MODE MATCHING TECHNIQUE FOR A LOSSY PILL-BOX CAVITY

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Abstract

Mode matching technique (MMT) has been applied to the study of a lossy pill-box cavity fed by a modulated charged particle beam current and by a coaxial wire. The main feature of the work is that we may give accurate numerical results for the longitudinal coupling impedance below the pipe cut-off frequency and around the resonances of the cavity for any particle velocity. Scattering matrix for the second configuration is also produced.

Cross checks, validating the theory, are presented. Both results are obtained with a light computation apparatus.

1 INTRODUCTION

Mode matching technique (MMT) has been already used for computing interactions between moving charges and surrounding equipment. Gluckstern [1] gave general formulas for tanks connected to the pipes of arbitrary shapes in the high frequency limit; Henke [2] studied the response of an array of pill-box cavities; Kheifets [3] concentrated his attention to the coupling impedance of a pill-box cavity above the pipes cut-off frequency.Up to now no one took into account finite conductivity in a non-perturbative way.

Even if MMT is in practice restricted to a limited number of cases, it has the advantages:

- to provide simple and handy formulas;
- to give quick answers for a large variety of cases with small computational efforts;
- not to be restricted to particles with $\beta = 1$.

In addition to these features we will show in this paper that it is generally possible to take into account the finite resistivity of the walls in a non perturbative way.

The object of this work is the response of a pill-box cavity fed by a charged particle beam moving with arbitrary velocity ($v = \beta c$) along the symmetry axis. The same method will be applied to the a pill-box fed by a coaxial wire. Both pill-box and wire may have finite resistivity, losses are taken into account resorting to the Leontovic boundary conditions. In the first case we will obtain the longitudinal coupling impedance, in the second one the scattering matrix.

This work has been stimulated by a collaboration between INFN Sezione di Napoli and the Dipartimento di Ingegneria Elettrica, with the RF group of the PS Division of CERN on the characterization of the choked-mode Shintake cavity.

2 THE COUPLING IMPEDANCE

The mode matching technique consists in expanding the e.m. fields in the pipes and in the cavity (Fig. 1) as a sum of eigenmodes of the structures multiplied by unknown coefficients $\mathbf{V_n^e}, \mathbf{I_n^e}$ and $\mathbf{V_n^i}, \mathbf{I_n^i}$ respectively, that is :

$$\mathbf{E^e} = \sum \mathbf{V_n^e\, e_n^e} \qquad \mathbf{H^e} = \sum \mathbf{I_n^e\, h_n^e}$$

$$\mathbf{E^i} = \sum \mathbf{V_n^i\, e_n^i} \qquad \mathbf{H^i} = \sum \mathbf{I_n^i\, h_n^i}$$

where the indices **e** and **i** stands for pipes and cavity.

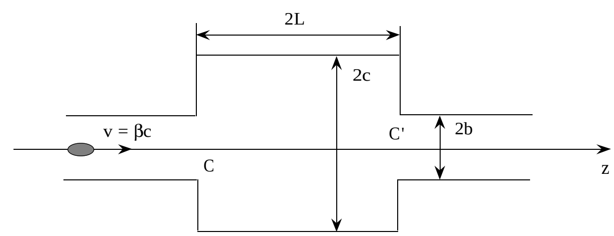

Figure 1: Geometry of the problem.

We remind that the eigenmodes may be defined only for homogeneous boundary conditions. However by means of an appropriate manipulation of the equations we are able to calculate the Green functions including losses on the bounderies. Details of the algebra of the problem will be found in an extended paper which will shortly appear [4].

By matching the boundary conditions on the ports (C and C') connecting the pill-box cavity to the pipes, we get four sets of infinite equations for the coeffiecients $\mathbf{I_n}$ and $\mathbf{V_n}$ Then we get the longitudinal electric field and introduce it into the definition of the coupling impedance.

We first remark that, in the case of vanishing resistivity, we could exactly reproduce the results given by Kheifets [3].

We give in Fig. 2 the longitudinal coupling impedance for a copper cavity for b = 34.35 mm, c = 125.10 mm, L = 188.55 mm and $\beta = 1$ in the range 0 - 4 GHz.

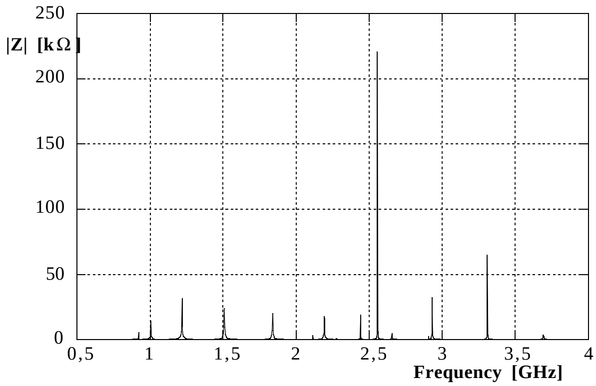

Figure 2: Longitudinal coupling impedance for a copper pill-box cavity for β equal 1.

As a cross check we separately computed some resonance frequencies and the relevant quality factors for the closed pill-box cavity; they are reproduced in Table 1, where the quality factors have been evaluated with standard perturbative formulas.

Table 1: Resonant frequencies and quality factors of the closed pill-box cavity (perturbative method)

TM_{ps}	Frequency [GHz]	Quality factor
1,0	0.9167	43350
1,1	0.9992	36379
1,2	1.2134	40090
1,3	1.5041	44634
2,0	2.1042	68725
2,1	2.1414	57345

The resonant frequencies and quality factors reported in Table 2 are those drawn by the results of the MMT.

Table 2: Resonant frequencies and quality factors of the closed pill-box cavity (MMT)

TM_{ps}	Frequency [GHz]	Quality factor
1,0	0.9209	43415
1,1	1.0065	36499
1,2	1.2194	39444
1,3	1.5087	43412
2,0	2.1147	73521
2,1	2.1618	59749

We note that the agreement is to within a few percent and, in some cases, even better.

Details concerning the second resonance are shown in Fig. 3, where the longitudinal coupling impedance is represented for different velocities of the particle, and for beta ranging between 0.9 and 1.

A drastic change of the behaviour takes place when β is reduced to lower values; for instance, in the case β = 0.2, the longitudinal coupling impedance is reduced to a few tens of ohms.

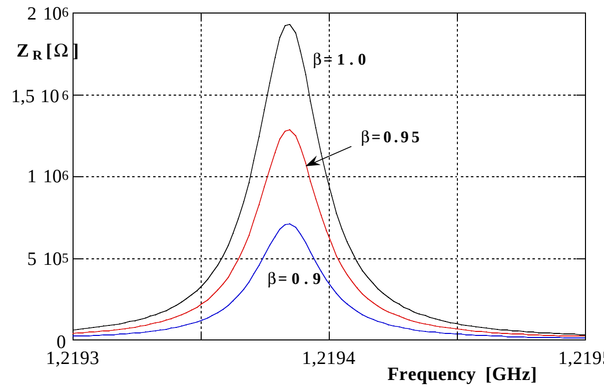

Figure 3: Longitudinal coupling impedance for various β around the second resonance.

Quite interesting is the impedance at the first resonance Due to the specific role of the transit time factor we have a reversed behaviour in the range 0.8< β <1 as shown in Fig 4.

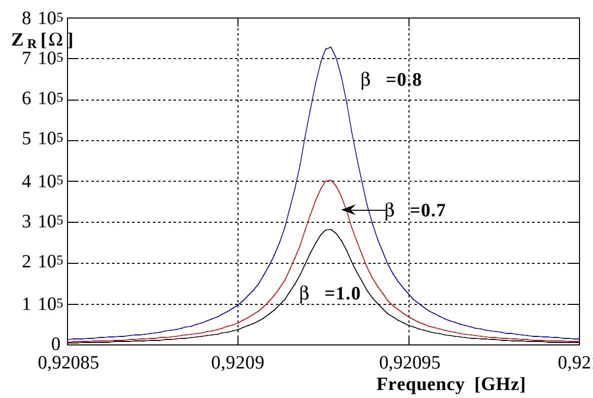

Figure 4: Longitudinal coupling impedance for various β around the first resonance.

Above the cut -off frequency the quality factor decays to lower values because more and more the power escaping through the pipes competes with the power lost by dissipation into the cavity. An examples is shown in Fig. 5.

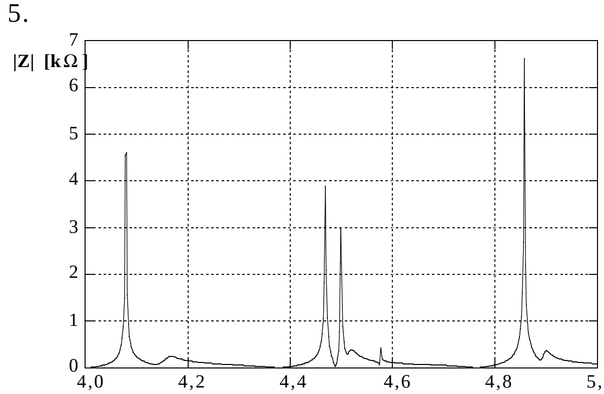

Figure 5: Longitudinal coupling impedance for a copper pill-box cavity for β equal 1, above the cut-off.

3 EVALUATION AND MEASUREMENT OF THE SCATTERING MATRIX

In order to look for an experimental validation of the MMT, we apply a similar procedure to the case of a coaxial wire cavity (the geometry is the same as in Fig. 1, where instead of the bunch there is a wire of radius a = 0. 375 mm). Particular attention has been given to the first element of the scattering matrix.

The coaxial wire method [5] is one of the most generally used bench methods for the measurement of the coupling impedance and of the scattering matrix. The technique is based on the assumption that a part of the vacuum chamber with a wire inserted on its center axis behaves like a coaxial line; the characteristic impedance of this coaxial line will change if we change the external environment (test device).

In particular we perform our measurements in the frequency domain [6], acquiring the transmission scattering matrix of the device (DUT) and of a reference line (REF), which has the same length as the cavity and radius equal to the small aperture shown in Fig. 1. The attention given to the scattering matrix is due to the fact that we are able to give an excact numerical evaluation of this quantity. A Network Analyzer Hp 8720C (50MHz – 13 GHz) has been used for the experiment. The Network gives directly in output the scattering parameters. A PC-Mac is connected through a HP-IP bus to the instrumentation for its control and for the acquisition of the data, real and imaginary components of the Sij parameters (Labview enviroment). An accurate calibration has been performed for each different part of the entire band of acquisition including all the cables and the connectors.

Both the REF and the DUT structures have geometrical and electrical impedance adapters in order to minimize the reflections and to improve the signal to noise ratio. The wire thickness of 0.75 mm was for us a good "compromise" between the possibility of acquire a good signal and the modification due to the wire to the bunch case e.m. field distribution.

A wide number of measurements was made in order to validate the MMT theory. Striking agreements were obtained. As an example, the comparison between theory and measure is given in Fig. 6.

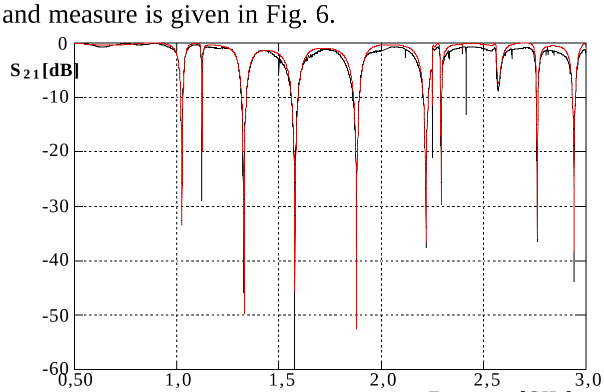

Figure 6: Comparison between theory and measure of the first element of the scattering matrix.

In fig. 6 the measured values are hardly detectable from the theoretical ones and in general are slightly below the latter ones.

Restricting ourselves to the first resonance, more accurate measurements have been done.We compare these data with the numerical simulations (with and without resistivity) in Fig. 7.

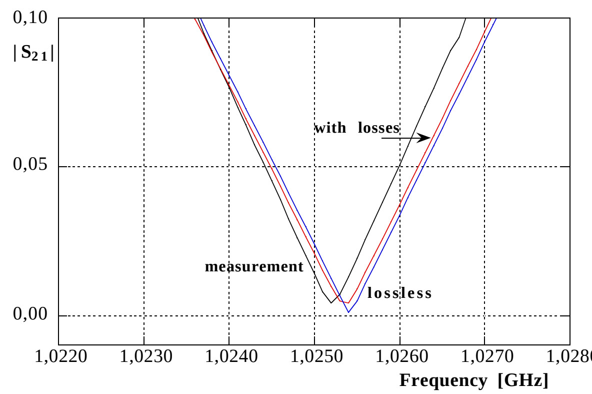

Fig. 7: Comparison between experimental and lossy/lossless simulations for the first resonance.

It is fair to say that the measurements validate the theory, as well as the theory validates the measurements.

4 CONCLUSIONS AND PERSPECTIVES

We presented a non perturbative method to introduce finite losses into a mode matching technique applied to structures which could be of interest for Accelerator Physics. The method can be applied to other configurations, as choked mode cavities, T-shaped cavities, thick irises, bellows, in a wide range of frequencies, especially in those ranges where heuristic formulas are no longer satisfactory.

The method can be extended also to the computation of the transverse coupling impedance.

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