MODELLING OF A LINEARLY COUPLED MACHINE USING THE COUPLED-RESPONSE MATRIX

R. Nagaoka, ESRF, Grenoble, France

Abstract

The paper describes an attempt to model a linearly coupled machine on the basis of the measured coupled orbit response. The main emphasis is on understanding the achievable resolution on the error skew field for the specific ESRF machine, by testing different skew configurations in the simulations. It is found that the locally integrated skew field is sufficiently accurate to describe the details of the linear coupling in all simulated cases. The developed scheme is applied to the real machine, examining in detail the obtained error skew distribution as well as reproducibility of the measured coupling characteristics. Correction of coupling is also attempted by utilising the obtained model.

1 INTRODUCTION

Coupling correction is of great importance for non diffraction limited light sources such as the ESRF. At the ESRF, the coupling is successfully corrected down below 1% with skew quadrupole correctors for the daily user operation. To obtain a full and consistent understanding of the coupling, as well as to pursue the limit of coupling correction, an attempt is made to utilise the coupled response matrix to model a linearly coupled machine [1]. The approach may be regarded as a direct extension of the linear optics modelling with the response matrix [2], applied to the linear coupling[♦].

$$\begin{pmatrix} \Delta u_H \\ \Delta u_V \end{pmatrix} = \begin{pmatrix} R_{HH} & R_{HV} \\ R_{VH} & R_{VV} \end{pmatrix} \bullet \begin{pmatrix} \Delta\theta_H \\ \Delta\theta_V \end{pmatrix} \tag{1.1}$$

where

Δu_U : Difference orbit measured at BPMs ($U = H, V$),
$\Delta\theta_U$: Kick angle given to steerers,
R_{HH} and R_{VV} : Diagonal response matrices,
R_{HV} and R_{VH} : Coupled response matrices.

The large amount of data contained in the coupled-response matrix that come along with the diagonal part in the usual response matrix measurement, are expected to depend on the details of the coupling of the machine. Since our goal is to construct a good model of the linearly coupled machine, we shall, instead of attempting to find the skew errors of the existing magnets, introduce certain number of *skew quadrupole flags* into the machine. Their strengths are then solved to best reproduce the measured coupled response. Prior to application to the real machine, different skew flag configurations and their validity shall be studied in the simulations.

♦After this work had been completed, we came across a work made in a similar spirit [3].

2 DEVELOPED SCHEME

2.1 Basic Relations

A shift Δy_{CO} of the vertical closed orbit due to an increment of the horizontal steerer strength satisfies

$$\begin{aligned} \Delta y_{co}'' - (b_1 + b_2 x_{co})\Delta y_{co} &= b_2 y_{co}\,\Delta x_{co} \\ &- a_1 \Delta x_{co} + b_2 \Delta x_{co} \Delta y_{co} + \ldots, \end{aligned} \tag{2.1}$$

where

κ : Normal dipole, b_1 : Normal quadrupole,
b_2 : Normal sextupole, a_1 : Skew quadrupole.

It follows that $(\Delta y_{co})_i$, the vertical displacement at i-th BPM due to j-th horizontal steerer may be expressed as,

$$\begin{aligned} (\Delta y_{co})_i &= \sum_s (b_2 l y_{co})_s \cdot R_{is}^{(V)} \cdot R_{sj}^{(H)} \cdot \Delta\theta_{Hj} \\ &- \sum_k (a_1 l)_k \cdot R_{ik}^{(V)} \cdot R_{kj}^{(H)} \cdot \Delta\theta_{Hj} \end{aligned} \tag{2.2}$$

where $R_{\alpha\beta}^{(U)}$'s ($U = H, V$) are matrices defined as

$R_{sj}^{(U)}$: From j-th steerer to s-th sextupole,
$R_{ik}^{(U)}$: From k-th skew quadrupole to i-th BPM,
$R_{is}^{(U)}$: From s-th sextupole to i-th BPM,
$R_{kj}^{(U)}$: From j-th steerer to k-th skew quadrupole.

A similar expression can be found for $(\Delta x_{co})_i$. The error skew field $\mathbf{z} \equiv [(a_1 \cdot l)_k, (b_2 \cdot l y_{CO})_s]$ can therefore be solved via the standard matrix inversion (e.g. SVD method).

2.2 Simulations

To study the effectiveness of the scheme, the machine simulator (RACETRACK) was utilised in two ways: 1) To represent the machine with unknown skew errors, from which the coupled-response matrix is *measured*. 2) To *reconstruct* the former machine with the skew distribution as obtained from the coupled-response matrix fitting. To study the reproducibility of the coupling characteristics, the normal mode decomposition and computation of equilibrium emittances were implemented, following respectively the works in Refs. 3 and 4. A least square fit to minimise any coupling related function with skew quadrupole correctors was also added to enable a coupling correction.

With the ESRF machine that consists of 32 Chasman-Green cells, having 7 BPMs per cell and 3 steerers per cell in each plane, it is found that the skew error of a single magnet cannot be identified even with a perfect fit of the coupled matrix. However, a locally integrated skew field so obtained is sufficiently accurate to describe even the position dependence of the linear coupling in all treated cases (Fig. 1). With the schemes of 4 to 10 skew flags per cell, coupling correction could also be efficiently made by minimising the coupling of the *reconstructed* machine and applying the resultant corrector strengths to the *original* machine.

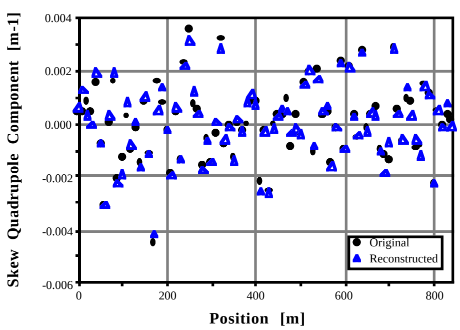


Fig. 1. Locally integrated skew strengths (every 10 meters) between the original and reconstructed machines.

3 APPLICATION TO THE REAL MACHINE

3.1 Obtained Skew Distribution

The analysis was made for two particular cases, the best corrected and uncorrected coupling. The response matrix fit was made according to the following steps: 1) Use of the optics and steerer calibration determined from the diagonal part of the response matrix fit [2]. 2) Use of 10 skew flags per cell. 3) Inversion of matrices each containing R_{VH} of two steerers. 4) Averaging over all computed distributions to minimise the imperfections.

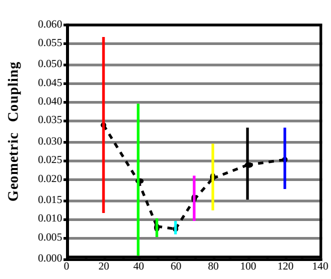


Fig. 2. Geometric coupling of the *reconstructed* machine versus *Neigen*. The optimum is around 60 in the shown case.

Unlike the simulation, the non-trivial dependence of the SVD inversion on the number of eigenvectors *Neigen* was carefully followed to find the optimum point (Fig. 2). The fit in most cases reproduced the measured coupled orbit down to a few microns in both planes.

The distribution of the uncorrected machine shows several localised peaks (Fig. 3). To check the consistency of the results, the difference of the two distributions is compared to the actually applied skew correctors, where a good agreement is obtained (Figs. 4).

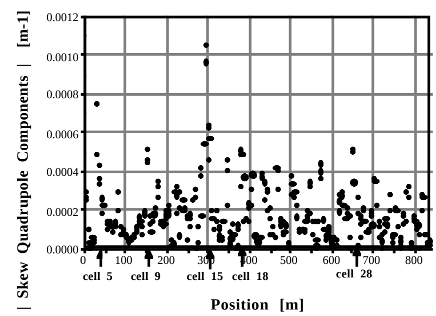
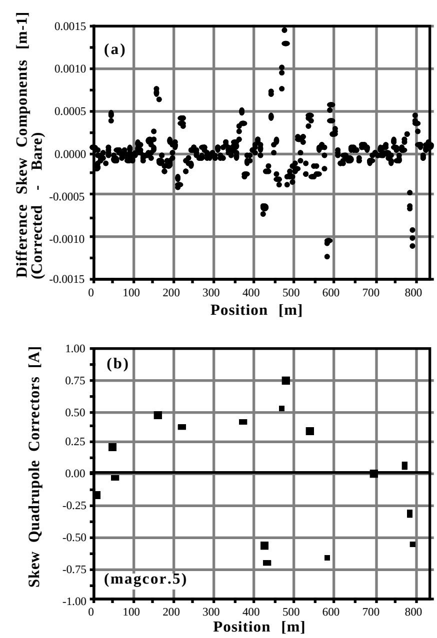


Fig. 3. Skew distribution of the uncorrected machine.



Figs. 4. Comparison of (a) the difference of the two distributions with (b) skew corrector distribution.

3.2 Description of the Coupled Machine

The obtained coupling parameters for the two cases are summarised in Table 1.

	Uncorrected	Corrected
Ratio of normal mode invariants	0.0459	0.0034
Ratio of geometric emittances	0.0754	0.0056
Ratio of H function based emittances	0.0125	0.0011
Betatron coupling coefficient $\lvert\kappa\rvert$	0.0068	0.0043

Table 1. Computed coupling parameters.

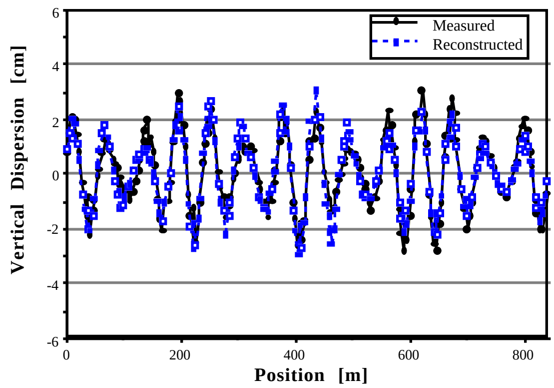
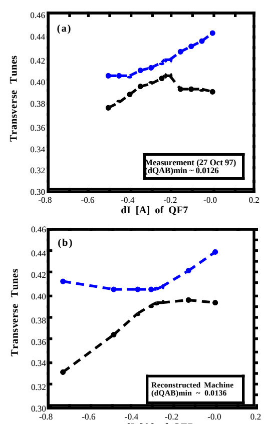


Fig. 5. Comparison of measured vertical dispersion with the calculated, for the uncorrected machine.



Figs. 6. Comparison of (a) measured tune separation with (b) the calculated, for the uncorrected machine.

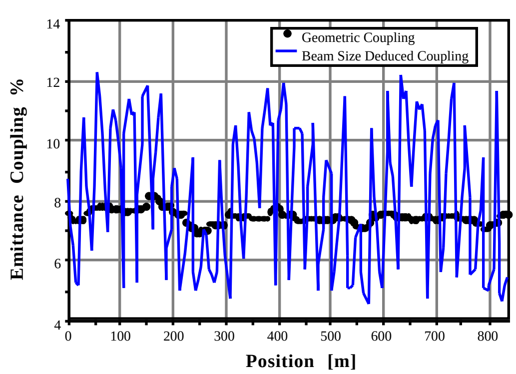


Fig. 7. Position dependence of the computed couplings, for the uncorrected machine.

The results for the corrected case are more or less within what expected from the measurements. For the uncorrected case, the vertical dispersion is particularly well reproduced (Fig. 5), while the tune separation to a slightly lesser extent, however due probably to a measurement error (Fig. 6). The ratio of the squared beam sizes divided by the unperturbed betas, which is expected to represent more closely the measured coupling, shows a larger variation than the geometric coupling (Fig. 7). However, it still underestimates the measured value by more than a factor of two, which is yet to be understood.

3.3 Coupling Correction

As the obtained skew distribution appears consistent, a coupling correction was attempted as in the simulation. Anticipating the need of iterations due to imperfections, both the number of skew flags and steerers for the matrix acquisition were reduced to minimise the required time. Starting from the uncorrected machine, the measured coupling reached ~1% from its initial value of ~30% in just two iterations that were performed.

4 CONCLUSION

With the aim of obtaining a consistent model of the linear coupling of the ESRF machine, by which to pursue the limit of coupling correction, an attempt was made to obtain an effective skew quadrupole distribution of the machine through a fit of the measured coupled-response matrix. It was found that, although the resolution may not be as good as to detect a single magnet skew error, a sufficiently accurate modelling can be made with a reduced number of skew components that represent the locally integrated strength.

The obtained error skew distribution appears satisfactory in all aspects considered: 1) Degree of fit of the measured coupled-response matrix. 2) Consistency between coupling corrected and uncorrected distributions. 3) Description of the measured coupling characteristics. The first test of the coupling correction based on the modelling had successfully brought down the coupling to ~1% level within few iterations.

ACKNOWLEDGEMENT

The author thanks L. Farvacque for having proposed the use of coupled-response matrix for the coupling analysis. He expresses his gratitude to J.L. Laclare for the stimulating discussion on the subject, and to J.M. Filhol for his support of the work. Lastly, he thanks the members of the theory group for useful discussions.

REFERENCES

[1] R. Nagaoka, "Modelling of A Linearly Coupled Machine Using the Coupled Response Matrix – Part 2: Application to the ESRF Machine without Coupling Correction", ESRF Machine Technical Note, 6-98/Theory and references therein.
[2] R. Nagaoka and L. Farvacque, "Calibration of Quadrupole Magnets via Response Matrix Fitting", this conference.
[3] J. Safranek, "Experimental Determination of Storage Ring Optics Using Orbit Response Measurements", Nucl. Instr and Meth. **A388** (1997) 27.
[4] D.A. Edwards and L.C. Teng, "Parametrization of Linear Coupling Motion in Periodic Systems", IEEE Trans. Nucl. Studies, **NS-20**, No. 3, June 1973.
[5] G. Guignard, "Betatron Coupling with Radiation", CERN 87-03, April 1987.