KINETICS OF 3-D IONIZATION COOLING OF MUONS

T.A.Vsevolozhskaya, BINP, Novosibirsk, Russia

Abstract

An analyzes of efficiency of 3–dimensional ionization cooling of muons is performed with use of transport equation approach in a scheme with bent lithium lenses and high field dipoles.

As it was already shown [1,2,3], the ionization cooling is efficient for transverse beam emittances and not at all for the longitudinal one. The natural longitudinal decrement is equal to the derivative of mean rate of energy loss with respect to particle energy. In a region of logarithmic grow of ionization loss, the value of $\delta_\parallel$ does not exceed 7% of $\delta_\perp$, whereas at momentum below $\sim 400 MeV/c$, where $\delta_\parallel$ is negative, its absolute value grows fast with energy decrease and becomes of the order of $\delta_\perp$ at $pc \sim 100 MeV$, thus resulting in a strong heating of muon beam instead of cooling.

Meanwhile, with no efficient cooling of longitudinal emittance, the energy spread in a beam is growing, and chromatic aberration in a lens, matching the amplitude functions of low-beta focus at slowing target and that of rather long-wave accelerator, results in sufficient increase in transverse emittance, which restricts from below its value, achievable without an efficient cooling in longitudinal direction.

Thus, the violent redistribution of summary decrement between transverse and longitudinal degrees of freedom has to be produced in a system for ionization cooling by way of creation of transverse gradient of energy loss rate, correlated with dispersion function.

With the use of transport (kinetic) equation for ionization cooling of muons [1] a system of differential equations is got for definition of mean square characteristics of particle beam and all correlations between them: $\langle \mathbf{r}^2 \rangle$, $\langle \mathbf{r}\theta \rangle$, $\langle \theta^2 \rangle$, $\langle \Delta E^2 \rangle$, and so on.

In common the full number of equations is rather big, and solution is to be got numerically. However, for analysis we can consider the free transverse motions and the longitudinal one separately.

For free radial motion, which is extracted through a change of variables $\rho = r - \psi \frac{\Delta E}{pv}$ and $\vartheta_r = \theta_r - \psi' \frac{\Delta E}{pv}$, the equations read:

$$\frac{\partial \langle \rho^2 \rangle}{\partial s} - 2\langle \rho \vartheta_r \rangle - 2(\delta_0 - \delta_r)\langle \rho^2 \rangle = \frac{\sigma_{st}^2 \psi^2}{p^2 v^2}$$

$$\frac{\partial \langle \rho \vartheta_r \rangle}{\partial s} + \delta_r \langle \rho \vartheta_r \rangle - \langle \vartheta_r^2 \rangle + k_r \langle \rho^2 \rangle = 0 \qquad (1)$$

$$\frac{\partial \langle \vartheta_r^2 \rangle}{\partial s} + 2\delta_0 \langle \vartheta_r^2 \rangle + 2k_r \langle \rho \vartheta_r \rangle = \frac{E_k^2 / X_0}{2(pv)^2}.$$

Here ψ and ψ' are the dispersion function and its derivative, $k_r = k + \frac{1}{R_0^2}$ with $k = \frac{e}{pc}\frac{dH}{dr}$; σ_{st}^2 stands for the mean square straggling of ionization loss of energy; $E_k^2 / X_0 = 4\pi e^4 (Z+1)n_e L_c$ defines the mean rate for multiple Coulomb scattering with logarithmic factor L_c, calculated for an effective depth of cooling medium $s_{eff} = \frac{1}{2\delta_0}$; δ_0 stands for the natural transverse decrement $\delta_0 = \frac{\xi}{pv}$, where $\xi = -\left(\overline{\frac{dE}{ds}}\right)_{ion}$, while δ_r is the real radial decrement

$$\delta_r = \delta_0 \left[1 - \psi \left(\eta + \frac{1}{R} \right) \right].$$

Here $\eta = \frac{1}{\xi}\frac{\partial \xi}{\partial r}$ presents the radial gradient of energy loss rate, R is the beam line curvature radius.

The equilibrium radial emittance, got from (1) in a form:

$$\epsilon_{r,eq} = \frac{E_k^2 \beta_r}{2\delta_r (pv)^2 X_0} \left(1 + \frac{\psi^2}{\beta_r^2}\frac{\gamma^2 + 1}{(Z+1)L_c} \right), \qquad (2)$$

gets an increase, caused not only by the decrement reduction, but also by an influence of the energy loss straggling. This puts a limit for dispersion function magnitude $\psi < \beta_r \sqrt{\frac{(Z+1)L_c}{\gamma^2 + 1}}$, which means, for instance, $\psi < 3\beta_r$ at particle momentum $\sim 200 MeV/c$.

The equations for mean square characteristics of longitudinal motion are:

$$\frac{\partial \overline{\Delta E^2}}{\partial s} + 2\delta_L \overline{\Delta E^2} - 2e\varepsilon_0' \overline{\Delta E \Delta s} = \sigma_{st}^2$$

$$\frac{\partial \overline{\Delta E \Delta s}}{\partial s} + \delta_L \overline{\Delta E \Delta s} - \frac{\overline{\Delta E^2}}{pv}\left(\frac{\psi}{R} - \frac{1}{\gamma^2} \right) - e\varepsilon_0' \overline{\Delta s^2} = 0$$

$$\frac{\partial \overline{\Delta s^2}}{\partial s} - 2\frac{\overline{\Delta E \Delta s}}{pv}\left(\frac{\psi}{R} - \frac{1}{\gamma^2} \right) = 0. \qquad (3)$$

Here δ_L denotes the longitudinal decrement:

$$\delta_L = \xi' + \delta_0 \psi \left(\eta + \frac{1}{R} \right), \qquad (4)$$

being a sum of natural longitudinal decrement ξ' and a fraction, transferred from the radial direction.

The equilibrium longitudinal emittance and mean square energy spread are found as:

$$\epsilon_{L,eq} = \frac{\sigma_{st}^2}{2pv\delta_L}\beta_L, \quad \overline{\Delta E^2}_{eq} \cong \frac{\sigma_{st}^2}{2\delta_L}$$

with β_L in expression for ϵ_L being an amplitude function for longitudinal motion, defined as:

$$\beta_L \cong \sqrt{\frac{pv\bar{\lambda}}{\gamma^2}\left[\frac{1}{e\varepsilon_0} + \frac{1}{\xi_0}\left(1 - \gamma^2\frac{\psi}{R}\right)\right]} \quad (5)$$

when degrader and accelerator units succeed each other. Here $e\varepsilon_0$ is an acceleration rate, and $\bar{\lambda} = \frac{\lambda}{2\pi}tg\varphi_a$ with λ being a wave length of accelerating voltage and φ_a – an acceleration phase.

Having got expressions for transverse and longitudinal emittances, we can evaluate the ultimate value of 6-dimensional equilibrium emittance, achievable by the use of bent current-carrying rod focus with optimized parameters. An optimization is applied, first of all, to the rod radius r_0 and to the product $\psi\eta^*(\eta^* = \eta + \frac{1}{R})$. With fixed magnitude of magnetic field at the rod surface H_m, restricted by a value of $10 \div 20T$ [4], the magnitude of r_0 defines the maximum achievable field gradient. From the other side, it is to be large enough to satisfy an evident relation $r_0^2 > \langle r^2 \rangle = \epsilon_{r,eq}\beta_r + \epsilon_{z,eq}\beta_z$. The ultimate value of r_0, with supposed valid a relation $\delta_r \cong \delta_z$, is found as:

$$r_{0,ult} = R_t^2\beta \frac{mc^2\,(Z+1)L_c}{eH_mL_i}\bigg/\left(1 - \frac{\psi\eta^*}{2}\right)$$

Here R_t is a ratio of rod radius to r.m.s. size of particle beam, β denotes the particle velocity in c units, and L_i – the logarithmic factor in the mean rate of ionization loss.

Optimization of $\psi\eta^*$ consists in maximization of a product $\Pi = \left(1 - \frac{\psi\eta^*}{2}\right)^3\left(\psi\eta^* + \frac{\xi'}{\delta_0}\right)$, which the ultimate 6-dimensional emittance is inversely proportional to, owing to the products of decrements and of transverse beta-functions ($\beta_{\perp_{ult}} \propto \sqrt{r_{0,ult}}$). The optimum value of $\psi\eta^*$ appears to be: $(\psi\eta^*)_{opt} = \frac{1}{2} - \frac{3}{4}\frac{\xi'}{\delta_0}$ and decrements correlation: $\delta_r = \delta_z = \frac{3}{2}\delta_L = \frac{3}{8}\delta_\Sigma$.

Resulting expression for the ultimate value of normalized 6-dimensional emittance reads:

$$\epsilon_{ult,N}^{(6)} = \frac{16}{27}\left(\frac{R_t}{H_m}\right)^2\frac{m^4c^4(Z+1)^3L_c^3\gamma(\gamma^2+1)}{e^2M^2\beta^3(L_i+2)^4}\beta_L. \quad (6)$$

Near its minimum, at $pc \cong 100MeV/c$, the value of $\epsilon_{eq,N}^{(6)}$ is estimated as

$$\epsilon_{ult,N}^{(6)} \cong 1.4 \cdot 10^{-5}\left(\frac{R_t}{H_m}\right)^2\sqrt{\bar{\lambda}\frac{1}{e\varepsilon_{\text{eff}}}}\,cm^3$$

with H_m to be taken in $Tesla$, $\bar{\lambda}$ – in cm and $e\varepsilon_{\text{eff}}$ – in MeV/cm. Denoted with $\frac{1}{e\varepsilon_{\text{eff}}}$ here is an expression in
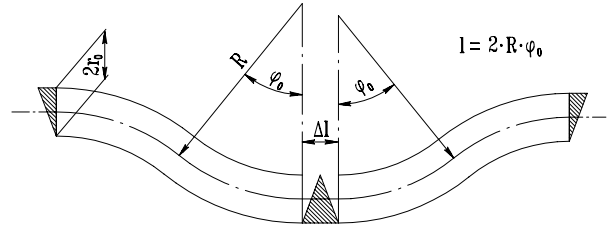

Figure 1: Degrader unit.

square brackets in (5). When $R_t \cong 2.5$ ($r_0 \cong 3.5\sigma_\perp$), $H_m \cong 10$, $\bar{\lambda} \cong 1.6$ ($\lambda \cong 10\,cm$, tg $\varphi_a \cong 1$) and $e\varepsilon_0 \cong 1$, the ultimate value of 6-dimensional normalized emittance $\epsilon_{ult,N}^{(6)}$ is got equal to $\sim 1.2 \cdot 10^{-6}\,cm^3$.

At particle momentum $200MeV/c$, and with more moderate accelerator parameters – $\bar{\lambda} \cong 5cm$ and $e\varepsilon_{\text{eff}} \cong 0.5MeV/cm$ – this value is: $\sim 4 \cdot 10^{-6}\,cm^3$.

As a beam line for 3-dimensional cooling we consider that composed of bent current-carrying rods of lithium with wedges of more heavy material inserted between them. When a length of lithium rod is l, and wedge length (by the bottom) – Δl, this is equivalent to creation of an effective density gradient with a relative value $\eta_{\text{eff}} = \left(\frac{1}{n_e}\frac{dn_e}{dr}\right)_{\text{eff}} \cong \frac{1}{r_0}\frac{n_{e,w}\Delta l}{2ln_{e,li} + \Delta ln_{e,w}}$.

A choice of material for wedges is limited by a necessity of low nuclear number, so that the beryllium and carbon seem to be practically the only candidates.

The length of wedges is also restricted by the betatron function modulation arising. The ratio $\Delta l/l$ practically can not be taken more than ~ 0.2, and thus the product $\psi\eta_{\text{eff}}$ appears to be far from optimum.

It is possible to amplify the effect of wedges, by increasing the dispersion function at them. This is achieved when between two subsequent wedges the beam gets a parallel shift (see fig. 1). When $\nu\varphi_0 = \frac{2}{3}\pi$, for instance, the absolute value of ψ at wedges is by 3 times larger than the average value in a rod R/ν^2. By that, with wedges made of beryllium, the value of $\Delta l/l$ for optimum $\psi\eta_{\text{eff}}$ by $H_0 \sim H_m$ is estimated equal to ~ 0.14 at beam momentum $200MeV/c$.

Cooling of 200 MeV/c momentum muons in a beam line, composed of such degrader units, separated with linear accelerator sections, compensating the mean lost energy, is illustrated in figure 2. Curves 1 and 2 show, accordingly, the radial and axial normalized emittances in $MeV/c\,cm$ and 3 – the longitudinal emittance in $MeV\,cm$ versus the number of degrader-accelerator cells. Thin lines present the numerical solution of system of equations, got from the transport equation [5], while the thick ones – the result of simulation with Moliere formulae used for the Coulomb scattering angle distribution, and Vavilov formulae – for that of the energy loss.

Dashed line shows the ratio R_t, got from the numerical solution, while the dashed squares at figure bottom show
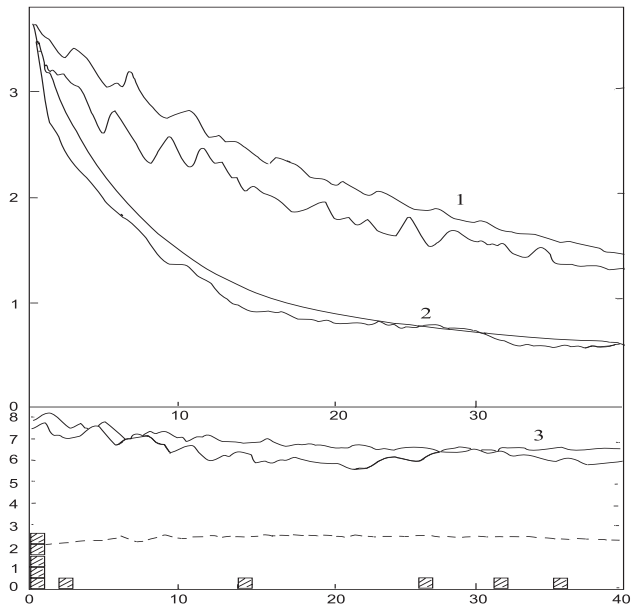
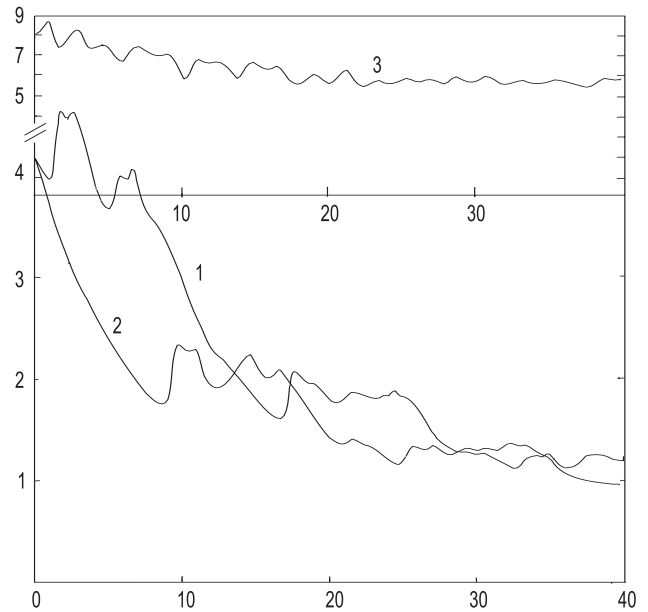Figure 2: Cooling of 200 MeV/c muons.

Figure 3:

the particles (from 100 initial), found lost by the simulation.

Both dipole and maximum focusing fields are taken equal to $10T$, acceleration rate – to $0.5MeV/cm$, and $\bar{\lambda}$ – to $5cm$. The initial r.m.s. momentum spread is $\pm 4.5\%$ and longitudinal coordinate – $\pm 1cm$.

The rod radius is gradually reduced with beam cooling from $\sim 8mm$ down to $\sim 3.5mm$, which provides with almost constant value of R_t, equal to $\sim 2.5(r_0 \cong 3.5\sigma_\perp)$. The rod length is also decreased in proportionality with $\sqrt{r_0}$ to keep constant value $\nu\varphi_0 = \frac{2}{3}\pi$.

The final radial emittance is more than two times larger than the axial one, which proves, that more than half of radial decrement is transferred to the longitudinal direction. The initial longitudinal emittance is taken close to the equilibrium value and thus only slightly decreases with cooling.

The magnitude of normalized 6-dimensional equilibrium emittance, got in the end of 40-cells cooling, is equal to $\sim 4.5 \cdot 10^{-6}\ cm^3$ in good accordance with estimation (6).

To avoid a significant difference in equilibrium values of radial and axial emittances by close to optimum transfer of decrement, the degrader units can be turned around the longitudinal axis through $90°$ several times over length of cooling. Figure 3 shows the result of such a procedure, performed after each eight cells.

The dipole field H_0, providing the beam bend, can be produced in a way, presented in fig.4. Here the lithium rod of radius r_0, is disposed inside a conducting rod of enlarged radius, being shifted from its central line in direction to the bend center. The residual part of enlarged rod, being supplied with a current, produces a homogeneous dipole field inside the lithium rod. By supplying in series, to get the necessary correlation between the dipole and focusing fields, the inner rod is connected to cylindrical envelope,
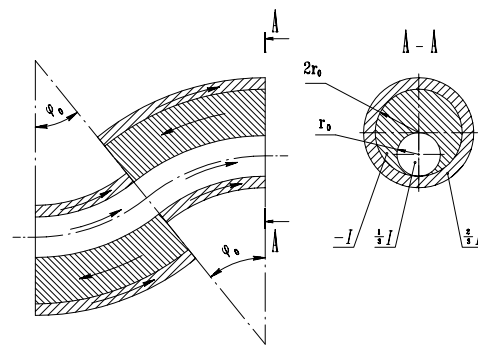

Figure 4: Current carrying rod with dipole field

which shunts necessary part of current.

1 REFERENCES

1. T. A. Vsevolozhskaya,9th Advanced ICFA Beam Dynamics Workshop, Montauk, NY October 15-20, 1995, AIP Conference Proceedings 372, p. 159.
2. A. N. Skrinsky and V. V. Parkhomchuk, *Methods of Cooling of Charged Particle Beams*, Physics of Elementary Particles and Atomic Nuclei, v. 12, $N3$, Moscow, Atomizdat,1981.
3. D. V. Neuffer, Particle Accelerators, v. 14 p.75, 1983
4. G. I. Silvestrov,9th Advanced ICFA Beam Dynamics Workshop, Montauk, NY October 15-20, 1995, AIP Conference Proceedings 372, p. 168.
5. T. A. Vsevolozhskaya, Preprint BINP, being published.