TRANSVERSE EMITTANCE BLOW-UP FROM DIPOLE ERRORS IN PROTON MACHINES

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Abstract

A set of emittance blow-up formulae for proton beams is derived. The starting point is the classical problem of the emittance increase provoked by a single transverse kick, p.e. an injection error. The degree of complication gradually increases by treating sequentially the case of a series of random kicks, a single kick with active damping and finally the case of coherent excitation. In all cases the phenomenon of decoherence is assumed. Illustrative experimental results will also be presented.

1 INTRODUCTION

Emittance preservation is an essential issue for proton colliders and their injectors. A variety of mechanisms can lead to an increase in the transverse beam size. One particular family will be studied in this paper, that is the important family of dipole errors. Many different sources belong to it: injection errors, single or multiple deflections for tune and chromaticity measurements, ground motion vibrations and all of these situations in presence of a transverse feedback system with finite noise properties. It is clear that there exist many opportunities for growth of the beam emittance. Therefore it is very important to be able to compute or predict the emittance growth in all of the circumstances that were mentioned.

2 EMITTANCE GROWTH FROM A SINGLE DIPOLE DEFLECTION

This problem has been treated in an elegant way by H. Hereward in [1]. In this case, as in the following, we are dealing with a multi-particle problem and the process of decoherence or filamentation is assumed to be active. That process will cause an increase in the *rms* beam oscillation amplitudes or *rms* beam size. It is characterised by a decoherence time τ_{dc} which takes its origin from a tune spread ΔQ (measured at the base of the tune or frequency distribution). It can be shown [2] that , within 10% :

$$\frac{\tau_{dc}}{T} = \frac{1}{\Delta Q} \qquad (1)$$

where T is the revolution period of the accelerator. For a kick of magnitude Δx a total transverse energy of Δx^2 is available to the beam. Since the beam transverse movement has two degrees of freedom, angle and position,

it is expected that after sufficient time this energy is equally distributed between them. Hence:

$$\Delta\sigma^2 = \frac{1}{2}\Delta x^2, \qquad (2)$$

where σ is the *rms* oscillation amplitude of the ensemble and $\Delta\sigma$ its increment. This equation is true for $\Delta x < \sigma$. Its validity has been verified in many experiments.

3 EMITTANCE GROWTH FROM A SERIES OF RANDOM DIPOLE DEFLECTIONS

This problem has been treated by Hereward and Johnsen [3]. The analysis of previous paragraph remains valid when the beam is subjected to a series of kicks which are distributed randomly in time. The power of the noisy kicker is $<\Delta x^2>/T$. Many generators around the circumference can always be combined into a single one with the same effect. It is important to note that the beam has one chance every turn to be affected by the noise of the generator. Hence the increase in *rms* amplitude per turn is given by:

$$\Delta\sigma^2 = \frac{1}{2}\left\langle \Delta x^2 \right\rangle \qquad (3)$$

and the emittance growth rate follows:

$$\frac{\Delta\sigma^2}{\Delta t} = \frac{1}{2}\frac{\left\langle \Delta x^2 \right\rangle}{T}, \qquad (4)$$

$$\tau^{-1} = \frac{1}{T}\frac{\left\langle \Delta x^2 \right\rangle}{2\sigma^2}. \qquad (5)$$

It has been assumed all along that σ and Δx are defined with the same optical functions. An example is shown in Fig. 1.

4 COMBINED EFFECT OF SINGLE KICK AND ACTIVE DAMPING

The process that is studied now is again a single kick excitation of a beam. The initial displacement is Δx_0. Due to the filamentation the coherent signal decreases as :

$$\Delta x(t) = \Delta x_0 e^{-t/\tau_{dc}}. \qquad (6)$$

This can also be written in another form:

$$\frac{d\Delta x}{dt} = -\frac{\Delta x}{\tau_{dc}}. \qquad (7)$$

An active feedback system will add a second damping term τ_d :

$$\frac{d\Delta x}{dt} = -\frac{\Delta x}{\tau_{dc}} - \frac{\Delta x}{\tau_d} = -\frac{\Delta x}{\tau} \quad \text{and} \quad \Delta x(t) = \Delta x_0 e^{-t/\tau}. \quad (8)$$

The part of the amplitude that goes into the decoherence is Δx_{dc}:

$$\frac{d\Delta x_{dc}}{dt} = -\frac{\Delta x_0}{\tau_{dc}} e^{-t/\tau}. \qquad (9)$$

The integral of this yields the total amplitude not corrected by the active feedback :

$$\Delta x_{dc} = \Delta x_0 \frac{\tau}{\tau_{dc}}. \qquad (10)$$

This uncorrected amplitude goes into beam size increase according to:

$$\Delta \sigma^2 = \frac{1}{2}\Delta x_{dc}^2 = \frac{1}{2}\Delta x_0^2 \left(\frac{1}{1+\dfrac{\tau_{dc}}{\tau_d}}\right)^2. \qquad (11)$$

This formula illustrates very clearly the competing tendencies between filamentation and active damping.

5 EMITTANCE GROWTH DUE TO COHERENT EXCITATION

The basic ingredient of the excitation is a series of kicks. They arrive once per turn modulated as the β-oscillation. This case has been studied by Hereward [4].

It may be enlightening to derive the emittance growth in two different ways.

a) This derivation is based on the well known fact that a stable coherent stimulation makes the beam oscillate with an equally stable coherent amplitude which is called $\bar{x}$. For a single kick the signal decoheres like τ_{dc}. The power of the signal will decohere twice as fast like $\tau_{dc}/2$. The power of the signal that disappears in the decoherence will increase the beam size. The beam size increase per turn follows in a straightforward manner:

$$\Delta \sigma^2 = \frac{\bar{x}^2}{2}\frac{2T}{\tau_{dc}} \quad \text{and} \quad \frac{\Delta \sigma^2}{\Delta t} = \frac{\bar{x}^2}{T}\Delta Q \qquad (12)$$

followed by the growth rate :

$$\tau^{-1} = \frac{1}{T}\frac{\bar{x}^2 \Delta Q}{\sigma^2}. \qquad (13)$$

b) The starting point of this derivation is the equation of motion :

$$\ddot{x} + Q^2\Omega^2 x = \frac{F}{m} \qquad (14)$$

where F is the driving force and m the mass of the particle and $\Omega = 2\pi/T$. The driving term can be expressed as a function of a deflection θ:

$$\frac{F}{m} = \frac{\Omega c}{2\pi}\theta. \qquad (15)$$

The stimulus θ is given at a frequency in the β-band of the beam. The beam response is :

$$|\bar{x}| = \frac{F/m}{Q\Omega^2 \Delta Q}|f(\xi)| \qquad (16)$$

where $f(\xi)$ is the (complex) dispersion integral of the ensemble and ξ is a normalised frequency taking values of 1 and -1 at the edges of the frequency distribution. The absolute value of $f(\xi)$ varies from 0.7π to 1.1π for reasonable distribution functions. For an elliptic distribution p.e. $f(\xi)$ is exactly equal to π. An approximation with about the same accuracy as the one that was applied to the computation of the decoherence time is proposed , that is $|f(\xi)| = \pi$, so that :

$$\bar{x} = \frac{\pi F/m}{Q\Omega^2 \Delta Q} = \frac{c\theta}{2Q\Omega\Delta Q} = \frac{\langle\beta\rangle\theta}{2\Delta Q} = \frac{\Delta x}{2\Delta Q}. \qquad (17)$$

It should be recalled that Δx is restricted to the β-spectrum of the beam. Both $\bar{x}$ and Δx can be taken at a single frequency or as rms values if the full β-band is involved. In a previous paragraph the blow-up of the beam size was computed for an excitation by white noise. Assume noise power that covers one revolution band of frequency. The power is $<\Delta x^2>$. The revolution band contains two β-bands, one slow wave and one fast wave. The power that goes in the two bands is:

$$\Delta x^2 = 2\Delta Q\langle\Delta x^2\rangle \qquad (18)$$

and also:

$$\bar{x}^2 = \frac{2\Delta Q\langle\Delta x^2\rangle}{4\Delta Q^2} = \frac{\langle\Delta x^2\rangle}{2\Delta Q}, \text{ and } \langle\Delta x^2\rangle = 2\Delta Q\bar{x}^2. \quad (19)$$

The blow-up rate for $<\Delta x^2>$ has been computed before:

$$\tau^{-1} = \frac{1}{T}\frac{\bar{x}^2\Delta Q}{\sigma^2} = \frac{1}{4T}\frac{\Delta x^2/\Delta Q}{\sigma^2}. \qquad (20)$$

The result is identical to the one that was found with the exponential decay approach. Note that the term $\Delta x^2/\Delta Q$ is nothing else but the (normalised) power density of the exciter in a β-side-band.

6 EMITTANCE BLOW-UP FROM RESIDUAL COHERENT OSCILLATION IN THE PRESENCE OF TRANSVERSE FEEDBACK

This is a special case of the previous one. Consider a feedback system with electronic gain g and at the input white noise with power density dx_n^2/df in position units. The noise power in one revolution frequency band is:

$$x_n^2 = \frac{dx_n^2}{df}\frac{2\Delta Q}{T} \text{ and } \overline{x}_n^2 = \frac{dx_n^2}{df}\frac{g^2}{2\Delta QT}. \quad (21)$$

The growth rate follows from (20):

$$\tau^{-1} = \frac{1}{2T^2}\frac{(dx_n^2/df)g^2}{\sigma^2}. \quad (22)$$

If the feedback loop is closed with the condition that $g/2\Delta Q > 1$, i.e. the loop reduces the noise efficiently, then, for a pure analog system with a bandwidth W:

$$\tau^{-1} = \frac{W}{2T}\frac{(dx_n^2/df)(2\Delta Q)^2}{\sigma^2}. \quad (23)$$

The same formula can be derived when starting from the theory of stochastic cooling[5].

In a digital system quantization noise is in general dominant. If this is the only excitation present in the beam previous formula remains valid with the substitution: $W(dx_n^2/df) = \Delta_D^2/12$, where Δ_D is the quantization referred to the input of the system. On the other hand, a beam that is excited (instability, ground vibrations) will oscillate up to the limit of the quantization. A persistent coherent oscillation amplitude is set up with amplitude $\Delta_D/\sqrt{12}$ and the growth rate becomes :

$$\tau^{-1} = \frac{1}{T}\frac{(\Delta_D^2/12)\Delta Q}{\sigma^2}. \quad (24)$$

An example of such a coherent oscillation is shown in Fig. 2. The measured blow-up was in agreement with the formula.

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[4]   H. Hereward, "The Elementary Theory of Landau Damping", CERN 65-20, 1965.

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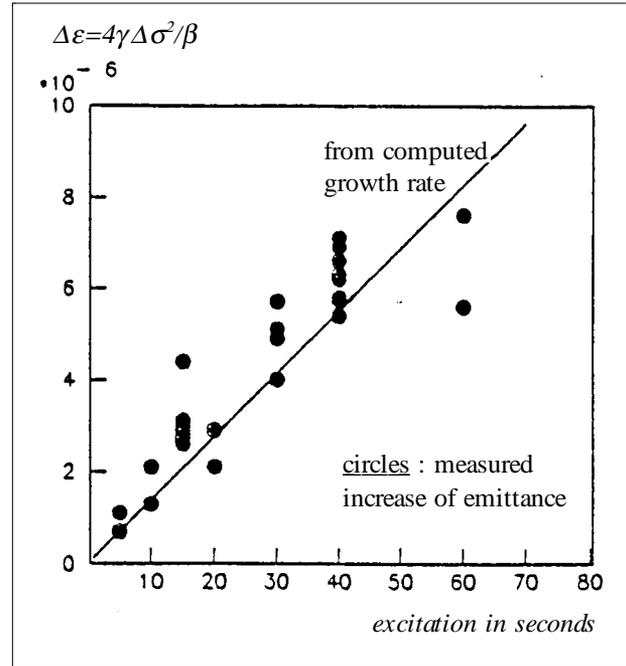
Figure 1 : Blow-up of emittance in the SPS collider. $\Delta\varepsilon = \gamma\, 4\Delta\sigma^2/\beta$, where γ is the relativistic factor and β the optical Twiss parameter at the azimuth of the emittance measurement.
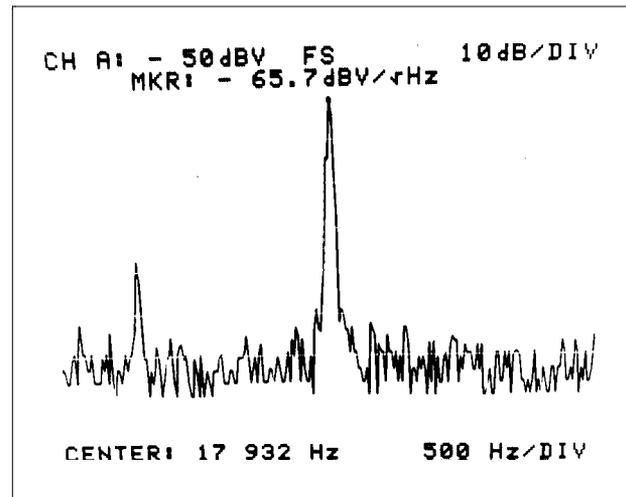

Figure 2 : Coherent oscillation spectrum of a beam in the SPS that is stabilised by a digital feedback system.