THE PARTICLE DYNAMICS IN THE LOW ENERGY STORAGE RINGS WITH LONGITUDINAL MAGNETIC FIELD

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Abstract

The particle dynamics of the low energy positrons and electrons in the storage ring with longitudinal magnetic field is discussed. Storing and acceleration is considered. Such rings are of interest for two physics problem of fundamental character: antihydrogen and positronium generation and electron cooling of heavy particles in intermediate energy range (of the order of several GeV).

The positron and electron energy lies in the range of several keV – several MeV. The use of a longitudinal magnetic field is very attractive for the beam focusing. In this case the stability of the beam can be provided with additional spiral coils. The acceleration of the beam can be provided by using induction acceleration.

The stability of the particle motion is determined by competition of two effects - drift displacement inside the toroidal bending magnet and focusing action of the spiral quadrupole coils.

The upper limit of the beam intensity is determined by longitudinal instability due to very small momentum spread of the particles.

1 INTRODUCTION

The traditional configuration of an electron cooling system is associated with electrostatic acceleration of a magnetically confined electron beam and recuperation of its energy in a collector. At the electron energy in the range from 1 up to 10 MeV the electrostatic acceleration of the electrons can be provided with various types of Van de Graaff generators, however such a machine requires to solve serious technical problems to realise this system.

To avoid the problems of a traditional electron cooling system and cool down the antiprotons at energy of 8 GeV electron cooling system with circulating electron beams was proposed in [1,2]. In this system the Recycler ring is equipped with an additional electron one, which is periodically filled up with new portion of cold electrons. The electron beam circulates in longitudinal (quasitoroidal) magnetic field, and the long term stability of the beam is provided with additional spiral coils, which form a quadrupole magnetic field. Such a focusing system is similar to the “stelarator” one. The acceleration of the electron beam without the distortions caused by RF system of linear accelerators is achieved by using of induction acceleration. Similar cycling electron accelerator, called “modified betatron”, was proposed by N.Rostocker [3].

In order to test the medium energy electron cooling system based on modified betatron the design of such a system prototype was started at Joint Institute for Nuclear Research (Dubna, Russia). The Modified Betatron Prototype (MOBY) [4] is an electron induction accelerator with electron energy of 4.36 MeV and with longitudinal magnetic field of 1 kG and additional quadrupole spiral magnetic field. The main goal of MOBY creation is to study the problems of particle dynamics in it.

For generation of antihydrogen atoms an antiproton source (like the AA at CERN or the antiproton source at FNAL) has to be supplemented by two small rings – one to store low energy antiprotons and another to store positrons, respectively [5]. The first ring is a conventional strong focusing storage ring for antiprotons with an energy in the range of 0.5 – 50 MeV. The second ring for positrons with energy of 5-10 keV is proposed [5-6] to have a focusing system with guiding longitudinal magnetic field and spiral quadrupole field. Such a magnetic system is similar to the one used in a modified betatron [1-2]. The description of particle dynamics in the ring and first results of the numerical simulation of the particle motion stability are presented here.

2 THE PARTICLE MOTION

2.1 Electron motion in the toroidal section

Taking into account the influence of the electron space charge, one can write the equations of electron motion in the toroidal section with guiding magnetic field B and bending transverse field B_x :

$$y'' - \frac{x'}{\rho} + \left(\frac{1}{R^2} - \frac{1}{{r_d}^2}\right) y = \delta\ ,\ \ x'' + \frac{y'}{\rho} - \frac{x}{{r_d}^2} = 0\ ,$$

$$\delta = \frac{1}{R}\left(\frac{\delta p}{p} - \frac{\delta B_y}{B_y}\right),$$

where $(\ )' = d/ds$, s is the coordinate along electron trajectory, x is the horizontal coordinate, $B=B/(1+y/R)$ is the guiding magnetic field in the toroidal section, R is the radius of the axial electron trajectory in the toroid section, y is the radial displacement of an electron relatively to the axial trajectory, $B_x = pc/eR + \delta B_x$ is the transverse bending magnetic field compensating the electron drift in the toroidal magnet, δB_x is an “error” of the transverse magnetic field, ν is the longitudinal electron velocity, $\gamma = 1/\sqrt{1 - v^2/c^2}$, p is the momentum of electron and δp is it momentum spread, ρ is Larmor radius: $\rho = v/\omega_B$, $\omega_B = eB/\gamma mc$ is the electron cyclotron frequency, r_d is Debye radius: $r_d = v/\omega_p$, $\omega_p = \sqrt{2\pi e^2 n_b/\gamma^3 m}$ is plasma (Lengmoir) frequency, n_b is the electron beam density.

The solution of the electron equation motion in toroidal section is described by the formulae:

$$y = A_1 \cos(\Omega_{t1} s + \varphi_1) + A_2 \cos(\Omega_{t2} s + \varphi_2) + \delta \cdot r_d^2 / \left(1 + r_d^2 / R^2\right)$$

$$x = \frac{A_1 \Omega_{t1} \sin(\Omega_{t1} s + \varphi_1)}{\rho\left(1/r_d^2 - \Omega_{t1}^2\right)} + \frac{A_2 \Omega_{t2} \sin(\Omega_{t2} s + \varphi_2)}{\rho\left(1/r_d^2 - \Omega_{t2}^2\right)},$$

where Ω_{t1}, Ω_{t2} are the "spatial" eigenvalues frequencies of electron rotation in the toroidal section:

$$\Omega_{t1,2}^2 = \frac{1}{2}\left(\frac{1}{R^2} - \frac{2}{r_d^2} + \frac{1}{\rho^2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{1}{R^2} - \frac{2}{r_2^2} + \frac{1}{\rho^2}\right)^2 + \frac{1}{r_{d^2}^2}\left(\frac{1}{R^2} - \frac{1}{r_d^2}\right)}$$

The constants $A_1, A_2, \varphi_1, \varphi_2$ are determined from the values of electron coordinates and transverse velocities at the toroidal section entrance.

2.2 Electron motion in the spiral quadrupole section

The equations of the electron motion in the quadrupole spiral section with guiding magnetic field B are the following [5]:

$$y'' - \frac{x'}{\rho} - \frac{y}{r_d^2} - \frac{G}{B\rho}\left(y\cos(2ks) + x\sin(2ks)\right) = 0,$$

$$x'' + \frac{y'}{\rho} - \frac{x}{r_d^2} - \frac{G}{B\rho}\left(y\sin(2ks) - x\cos(2ks)\right) = 0,$$

where G is the gradient of quadrupole spiral field, $k = m\pi/h$ is the period of the quadrupole spiral coil, h is the length of quadrupole section, $m=1,2,3...$ One can rewrite these equations using the complex value $u=y+ix$. The equation for u has the following solution:

$$u = \sum_{n=1}^{2} \alpha_n \left(\sin(\Omega_n s) - \frac{i\Omega_n (2k + 1/\rho)\cos(\Omega_n s)}{\tilde{\Omega}_n^2}\right) e^{iks} + \sum_{n=1}^{2} \gamma_n \left(\cos(\Omega_n s) - \frac{i\Omega_n (2k + 1/\rho)\sin(\Omega_n s)}{\tilde{\Omega}_n^2}\right) e^{iks}$$

where Ω_1, Ω_2 are the "spatial" eigenvalues frequencies of electron in the quadrupole section:

$$\Omega_{1,2}^2 = -\frac{1}{r_d^2} + \frac{1}{4}\left[\left(2k + \frac{1}{\rho}\right)^2 + \frac{1}{\rho^2}\right] \pm \sqrt{\left(2k + \frac{1}{\rho}\right)^2 \left(\frac{1}{4\rho^2} - \frac{1}{r_d^2}\right) + \left(\frac{G}{B\rho}\right)^2}$$

$$\tilde{\Omega}_n^2 = \Omega_n^2 + k^2 + \frac{k}{\rho} - \frac{G}{B\rho} - \frac{1}{r_d^2}, \quad n=1,2 .$$

The constants $\alpha_{1,2}$ and $\gamma_{1,2}$ are determined from the initial conditions at the entrance of the quadrupole section.

3 THE STABILITY OF ELECTRON MOTION

The stability of the particle motion in the section modified betatron is related to two effects. The first one is a strong coupling between horizontal and vertical degrees of freedom. This coupling appears due to spiral quadrupole coils. The second one is a difference of the field geometry in toroidal and straight section of the betatron. It produces the parametric resonances, which have an island structure on the plane Gradient – Electron energy (Fig.1). These islands of instability appear, when two of four eigenvalues λ of the betatron matrix either all four eigenvalues become not equal to unit: $|\lambda_i| \neq 1$, i=1,2,3,4.

To avoid the resonances during acceleration one has to change the quadrupole gradient or guiding magnetic field faster, than electron energy. In this case one can cross through the islands of instability faster than an instability develops. The value $\tau_{life} \approx T/(n_s \delta\lambda)$ characterises the lifetime of electron in betatron, where T is the revolution period, n_s- number of supperperiods in the ring lattice.

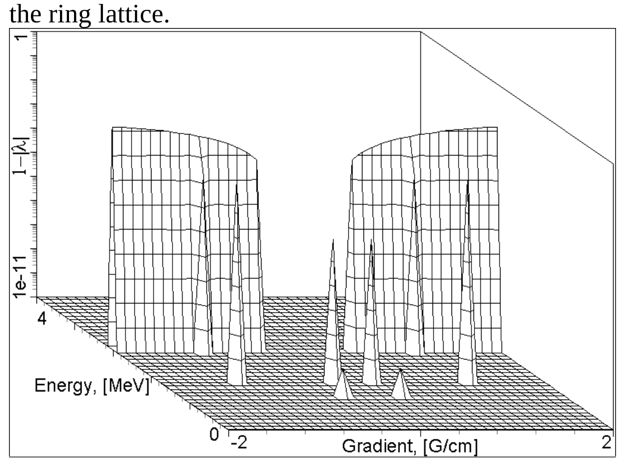

Fig.1. The electron beam instability islands for the MOBY structure when one straight kicker sections have no quadrupole coil. I=0.1 A, B=1 kG

The modified betatron with an uniform quadrupole coil consists of two section: toroidal section with uniform spiral quadrupole coil and straight section with the same uniform spiral quadrupole coil. This structure promises a good stability for the electron beam with $\delta\lambda < 10^{-14}$. The requirement of uniform for the spiral quadrupole coil along electron trajectory is base point at design of the section modified betatron let us to avoid parametric resonances (Fig. 2). However, a design of MOBY with the uniform quadrupole has a lot of technical problems.

4 THE LONGITUDINAL STABILITY

The main limitation of the electron beam current is related to the longitudinal motion stability, when very small momentum spread of the electrons is required to obtain a short cooling time. The momentum spread of the circulating electron beam is limited by well-known Keil-Schnell criterion:

$$\frac{\Delta p}{p} \geq \sqrt{\frac{eI_e\left|\frac{Z_n}{n}\right|}{mc^2\gamma\beta^2|\eta|}}\ ,\ \eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$$

At electron energy of few MeV the beam impedance

$$\left|\frac{Z_n}{n}\right| = \frac{1}{2\beta\gamma^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(1 + 2\ln\frac{b}{a}\right)$$

is of the order of tenth Ohms, where b and *a* are the radii of vacuum chamber and electron beam. The transition energy for uniform betatron structure is determined by frequency magnitudes of Larmor and betatron oscillations [5]:

$$\gamma_{tr} = \frac{GR}{B}\sqrt{\frac{Rn_1}{4\pi\rho}}\ ,$$

where n_1 is number of spiral winding turns per ring circumference. Therefore at the electron energy of a few MeV the electron beam with current of hundred mA can be stable, when momentum spread is larger than $2 \cdot 10^{-4}$.

5 THE ANGULAR SPREAD OF THE ELECTRON BEAM

The main parameter of electron beam, generated by electron cooling system is its angular spread. It should be less than one of antiproton beam. In the case we discuss the angular spread of antiprotons in the Recycler is about $2 \cdot 10^{-4}$. The simulated angular spread of the electron beam in the Recycler electron storage ring with longitudinal magnetic field of 2 kG also corresponds to $2 \cdot 10^{-4}$

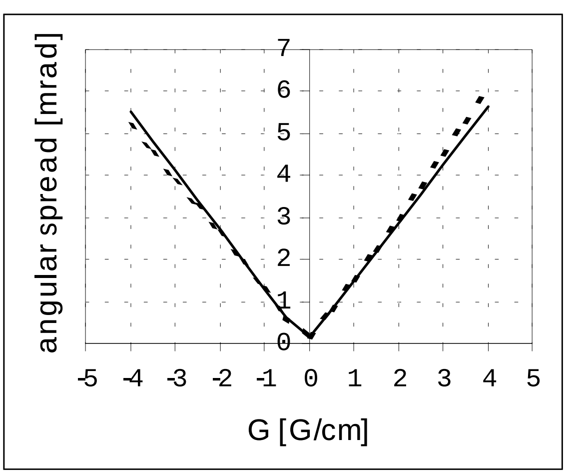

Fig.2. The dependence of angular spread of the electron beam on quadrupole gradient for section modified betatron. *E*=4 MeV, *B*=1 kG, *I*=0.1 A. dashed line is the horizontal angular spread, solid line is vertical one.

For the stable conditions the angular spread is determined by the quadrupole gradient magnitude and drift motion in toroidal sections. It increases at large quadrupole gradient (Fig.2).

For typical parameters of $G\cong 1$ G/cm the angular spread of electron at the energy of a few MeV in magnetic field of 1kG is about 10^{-3}. The choice of electron the quadrupole gradient magnitude is determined also by the values of the electron momentum spread and errors of bending toroidal magnetic field δB_x. The reduction of the quadrupole gradient value leads to an increase of the amplitude of the transverse oscillations of electrons (Fig.3). At the low level of the quadrupole gradient the aperture of electron beam is very high. At the typical magnitude of the bending magnetic field error $\delta B_x/B_x \approx 10^{-2}$ the electron beam size increases by 3 times at $G\cong 1$ G/cm.

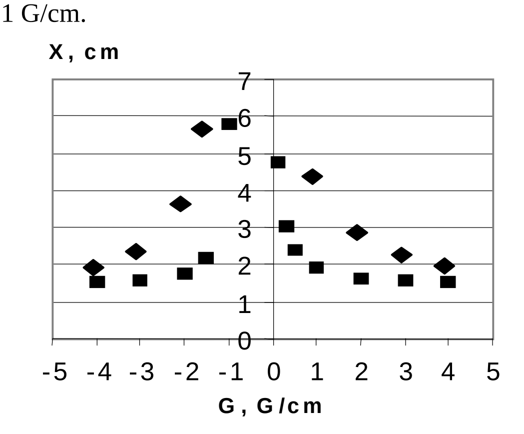

Fig.3. The dependence of horizontal coordinate on quadrupole gradient for modified betatron with quadrupole section. ◆-$\delta B_x/B_x \approx 10^{-2}$, ■-$\delta B_x/B_x \approx 10^{-3}$, initial horizontal coordinate x_0=1.5 cm, *B*=1 kG, *E*=3 MeV, *I*=0.2 A.

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