SPIN DEPOLARIZATION BY THE BEAM-BEAM EFFECT

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Abstract

Particle colliders with polarized beams require careful control of spin depolarization. During acceleration spin is subjected to intrinsic and imperfection resonances resulting in depolarization. Extra source of depolarization is beam-beam collisions. Due to beam-beam interaction, particle motion become essentially nonlinear and under some circumstances unstable. In present paper effect of beam-beam collision on spin depolarization in a proton- proton collider is studied. Performed study indicates, that spin depolarization due to beam-beam collisions is suppressed if beam-beam interaction is stable and if operation point is far enough from spin resonances. Meanwhile, under beam-beam instability, spin is a subject of strong depolarization. Analytical estimations are confirmed by results of computer simulations.

1 PARTICLE BETATRON MOTION

Let us consider a collider ring with two installed Siberian Snakes. We use a two-dimensional particle model in phase space (x, p_x), (y, p_y), where x and y are particle positions, $p_x = \beta_x^*(dx/dz)$ and $p_y = \beta_y^*(dy/dz)$) are particle momentum, $\beta_x^* = R/Q_x$ and $\beta_y^* = R/Q_y$ are average values of beta-functions of the ring, R is a ring radius, Q_x and Q_y are betatron tunes. Particle motion between subsequent collisions combines linear matrix with betatron angles $\bar{\theta}_x = 2\pi Q_x$, $\bar{\theta}_y = 2\pi Q_y$, perturbed by beam-beam interaction:

$$\begin{pmatrix} x_{n+1} \\ p_x^{n+1} \\ y_{n+1} \\ p_y^{n+1} \end{pmatrix} = \begin{pmatrix} \cos\bar{\theta}_x & \sin\bar{\theta}_x & 0 & 0 \\ -\sin\bar{\theta}_x & \cos\bar{\theta}_x & 0 & 0 \\ 0 & 0 & \cos\bar{\theta}_y & \sin\bar{\theta}_y \\ 0 & 0 & -\sin\bar{\theta}_y & \cos\bar{\theta}_y \end{pmatrix} \begin{pmatrix} x_n \\ p_x^n + \Delta p_x^n \\ y_n \\ p_y^n + \Delta p_y^n \end{pmatrix} . \quad (1)$$

Beam-beam kicks Δp_x^n, Δp_y^n are expressed as a result of interaction of particles with opposite beam with Gaussian distribution function

$$\Delta p_x^n = 4\pi\xi x_n \frac{1 - \exp[-r_n^2/(2\sigma_n^2)]}{[r_n^2/(2\sigma_n^2)]} , \quad (2)$$

and similar for Δp_y^n. Parameter ξ is a beam-beam parameter, which characterizes the strength of interaction:

$$\xi = \frac{N r_o \beta^*}{4\pi\sigma^2\gamma} , \quad (3)$$

where N is a number of particles per bunch, $r_o = q^2/(4\pi\varepsilon_o mc^2)$ is a classical particle radius, σ is a transverse standard deviation of the opposite beam size and γ is a particle energy.

2 SPIN MATRIX

Rotation of spin $\vec{S} = (S_x, S_y, S_z)$ in a ring is calculated as a product of spin matrixes in a lattice arc, in Siberian Snakes and in the interaction region. Matrix of spin advance in an lattice arc is described as a matrix of spin rotation in dipole magnet with bending angle υ:

$$D_\upsilon = \begin{vmatrix} \cos(\omega\delta z) & 0 & -\sin(\omega\delta z) \\ 0 & 1 & 0 \\ \sin(\omega\delta z) & 0 & \cos(\omega\delta z) \end{vmatrix} , \quad (4)$$

where $\omega\delta z = (1+G\gamma)\upsilon$ and G = 1.79285 is the anomalous magnetic moment of the proton. Matrixes of Siberian Snakes are given by

$$S_1 = \begin{vmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}, \quad S_2 = \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} . \quad (5)$$

Spin advance after crossing the interaction point is described as follow [1] :

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} 1-a(B^2+C^2) & ABa + Cb & ACa - Bb \\ ABa - Cb & 1-a(A^2+C^2) & BCa + Ab \\ ACa + Bb & BCa - Ab & 1-a(A^2+B^2) \end{vmatrix} \begin{vmatrix} S_{x,o} \\ S_{y,o} \\ S_{z,o} \end{vmatrix} , \quad (6)$$

$$A = \frac{P_x}{P_o}, \quad B = \frac{P_y}{P_o}, \quad C = \frac{P_z}{P_o}, \quad P_o = \sqrt{P_x^2 + P_y^2 + P_z^2} , \quad (7)$$

$$a = 1 - \cos(P_o\,\delta z) , \quad b = \sin(P_o\,\delta z) , \quad (8)$$

$$P_x = 16\pi G \gamma\xi \frac{\sigma^2}{\beta^* l} \frac{y}{r^2} [1 - \exp(-\frac{r^2}{2\sigma^2})] , \quad (9)$$

$$P_y = -16\pi G \gamma\xi \frac{\sigma^2}{\beta^* l} \frac{x}{r^2} [1 - \exp(-\frac{r^2}{2\sigma^2})] , \quad P_z = 0, \quad (10)$$

where $\delta z = l/2$ is an interaction distance and l is a bunch length.

3 ANALYTICAL TREATMENT OF SPIN DEPOLARIZATION

Analytical treatment of spin depolarization is possible with additional approximations. Suppose, the betatron tunes in x and y directions are equal each other $\bar{\theta}_x = \bar{\theta}_y = \bar{\theta}$. Consider particle motion far enough from low order resonances, therefore, particle trajectory can be expressed as a linear oscillator,. $x = r\cos(n\theta + \Psi)$, $y = r\sin(n\theta + \Psi)$, with perturbed betatron tune $\theta = \bar{\theta} + \Delta\theta$, where Ψ is an initial phase of betatron particle oscillations and $\Delta\theta$ is a tune perturbation due to beam-beam collisions. Under that assumptions, the following matrix of spin advance has been attained [1]:

$$\begin{vmatrix} S_x \\ S_y \\ S_z \end{vmatrix} = \begin{vmatrix} (-1)^{n+1}\{1 - \frac{\varphi^2}{2}[\sum_{i=0}^{n}\cos(i\theta+\Psi)]^2\} & \frac{\varphi^2}{4}[\sum_{i=0}^{n}(-1)^{i+n}\sin 2(i\theta+\Psi) + ...] & (-1)^{n+1}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) \\ \frac{\varphi^2}{4}[\sum_{i=0}^{n}(-1)^{i+1}\sin 2(i\theta+\Psi) + ...] & 1 - \frac{\varphi^2}{2}\{\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi)\}^2 & \varphi\sum_{i=0}^{n}(-1)^{i}\sin(i\theta+\Psi) \\ (-1)^{n}\varphi\sum_{i=0}^{n}\cos(i\theta+\Psi) & \varphi\sum_{i=0}^{n}(-1)^{i+n}\sin(i\theta+\Psi) & (-1)^{n+1}[1 - \frac{n+1}{2}\varphi^2 + ...] \end{vmatrix} \begin{vmatrix} S_{x,o} \\ S_{y,o} \\ S_{z,o} \end{vmatrix}, \quad (11)$$

where n is a turn number and φ is a small parameter:

$$\varphi = 4\pi G \gamma \xi \frac{r}{\beta^*} << 2\pi \ . \quad (12)$$

From attained matrix, it follows that spin depolarization is not taking place or suppressed if betatron tune values are chosen far away from low-order resonances, and that beam-beam collisions are stable. Actually, suppose that the initial spin has only one transverse component, i.e., $S_x = 0$, $S_y = 1$ and $S_z = 0$. After n turns, the average and rms values of spin components are as follows [1]:

$$\bar{S}_x = 0, \quad \bar{S}_z = 0, \quad \bar{S}_y = 1 - \frac{1}{8}\left(\frac{\tilde{\varphi}}{\cos(\tilde{\theta}/2)}\right)^2, \quad (13)$$

$$\overline{<S_x^2>} = \left[\frac{\tilde{\varphi}^2}{8(\cos\tilde{\theta})}\right]^2, \quad \overline{<S_y^2>} = \frac{3}{256}\left[\frac{\tilde{\varphi}}{\cos(\tilde{\theta}/2)}\right]^4,$$

$$\overline{<S_z^2>} = \left[\frac{\tilde{\varphi}}{2\cos(\tilde{\theta}/2)}\right]^2, \quad (14)$$

where $\tilde{\varphi}$ and $\tilde{\theta}$ are the average values of parameters φ, θ among all particles. In Fig. 1 results of numerical study of particle motion and suppressed spin depolarization in the presence of stable beam-beam interaction are presented for the values of betatron tunes $Q_x = Q_y = 14.43$.

4 SPIN DEPOLARIZATION AT BEAM-BEAM INSTABILITY

There are several mechanisms which lead to particle instability under beam-beam collisions. Excitation of nonlinear resonances and unstable stochastic particle motion due to overlapping of resonance islands are the fundamental phenomena in beam-beam interaction [2]. Another mechanism of unstable particle motion is a diffusion created by random fluctuations in distribution of the opposite beam. In Ref. [3] the noise beam-beam instability was studied for the case of random fluctuations in the opposite beam size

$$\sigma_n = \sigma_o\left(1 \pm \frac{u \cdot u_n}{2}\right), \quad (15)$$

where u is a noise amplitude and u_n is an uniform random function with unit amplitude. It was shown that in the presence of noise, beam emittance increases with the turn number n as

$$\frac{\varepsilon_n}{\varepsilon_o} = \sqrt{1 + D\,n} \ . \quad (16)$$

where D is a diffusion coefficient.

In Fig. 2 the results of beam dynamics and spin depolarization in the presence of noisy beam-beam instability are presented. Parameters of the process were chosen the same as for the stable beam-beam interaction without noise as presented in Fig. 1. The value of noise amplitude $u = 0.025$ was chosen arbitrarily, to demonstrate the main features of diffusion beam-beam instability. As is seen, increase of beam sizes due to beam-beam collisions results in spin depolarization. It is also expected from analytical formulas (13) - (14), where the average and rms beam parameters are proportional to the powers of the small parameter φ, which, is proportional to beam size according to Eq. (12). Therefore, if beam is subjected to beam-beam instability, it causes spin depolarization.

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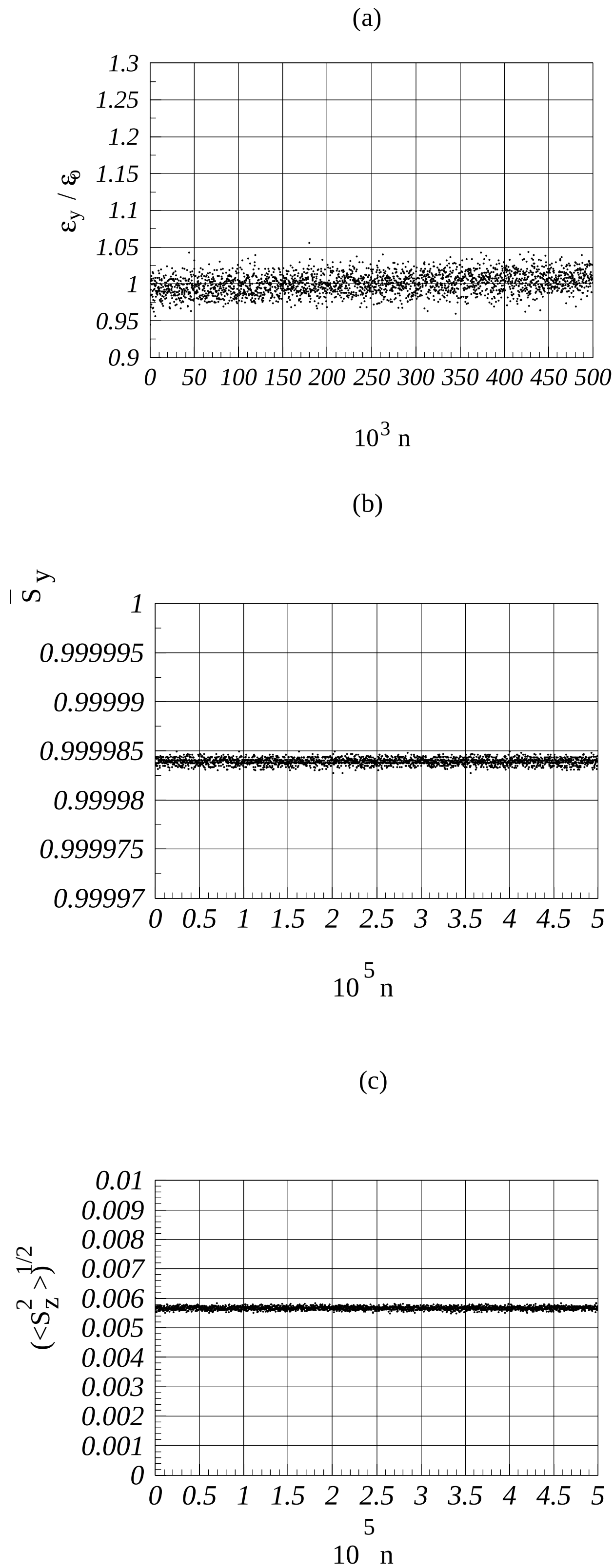

Fig. 1. (a) Beam emittance, (b) average value of $\overline{S}_y$, (c) rms value of $(\overline{<S_z^2>})^{1/2}$ as a function of the turn number for stable beam-beam interaction.

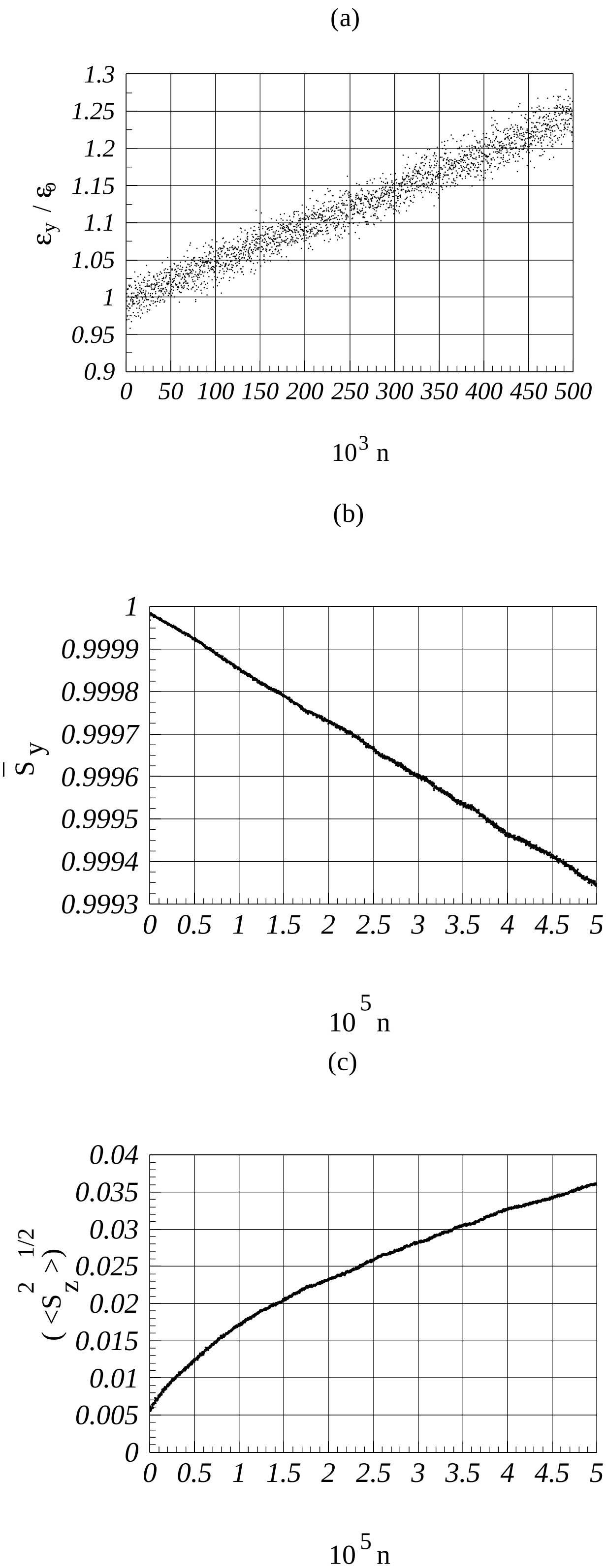

Fig. 2. (a) Beam emittance, (b) average value of $\overline{S}_y$, (c) rms value of $(\overline{<S_z^2>})^{1/2}$ as a function of the turn number for unstable beam-beam interaction.