

# NONLINEAR DYNAMICS IN NUCLOTRON

D. Dinev

Institute for Nuclear Research and Nuclear Energy,  
72 Tzarigradsko chaussee, 1784 Sofia, Bulgaria

## Abstract

The paper represents an extensive study of the nonlinear beam dynamics in the Nuclotron. Chromatic effects, including the dependence of the betatron tunes on the amplitude and chromatic perturbations have been investigated taking into account the measured field imperfections. Beam distortion, smear, dynamic aperture and nonlinear acceptance have been calculated for different particle energies and betatron tunes.

## 1. INTRODUCTION

The Nuclotron is an accelerator of relativistic nuclei working at the Joint Institute for Nuclear Research in Dubna, Russia - [1]. This is a superconducting heavy ion synchrotron. It has FODO magnetic structure with 32 periods and 8 superperiods. The main accelerator parameters are:

Circumference	251.52 m
Injection energy per nuclei	5 MeV/ A
Maximum energy per nuclei (Z/A=0.5)	6 GeV/ A
Beam intensity	$5 \cdot 10^{11}$ A/Z <sup>2</sup> ppp

This paper represents an extensive study of the nonlinear dynamics in the Nuclotron.

## 2. CHROMATIC EFFECTS

The natural chromaticity (taking into account the influence of dipoles) and the chromaticity under systematic sextupole and octupole errors in dipoles are given in Table 1.

At injection (Bρ=1.0Tm) the relative momentum spread is  $\delta = \pm 4 \cdot 10^{-3}$  which results in  $\Delta Q_x = \pm 0.04$  and  $\Delta Q_y = \pm 0.02$ . At maximum energy (Bρ=45.83 Tm) the relative momentum spread is  $\delta = \pm 8 \cdot 10^{-4}$  which results in  $\Delta Q_x = \pm 0.004$  and  $\Delta Q_y = \pm 0.018$ . Therefore the chromaticity must be corrected.

Table 1.

Parameter	Value
Natural chromaticity	
$Q'_x$	-7.735
$Q'_y$	-7.937
Chromaticity at Bρ=1.0Tm (systematic errors in dipoles)	
$Q'_x$	-10.206
$Q'_y$	-5.346
Chromaticity at Bρ=45.83 Tm (systematic errors in dipoles)	
$Q'_x$	4.889
$Q'_y$	-22.398

Two families of sextupole lenses are available in the Nuclotron for correction of chromaticity. They are placed just before the focusing and defocusing quadrupoles in the strait sections of each superperiod.

The strengths of these sextupole lenses are given in Table 2.

Table 2.

B''	Value, T/m <sup>2</sup>
without errors	
SF	1.50
SD	-2.55
with systematic errors in dipoles at Bρ=1.0Tm	
SF	1.74
SD	-2.05
with systematic errors in dipoles at Bρ=45.83Tm	
SF	8.27
SD	-249.37

We have calculated the dependence of betatron tunes on the amplitude which appears in sextupole (second order effect) and octupole fields. The results obtained at  $B\rho=1.0$  Tm are:

$$\frac{\Delta Q_x}{d\varepsilon_x} = 101.4 \quad \frac{\Delta Q_y}{d\varepsilon_y} = -12.4 \quad \frac{\Delta Q_x}{d\varepsilon_y} = \frac{\Delta Q_y}{d\varepsilon_x} = 4.4$$

As the emittances at  $B\rho=1.0$  Tm are  $\varepsilon_x=\varepsilon_y=30 \cdot 10^{-6} \pi$  m the corresponding tune shifts are small.

The chromatic perturbations have been studied by means of the Montague chromatic functions B, A and the vector  $W=(B,A)$ -[3]. The calculated maximum value of the vector W taking into account the magnet imperfections are given in Table 3.

Table 3

max Montague chr. functions	value
Without errors	
$W_{x,max}$	1.6 $\delta$
$W_{v,max}$	1.6 $\delta$
$B\rho=1.0$ Tm, systematic errors in dipoles	
$W_{x,max}$	13.1 $\delta$
$W_{v,max}$	9.8 $\delta$
$B\rho=45.83$ Tm, systematic errors in dipoles	
$W_{x,max}$	25.8 $\delta$
$W_{v,max}$	42.3 $\delta$

The maximum relative chromatic error in the amplitude function  $\beta(s)$  taking into account the systematic errors in the dipoles at  $B\rho=1.0$ Tm is  $\Delta\beta/\beta = \pm 6\%$  and at  $B\rho=45.83$  Tm  $\Delta\beta/\beta = \pm 4\%$

### 3. SMEAR

The beam distortion have been investigated by means of T. Collins's distortion functions method-[4].

The figure of merit is called SMEAR and is defined through:

$$SMEAR = \frac{\sigma(A)}{\langle A \rangle} , A = \sqrt{A_x^2 + A_z^2} \quad (1)$$

where  $A_x$  and  $A_z$  are the horizontal and vertical amplitudes,  $\sigma$  denotes the standard deviation and  $\langle \rangle$  - the mean value.

For Nuclotron we have calculated that at the working point  $Q_x = 6.8$  and  $Q_y = 6.85$   $SMEAR=5.3\%$ . This is a rather small value. It was decided (SSC, LHC, HERA) that if  $SMEAR < 6.4\%$  the machine is considered to be sufficiently linear.

### 4. DYNAMIC APERTURE

The Nuclotron dynamic aperture has been calculated by particle tracking. The computer code MAD-[5] has been used for the tracking.

Due to the limited computer power the number of tracked turns in our calculations was set to 500, which is a commonly used value. The dynamic aperture calculated for such a low number of turns is referred as short-term dynamic aperture.

One should distinguish between the area of stable motion obtained with imposing of aperture limitation representing the real vacuum chamber sizes and without such limitations. We will call the area of stable motion 'nonlinear acceptance' in the first case preserving the name 'dynamic aperture' for the latter case when no real physical aperture but rather artificial limiting value is used. In our numerical calculations we use an amplitude limit of 1 meter for obtaining of the dynamic aperture and aperture limits of 0.04m in quadrupoles (radius) and of 0.056m in dipoles (full poles gap) for obtaining the nonlinear acceptance.

Fig.1. shows the dynamic aperture and the nonlinear acceptance for the injection energy  $E=12$  MeV/A ( $B\rho=1.0$  Tm) while Fig.2. shows all these for the maximum beam energy  $E=6$  GeV/A ( $B\rho=45.83$  Tm).

Fig.3. depicts the dependence of the nonlinear acceptance on the particle momentum (energy).

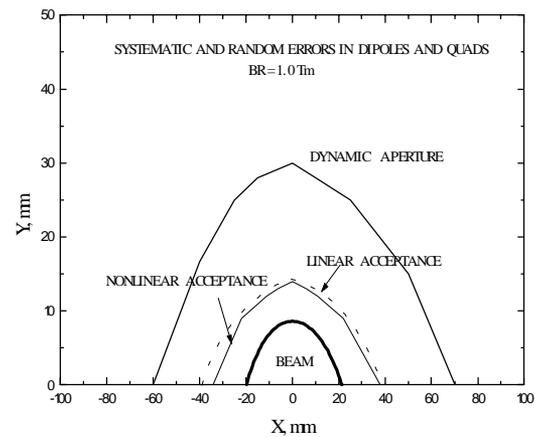
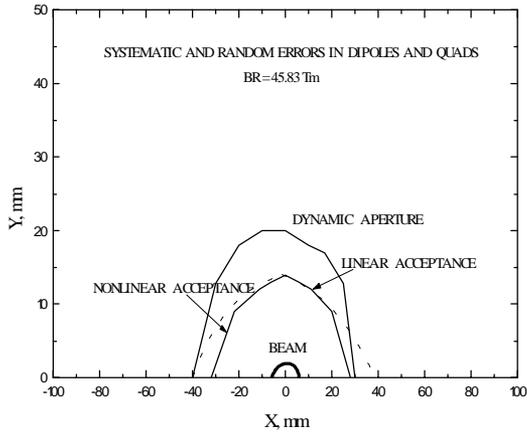
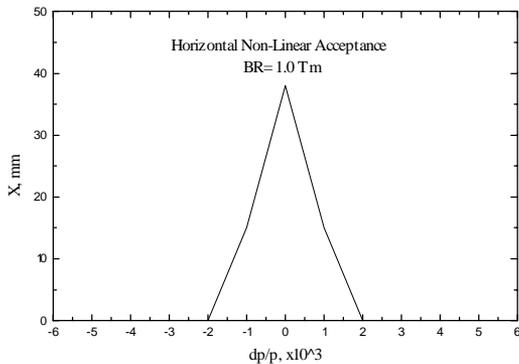


Figure 1. Dynamic aperture and nonlinear acceptance at  $B\rho = 1.0$  Tm.



**Figure 2.** Dynamic aperture and nonlinear acceptance at  $B\rho = 45.83$  Tm.



**Figure 3.** Dependence of nonlinear acceptance on particle momentum.

## 5. RESONANCE EFFECTS

Whereas systematic field imperfections excite only resonance harmonics with numbers  $p$  multiple to the number of accelerator periods  $P$ ,  $p=k.P$ , the so-called fundamental or structural resonances random field imperfections excite all resonance harmonics  $p$ .

Both are quite dangerous as under resonance conditions the amplitude of the particle oscillations grows boundlessly and ultimately the particle will be lost on the vacuum chamber walls.

The widths of some resonances which lie close to the working point and are excited by random field imperfections are given in Table 4. They have been calculated applying G. Guignard's formulae-[6].

Table 4.

Resonance	width x $10^4$ $B\rho = 1.0$ Tm	width x $10^4$ $B\rho = 45.83$ Tm
$3Q_x = 20$	10.8	7.2
$2Q_x - Q_y = 7$	22.6	8.6
$2Q_x - Q_z = 7$	8.8	6.0
$4Q_x = 27$	2.1	0.28
$3Q_x - Q_z = 27$	9.0	0.89

In the Nuclotron the resonance  $3Q_x=20$  will be used for resonance extraction-[7]. It will be excited by means of four sextupole lenses with field gradients:

$$\frac{\partial^2 B_z}{\partial x^2} = 233 \text{ T/m}^2 \quad \text{for LS1 and LS3}$$

$$\frac{\partial^2 B_z}{\partial x^2} = 110 \text{ T/m}^2 \quad \text{for LS2 and LS4}$$

The electrostatical septum is deployed at  $x=30$  mm. The step of particles towards the septum is 5 mm. Taking into account that the septum edge thickness is 0.3 mm this results in an extraction efficiency of 95%.

## REFERENCES

1. B. V. Vasilishin et al. Preprint of JINR 9-86-512, Dubna, 1986.
2. A. M. Donyagin et al. Preprint of JINR P9-94-329, Dubna, 1994
3. B. W. Montague Preprint CERN, LEP Note 165, Geneva, 1979
4. T. Collins. Preprint of FNAL 84-114, Batavia, 1984.
5. F. Iselin, J. Niederer. The MAD Program - User's Reference Manual. CERN/LEP-TH/ 88-38, Geneva, 1988.
6. G. Guignard. Preprint of CERN ISR-MA 75-23, Geneva, 1975
7. B. V. Vasilishin et al. Preprint of JINR 9-86-511, Dubna, 1986.