}$ to subtract off the unwanted term. A precise α value is not needed here since one may fit and extract the dispersive part by minimising

$$F_j^{(u)} \equiv \sum_{i=1}^{M} [dU_{ij} - (D_i)_{meas} \cdot \delta_j]^2, \qquad (3.7)$$

which defines the corresponding energy deviation parameter

$$\delta_j = \frac{\sum_{i=1}^{M} dU_{ij} \cdot (D_i)_{meas}}{\sum_{i=1}^{M} (D_i)_{meas}{}^2}. \qquad (3.8)$$

3.3 Simulation

What was described above has been implemented into the computer code CATS [4]. An example of simulation is shown that confirms the effectiveness (Fig. 1). In the example, the quadrupoles are randomly varied to less than 1%, the steerer coefficients by ±20%, with which a response matrix is generated. The fitting is then performed on this matrix starting from a certain point. One sees that all are converging in the right directions. As one may expect, the response matrix fitting is found to be particularly sensitive to the tunes of the optics.

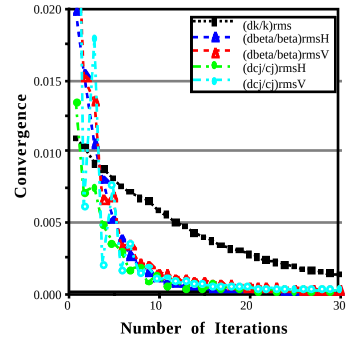

Fig. 1: Simulation of response matrix fitting.

4 ACTUAL RESULTS

4.1 Choice of the Optics and the Response Matrix Measurements.

To cover a wide range in the quadrupole settings, the standard optics at a different energy as well as an exotic low α optics were included. Orbit displacements were chosen to stay within the *linear* range: *rmsH* ~ 0.3 mm and *rmsV* ~ 0.2 mm.

4.2 Characteristics of the Performed Response Matrix Fittings.

The converged solutions mostly reproduced the measured matrices down to a few percent, always resulting in better fits vertically. In terms of displacement, typically ~7 µm horizontally and ~2 µm vertically. With the low α optics that has a large horizontal dispersion, the path length correction turned out to be significant to get a good convergence (Fig. 2). There was an improvement of a factor of five in the fit after removal of the dispersive contribution. The effect was however negligible in the remaining cases.

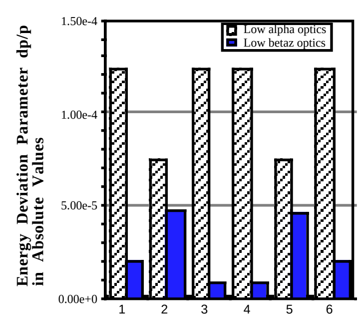

Fig. 2: The obtained energy deviation parameters.

4.3 Steerer Calibration.

The response matrix fit quasi uniquely determined the calibration coefficients c_j's. The results reveal a saturation like effect of the steerer field, giving variations up to

~15% (horizontal) and ~5% (vertical), as a function of the sextupole current in the combined function magnets (Fig. 3).

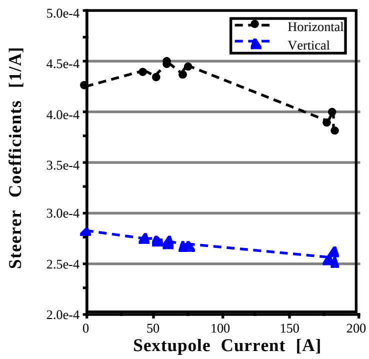

Fig. 3: Steerer calibration versus sextupole current.

4.4 *Obtained Quadrupole Coefficients.*

Thanks to the corrections of steerer calibrations and the path length that contributed non-negligibly, the necessary accuracy of $<10^{-3}$ level was reached in identifying the quadrupole coefficients. Plotting the best fit coefficients versus current, one notices a global trend that effective quadrupole strength weakens with the increasing current, which may arise from proximity effects in the magnet assembly (Fig. 4).

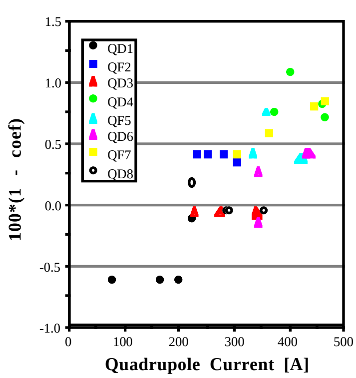

Fig. 4: Quadrupole coefficients versus current.

4.5 *Comparison of the Two Different Calibration Models.*

The results of the fits in the modified calibration model turned out to be nearly identical to the original model (Fig. 5). The fact signifies that what counts in the fit is the effective integrated strength. The independent fitting made in parallel also assures the correctness of the obtained results.

5 CONCLUSION

To obtain an accurate calibration of the storage ring quadrupoles, the fitting of the orbit response matrix was attempted by varying the strength of the eight quadrupole families. As any existing asymmetry is out of the present

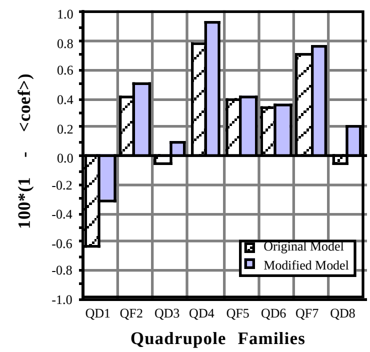

Fig. 5: Quadrupole coefficients for the two models.

concern, averaging is made over the matrix elements prior to the fit to remove the imperfections. The steerer calibration turned out to be quasi uniquely determined in course of the response matrix fit. The steerers being combined with sextupoles in the ESRF machine, the results revealed a certain dependence of the dipole field on the sextupole current. The path length effect was found to be significant with optics that have large horizontal dispersions.

Thanks to the two significant corrections above, the response matrix fitting provided precision on the quadrupole calibration down to the necessary 10^{-4} level. The coefficients averaged over the best fits predict the phase advance per unit cell in almost any optics loaded on the machine up nearly to one degree accuracy even in the horizontal plane.

ACKNOWLEDGEMENT

The authors especially thank A. Ropert for helpful discussions and active participation in the measurement. They also thank J.M. Filhol and the members of the theory group for discussions. Thanks are also for those of the operation group, who provided the basic calibration data under the new cycling.

REFERENCES

[1] A. Ropert with the participation of the Operation Group (G. Chazot, J.M. Filhol, M. Paulin, L. Petit) and L. Farvacque, “Calibration of Quadrupoles and Modelling of the Storage Ring Lattice”, ESRF Machine Technical Note, 8-95/Theory.

[2] J. Safranek, “Experimental Determination of Storage Ring Optics Using Orbit Response Measurements”, Nucl. Instr and Meth. **A388** (1997) 27.

[3] R. Nagaoka and L. Farvacque, “Calibration of Quadrupoles and Modelling of the Storage Ring Lattice Under the Upgraded Cycling”, ESRF Machine Technical Note, 6-97/Theory.

[4] R. Nagaoka, “CATS: A Computer Code for the Optimisation of Sextupoles and Linear Optics in Circular Accelerators”, Sincrotrone Trieste ST/M-91/3.