

SEARCHING FOR BIG LINEAR PERTURBATIONS IN SYNCHROTRONS

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Abstract

A new algorithm of successive orbit approximation have been developed for searching for availability of large linear perturbations, i. e. field errors and element misalignments and tilts in synchrotrons. Such big perturbations often occur during the machine commissioning and may cause very large closed orbit distortions. The new algorithm has been applied to the superconducting heavy ion synchrotron Nuclotron in JINR-Dubna.

1. INTRODUCTION

During the commissioning of a synchrotron much larger linear perturbations than the usual random linear errors may occur. They are caused by unpredictable mistakes during magnetic elements and vacuum chamber assembling and by big quadrupole misalignments due to geodesic errors or local ground movement.

The main result of the big linear perturbations is a large closed orbit distortion. Such a large orbit distortion may cause significant beam losses. On the other hand the corrector strengths necessary to correct such a highly distorted orbit could become quite large and even to exceed the maximum available corrector power thus making the complete orbit correction impossible.

It follows from all this that in case of large orbit distortions special efforts should be made to reveal the sources of this large distortions. After that the suspected magnetic elements should be carefully examined.

For detection of large linear perturbations we have developed an algorithm of successive orbit approximation. This algorithm is described after some necessary introductory definitions. A description of a computer code LEA realizing the algorithm along with computational experiments follows.

2. LINEAR PERTURBATIONS

The linear perturbations are:

a.) field errors in dipoles $\Delta B = B - B_0$.

b.) element misalignments Δx and Δz ; x and z being the transverse particle coordinates.

c.) dipole tilts around the longitudinal s axis, θ .

d.) stray dipole fields ΔB_{st} .

All these cause linear about x and z members to appear in the transverse motion Hamiltonian-[1].

The quadratic part H_0 of the Hamiltonian describes betatron oscillations, as long as the linear part $\delta H^{(1)}$ gives rise an depending on s external force. Thus we face an forced oscillation problem. The external force causes closed orbit distortions, the closed orbit being a periodic solution of the Hamiltonian equations of motion. It is the most convenient to describe the closed orbit in the generalized variables-[2]:

a.) azimuth

$$\phi = \int_0^s \frac{ds}{Q \beta(s)} \quad (1)$$

where Q is the betatron tune and $\beta(s)$ is the Twiss's amplitude function.

b.) normalized deviation:

$$\eta = \frac{x}{\sqrt{\beta(s)}} \quad (2)$$

In this variables the equation of transverse motion becomes an equation of forced oscillations. In the case of only one error kick ϵ_0 deployed at $\phi = \psi_0$:

$$\frac{d^2 \eta}{d\phi^2} + Q^2 \eta = Q \sqrt{\beta_0} \epsilon_0 \delta(\phi - \psi_0) \quad (3)$$

The 2π periodic solution of (3) is:

$$\eta(\phi) = \delta_0 \cos Q(\pi - |\phi - \psi_0|) \quad (4)$$

where:

$$\delta_0 = -\frac{\sqrt{\beta_0} \epsilon_0}{2 \sin \pi Q} \quad (5)$$

From the superposition principle when M error kicks ϵ_j are deployed at ψ_j :

$$\eta(\phi) = \sum_{j=1}^M \delta_j a_j(\phi) \quad (6)$$

where:

$$a_j(\phi) = \cos Q(\pi - |\phi - \psi_j|) \quad (7)$$

3. ALGORITHM OF SUCCESSIVE ORBIT APPROXIMATION

3.1. Signal to Noise Ratio.

In presence of a large linear error one of the harmonics in the sum (8) will be prevalent. It will play the role of our 'signal'. Let prevalent be the harmonics with $j=k$, i.e. the big linear error is located at $\phi=\psi_k$. The remaining sum of harmonics over all $j \neq k$ will constitute the 'noise' for the fitting process. This noise is normally distributed and its dispersion could be easily calculated. In the case of full symmetry of the synchrotron with M dipoles having field errors with a standard deviation $\sigma(\Delta B/B)$ and a large error $(\Delta B/B)_k$ in the k -th. dipole:

$$\frac{\text{signal}}{\text{noise}} = \frac{(\frac{\Delta B}{B})_k}{\sqrt{2M} \sigma(\frac{\Delta B}{B})} \quad (8)$$

The numerical estimation of (10) shows that in the usual cases the signal to noise ratio is adverse thus making the revealing of the signal from the BPMs measurements a difficult task.

3.2. Algorithm.

For determining of large linear perturbations the following algorithm of successive orbit approximation could be applied.

1. Zero step:

At the beginning of the approximation process we set:

$$\eta^{(0)}(\phi) = \eta^{BPM}(\phi) \quad (9)$$

where $\eta^{BPM}(\phi)$ is the orbit measured by the beam position monitors.

2. k^{th} step:

At the k^{th} step we will approximate the curve $\eta^{(k-1)}(\phi)$ obtained at the previous $(k-1)$ step successively by the curves:

$$\Delta_p a_p(\phi) \quad , \quad p = \overline{1, M} \quad (10)$$

Δ_p will be the only parameter of each one of the M successive approximations. Let us introduce the vectors:

$$\vec{\eta}^{(k-1)} = (\eta_1^{(k-1)}, \eta_2^{(k-1)}, \dots, \eta_N^{(k-1)})^T \quad (11)$$

$$\vec{\eta}_p = (\Delta_p a_p(\phi_1), \Delta_p a_p(\phi_2), \dots, \Delta_p a_p(\phi_N))^T \quad (12)$$

where ϕ_i is the azimuth of the i^{th} BPM and $\eta_i^{(k-1)}$ is the orbit at this monitor obtained at the $(k-1)$ step of the algorithm under consideration.

We will distinguish between the usual random errors δ_j and the large errors Δ_j which we are looking for and which, as we have already said, are due to extraordinary imperfections in the elements. The former constitute the random vector:

$$\vec{\delta} = (\delta_1, \delta_2, \dots, \delta_M)^T \quad (13)$$

while the latter constitute the vector:

$$\vec{\Delta} = (\Delta_1, \Delta_2, \dots, \Delta_M)^T \quad (14)$$

In accelerators $N < M$ as a rule. Supposing that a big error Δ_p exists at the p^{th} dipole we can write:

$$\vec{\eta}^{(k-1)} = \vec{\eta}_p + A \cdot \vec{\delta} \quad (15)$$

$$A = \{A_{ij}\} = \{\cos Q(\pi - |\phi_i - \psi_j|)\} \quad (16)$$

The random variables δ_j are independent and normally distributed ($\delta_j \sim N(0, D_j)$). The covariance matrix of the random vector $\vec{\delta}$ is:

$$K_\delta = \{D_i \delta_{ij}\} \quad i = \overline{1, M} \quad j = \overline{1, M} \quad (17)$$

δ_{ij} being the Kronecker's symbol and D_i the dispersion of δ_i . The probability theory says that the random vector $\vec{\eta}^{(k-1)}$ is also normally distributed, $\vec{\eta}^{(k-1)} \sim N(\vec{\eta}_p, K_\eta)$. The PDF is:

$$p(\eta_1^{(k-1)}, \eta_2^{(k-1)}, \dots, \eta_N^{(k-1)}) = \frac{1}{\sqrt{(2\pi)^N |K_\eta|}} \times \exp\left\{-\frac{1}{2}(\vec{\eta}^{(k-1)T} - \vec{\eta}_p^T) \cdot K_\eta^{-1} \cdot (\vec{\eta}^{(k-1)} - \vec{\eta}_p)\right\} \quad (18)$$

The mean value is:

$$\vec{m}_\eta = \vec{\eta}_p \quad (19)$$

and the covariance matrix:

$$K_\eta = A \cdot K_\delta \cdot A^T \quad (20)$$

For determining of the parameters Δ_p we will apply the Fischer's maximum likelihood method. According to this method we should search for the maximum of the joint probability:

$$p(\eta_1^{(k-1)}, \eta_2^{(k-1)}, \dots, \eta_N^{(k-1)} | \Delta_p) \xrightarrow{\Delta_p} \text{Max} \quad (21)$$

where $\eta_i^{(k-1)}$ are the orbit deviations in the BPMs (at the $(k-1)$ step of the algorithm) and Δ_p the unknown parameter. The condition (21) is equivalent to:

$$(\vec{\eta}^{(k-1)T} - \vec{\eta}_p^T) \cdot C_\eta \cdot (\vec{\eta}^{(k-1)} - \vec{\eta}_p) \xrightarrow{\Delta_p} \text{Min} \quad (22)$$

3. COMPUTER CODE LEA AND COMPUTATIONAL EXPERIMENTS

$$C_{\eta} = K_{\eta}^{-1} \quad (23)$$

Formula (22) differ from the well-known LSQ criterion due to the fact that the components of the random vector $\vec{\eta}$ are correlated (see(15)).

To find Δ_p from (22) we must equal the first derivative of the left side to zero which yields:

$$\Delta_p^* = \frac{\zeta}{\xi} \quad (24)$$

where:

$$\zeta = \sum_{i,j=1}^N C_{\eta_{ij}} [\eta_i^{(k-1)} \cos Q(\pi - |\phi_j - \psi_p|) + \eta_j^{(k-1)} \cos Q(\pi - |\phi_i - \psi_p|)] \quad (25)$$

$$\xi = 2 \sum_{i,j=1}^N C_{\eta_{ij}} \cos Q(\pi - |\phi_i - \psi_p|) \cdot \cos Q(\pi - |\phi_j - \psi_p|) \quad (26)$$

Let:

$$q_p^{(k)} = \underset{\Delta_p}{\text{Min}} \left[(\vec{\eta}^{(k-1)T} - \vec{\eta}_p^{T}) C_{\eta} (\vec{\eta}^{(k-1)} - \vec{\eta}_p) \right] \quad (27)$$

From all the approximation curves $\Delta_p^* a_p(\phi)$, $p=1,2,\dots,M$ we will choose the one with the best fit:

$$q_p^{(k)} \xrightarrow{p} \text{Min} \quad (28)$$

Let:

$$p^* : q_{p^*}^{(k)} = \underset{p}{\text{Min}} \left[q_p^{(k)} \right] \quad (29)$$

Let us denote:

$$q^{(k)} = q_{p^*}^{(k)} \quad (30)$$

The optimum orbit at the k^{th} step of the algorithm will be:

$$\eta^{(k)}(\phi) = \eta^{(k-1)}(\phi) - \Delta_{p^*}^* a_{p^*}(\phi) \quad (31)$$

3. Exit:

Check whether:

$$q^{(k)} < \text{eps} \quad (32)$$

where eps is the chosen accuracy of the approximation.

If this is true the process of successive orbit approximation stops. All the optimum values $\Delta_{p^*}^*$ obtained at the optimization steps will be the most probable linear perturbations in the magnetic elements. If this is false we set $k=k+1$ and return to point 2.

A C++ computer code named LEA (Linear Error Analysis) has been developed for realizing of the described algorithm. The program computes also G. Guignard's Ψ -function-[3], which is another measure for availability of large linear errors and the error spectrum, which allows for assessment of the error (see below). LEA has off-line graphics.

We have carried out a large number of computational experiments on the basis of the JINR-Dubna superconducting heavy ion synchrotron NUCLOTRON-[4]. Fig. 1. shows the case of a large kick 0.327 mrad ($\Delta B/B=5.10^{-3}$) and random errors with $\sigma(\Delta B/B_0=5.10^{-4}$ in all the other dipoles. LEA reveals unmistakably $\varepsilon_2 = 0.4 \text{ mrad}$.

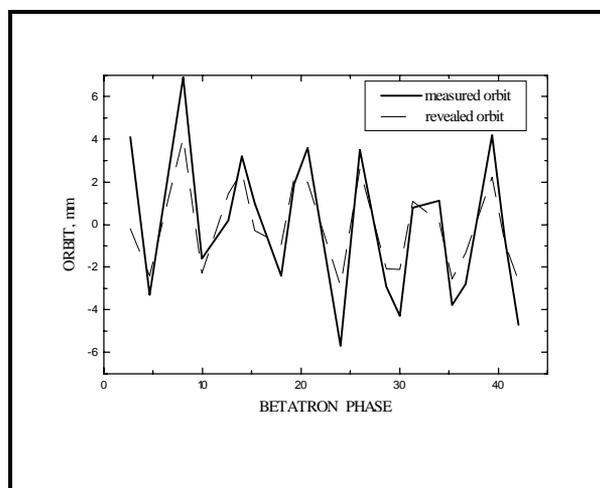


Figure 1. Measured and revealed orbit.

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