SOME PROBLEMS OF OPTIMIZATION PROCEDURE FOR BEAM LINES

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Abstract

This paper presents some approaches to optimization problems for beam line design. The matrix formalism for Lie algebraic tools is put in to these approaches. This allows to use all advantages of symbolic presentation of desired knowledge and create effective and flexible optimization procedures.

1 THE PROPAGATOR FOR BEAM DYNAMICS

1.1 Introduction

Beam dynamics in beam line systems is usually described by nonlinear motion equations. The corresponding maps (the so called time displacement operator or propagator), generated by these motion equations can be calculated with the help of the so called Lie algebraic tools (for Hamiltonian systems see [1]). In previous papers [2, 3] the main concepts of the matrix formalism for Lie algebraic tools are presented. Using this formalism one can build desired beam line map in a matrix form regardless of beam state descriptions.

1.2 The Basic Concepts of Matrix Formalism

Further we shall consider differential equations of motion in the general form

$$\frac{dX}{dt} = F(X, U; t), \tag{1}$$

where $X(t) \in R^n$ is a phase vector and $U(t) \in R^r$ – a control vector, t is an independent variable (for example, the length along the referenced trajectory) and the function $F(X, U, t)$ can be represented as a Taylor expansion

$$F(X, U; t) = \sum_{k=0}^{\infty} \mathbf{P}^{1k}(U; t) X^{[k]}, \tag{2}$$

Where $X^{[k]}$ is the so called Kronecker power of k-order for a phase vector X, $\mathbf{P}^{1k}(U; t)$ are matrices containing Taylor expansion coefficients. According to the matrix formalism [4] solutions of Eqs.(1)–(2) are sought in the form

$$X(t) = X(X_0, U; t|t_0) = \sum_{k=0}^{\infty} \mathbf{M}^{1k}(U; t|t_0) X_0^{[k]}, \tag{3}$$

where $X_0 = X(t_0)$ is an initial phase vector and $\mathbf{M}^{1k}(U; t|t_0)$ are solution matrices. In the previous papers ([2, 3, 4, 5, 6]) basic features of this approach were demonstrated and various examples of applications were described. Calculation of the matrices $\mathbf{M}^{1k}$ are made using the Lie algebraic methods and the matrix algebra tools expanded by the Kronecker operators. According to the Lie algebraic tools we can write solution of the Eqs.(1)–(2) in the form ($t \in [t_0, t_*]$):

$$X(U; t) = \mathcal{M}(U; t|t_0) \circ X_0 = \mathrm{T}\exp\left\{ \int_{t_0}^{t} \mathcal{L}_F \, d\tau \right\}, \tag{4}$$

where $\mathcal{M}$ is the propagator (Lie transformation), generated by the vector field (Lie operator) $\mathcal{L}_F$, associated with the function $F(U; t)$ ($\mathcal{L}_F = F^*(X, t)\partial/\partial X$). $\mathrm{T}\exp$ is the so called T-exponential operator (time ordered exponential operator). For non–autonomous cases this operator can be rewritten with the help of the Magnus's representation [7] as the routine exponential operator

$$\mathrm{T}\exp\left\{ \int_{t_0}^{t} \mathcal{L}_F \, d\tau \right\} = \exp\left\{ \hat{\mathcal{L}}(F; t|t_0) \right\}. \tag{5}$$

The new operator $\hat{\mathcal{L}}$ is associated with a new function $\hat{F}(F; t|t_0)$ which can be calculated using the continuous analogue of the CBH formula. The expansion (2) generates Taylor expansions of Lie operators $\mathcal{L}_F = \sum_{k \geq 0} \mathcal{L}_{F_k}$ and $\hat{\mathcal{L}}_F = \sum_{k \geq 0} \mathcal{L}_{\hat{F}_k}$. Here F_k and $\hat{F}_k$ are homogeneous vector polynomial functions, foe example $F_k = \mathbf{P}^{1k} X^{[k]}$. The sequences of $\{F_k\}$ and $\left\{\hat{F}_k\right\}$ are determined by the following matrices $\mathbf{P}^{1k}(t)$ and $\hat{\mathbf{P}}^{1k}(t|t_0)$. Rewrite the Eqs.(4)–(5) using the analogue of the Dragt–Finn factorization in the form

$$\mathcal{M}(U; t|t_0) = \ldots \circ \exp\left\{\tilde{\mathcal{L}}_2\right\} \circ \exp\left\{\tilde{\mathcal{L}}_1\right\}, \tag{6}$$

where $\tilde{\mathcal{L}}_k = \left(\mathbf{G}^{1k}(U; t) X^{[k]}\right)^* \partial/\partial X$. For new matrices $\mathbf{G}^{1k}(F; t|t_0)$ we can obtain the rather simple formulae using the Kronecker product and sum operations. Step by step applying of the factorized maps (5) on X_0 we evaluate the solution (3). The matrices $\mathbf{M}^{1k}$ can be written in closed forms as functions of $\mathbf{G}^{1k}$ [3].

1.3 *The Additional Features of the Matrix Formalism*

It is known that the above mentioned approach is one of the perturbation theory methods. The desired solution is created in the form of power series (see Eq.(3)). It is clear that this way can be realized only with truncated procedures for some chosen order of expansions. In the referred works the corresponding matrices $\mathbf{P}^{1k}$, $\hat{\mathbf{P}}^{1k}$, $\mathbf{G}^{1k}$ and $\mathbf{M}^{1k}$ are calculated up to seventh order in symbolic forms using the computer algebra codes ($REDUCE$, $MAPLE$). But we have to note that for this approach there appear two problems: *1) the support of the accuracy of truncated expansions, 2) the support of intrinsic properties (for example, symplecticity for Hamiltonian systems).* The second problem is solved with the help of the correction procedure [3] for the matrices $\mathbf{M}^{1k}$. For this correction one have to solve a chain of the linear algebraic equations and redefined some of the elements of $\mathbf{M}^{1k}$. These calculations one can make in symbolic forms too and they make matrix elements calculation more accurately and quickly, because such calculations are made only once and the results are used as required.

2 THE OPTIMIZATION PROBLEMS

2.1 *Formalization Procedures*

There are two approaches to the formalization procedure for optimization problems: the first is based on beam optics properties, such as the focusing distance, the magnification (demagnification), linear and nonlinear aberrations (geometrical, chromatic) and so on; the second is based on the description of the beam evolution as the evolution of the phase manifold occupied by some set of particles $\mathfrak{M}(t|t_0) = \mathcal{M}(U;t|t_0)\circ\mathfrak{M}_0$, where $\mathfrak{M}_0$ is a starting phase manifold.

The first approach allows to create mathematical optimal criteria regardless of beam state knowledge. This leads to nonlinear equations and inequalities which describe the corresponding properties of the system under study. To solve these equations and inequalities we use nonlinear programming methods. The second approach is based on optimal criteria, which are written in the following generalized form:

$$\mathrm{J}[U] = \int\limits_{t_0}^{t_*} \int\limits_{\mathfrak{M}(\tau|t_0)} \mathrm{g}_1(X,U;\tau)dXd\tau + \\ + \int\limits_{\mathfrak{M}(t_*)|t_0)} \mathrm{g}_2(X,U)dX.$$

In this presentation of the optimization procedure one can take into account more subtle effects of beam dynamics. The usual methods for solution of the corresponding problems are methods of the optimal control theory. In this work we use parametric descriptions of the control functions $U(t) = U(A,t) \in R^r$, $A \in R^m$ for almost all $t \in [t_0, t_*]$, where A is a parameters vector representing a class of the used control function $U(t)$. Some of these parameters have geometric nature and some of them describe focusing and deflecting forces in the beam line. Usually the computer experiments dictate to select a discrete subset for most geometric parameters in some technological desired set of its variations. The optimization procedure on this discrete subset is reduced to a tabulation procedure using an appropriate lattice. This tabulation procedure can be realized either using a regular multivariate lattice or using a random lattice. The first variant is suitable for some defined lattice of control parameters. This approach allows to create databases of valid systems. The second variant is most often used in the case when there is not any information about the starting point in the parameters space. The both variants are convenient to realize with the appropriate visualization procedure, which helps to detect those or another singularity. Similar approach was used for high solid angle mass–spectrometer modeling [8]. As to the control functions concerning to focusing and deflecting forces the control theory methods are more preferable. As we mention here above there are two approaches. In the first one the problem is formulated in the terms of mappings generated by the system and in the second approach the object function $\mathrm{J}[U]$ and constraint conditions are written using phase portrait of the beam. The first approach is often used in part for the linear approximation model. But using the computer algebra methods and codes we can compute the necessary conditions in symbolic form for nonlinear approximation models too. For this purpose it is more convenient to use the matrix formalism. Indeed in this case one can include nonlinear effects in the symbolic form, which allows to find a desired solution more effectively. For example, for the problem of nonlinear aberrations correction the optimal control problem is reduced to solving procedure for the system of linear algebraic equations [9].

2.2 *Methods of Solution*

Certainly the efficiency of any optimization procedure depends for the most part on the used methods of optimization problem solution. In this report we suggest the approach which is based on the representation of the control function $U(t)$ using some set of model control functions $U(A,t)$, where A is a parameters vector which describe the selected model function. As an example let us consider the gradient of a quadrupole lens $g(t)$, where t is measured along a referenced trajectory. Along the system this gradient function is a continuous function and nonvanishing values correspond to existing of physical quadrupole (or other multipole) lenses. But for each specific lens we can approximate this function using some model function $U(A,t)$. It is obviously that the selection of $U(A,t)$ depends on the experimental data. We can prepare some set of such kind of model functions in advance (in the form of a database of approximation functions for the fringe fields) and use them in optimization procedure. In this case the m–dimensional vector of parameters A is some part of variable parameters vector for the corresponding nonlinear programming problem. To solve this problem we use two direct methods: the modified method of sliding tolerances [10] and the stochastic method, the so called LP_τ–method [11]. The appropriate combination of these methods allows to solve enough complicated problems (see, for example, [12]).

3 ORGANIZATION OF CALCULATIONS

The usage of the matrix formalism for Lie algebraic tools allows us sufficiently to simplify the most problems of finding optimal structure of the beam lines. Let us enumerate the advantages of this approach:

⋆ *the possibility to write the object function and corresponding condition functions using symbolic representation for nonlinear problems too;*

⋆ *the effectiveness of all calculations is increased as one can use symbolic prepared in advance formulae, which are withdraw from corresponding databases if necessary;*

⋆ *the cross from one model of the beam line to another is realized in corresponding to the dynamic modeling approach* [13];

⋆ *the possibility to form enough simple symbolic formulae for estimation of efficiency of desired beam line.*

These possibilities permit to formulate in the most cases a nonlinear programming problem (instead of the optimal control problem) upon a class of control parameters which can include both some geometrical parameters (describing the starting manifold or starting distribution function, if it is necessary) and components of the vector A (see above).

Certainly that the symbolic presentation for the object function and the functions generated various bounded conditions (both on phase variables and the control parameters) permit to simplify the optimization procedure. This allows in many cases to use personal computers instead of workstations. The usage of the data–base management system and the dynamic modeling paradigm [13] gives a capability to manipulate by more large structure elements in comparison with the control parameters.

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