

# SIMULATION OF THE EFFECTS OF LONGITUDINAL BROAD-BAND IMPEDANCES ON AN ELECTRON-COOLED BUNCHED ION BEAM

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## Abstract

Electron-cooling bunching of a  $U_{238}^{92+}$  ion beam at 150 MeV/u was simulated, the space-charge impedance, the broad-band impedance, and the resistive wall impedance being taken into account longitudinally. It has been investigated how the broad-band impedance  $Z_{bb}/n = 5 \Omega$  affects the bunching under the dominant space-charge impedance  $Z_{sc}/n = 1600i \sim 1850i \Omega$ . It has been investigated whether the dispersion relation applied to coasting beams is applicable to cooled bunched beams or not.

## 1 INTRODUCTION

Collision experiments with ion-ion beams and ion-electron beams are planned in the double storage rings (DSR) of RI-beam factory project. Bunched ion beams will be formed to obtain a high luminosity. Previous simulations of electron-cooling bunching of beams[1] showed that even if the betatron tune spread of a cooled bunched beam is over 0.5, the transverse emittance is not enlarged by ions' trapping into resonances, and that the force induced through the transverse space-charge impedance has effects on increase of the transverse emittance in the equilibrium.

In the simulation where coupled-bunch phenomena are neglected, forces induced through the broad-band impedance and the resistive wall impedance have newly been taken into account longitudinally as forces acting on ions. The broad-band resonator model has been used to estimate the broad-band impedance:

$$R_s = \frac{Z_{bb} \omega_r}{n \omega_0}, \quad \omega_r = \frac{c}{b}, \quad Q = 1,$$

where  $\omega_0$  is the revolution frequency,  $\omega_r$  the resonance frequency of the impedance,  $n$  the harmonic number of  $\omega_0$ ,  $c$  the light velocity, and  $b$  the inner radius of the vacuum chamber. As the beam is planned to be injected into every RF bucket, the simulation has been carried out on the assumption that every beam intensity per bunch and every bunch behavior are equal, respectively. The parameters listed in Table 1 have been used as input data of the simulation program. The parameters of the electron cooling have been shown by the simulation to be such that a faint coasting beam is cooled with the cooling times of 5 ms for the momentum spread and 20 ~ 22 ms for the transverse emittances.

## 2 BEAM BUNCHING

A coasting beam is electron-cooled, and after the momentum spread reaches a given spread, the beam is bunched

Table 1: Parameters of the ring, a coasting beam, and the electron cooling.

Ring	
Circumference	260 m
Momentum compaction factor	0.03772
Betatron tune ( $\nu_h / \nu_v$ )	7.38/5.8
Natural chromaticity ( $\xi_h$ )	-35
Twiss parameters at the cooling section	
$\alpha_h^{ec} = \alpha_v^{ec}$	0
$\beta_h^{ec} = \beta_v^{ec}$	7 m
RF harmonics $h$	87
Inner radius of the vacuum chamber $b$	4 cm
Coasting beam	
Momentum spread ( $6 \times \text{rms}$ )	$10^{-3}$
Rms transverse emittance ( $\epsilon_h = \epsilon_v$ )	$10^{-6} \pi$ m-rad
Electron cooling	
Electron current	5 A
Cathode temperature $kT_c$	0.1 eV
Length of the cooling section	3 m
Electron-beam radius at the section	25 mm
Longitudinal magnetic field at the section	1 kG

by applying the RF voltage. The RF voltage is increased until the bunch length reaches a given length. After then, the beam goes to an equilibrium. The simulation of such a bunching process shows the following results.

The transition of the bunching is shown in Figure 1, which is for the simulation case with consideration of the broad-band impedance and resistive wall impedance. There is no remarkable difference between the results of the simulation cases with the consideration and without the consideration. At the end of the simulation, or at 37 ms of the bunching, the bunch length hardly decreases, but the momentum spread decreases still. The peak current  $I_p$ , or the current at the bunch center is 23 mA. The RF voltage is 16.3 kV. The horizontal emittance is larger than the vertical one because the transverse space-charge impedance was taken into account only horizontally. There is no more interest in continuing of the simulation alone, because the intra-beam scattering is not taken into account in the simulation.

### 2.1 Distortion of the RF wave form

Bunch profiles of space-charge dominated beams are known to be approximately parabolic and to be coupled with the distortion of the RF wave form[2]. The bunch profile at 37 ms of the bunching is shown in Figure 2-a. As the RF-like voltage induced through such a parabolic profile is

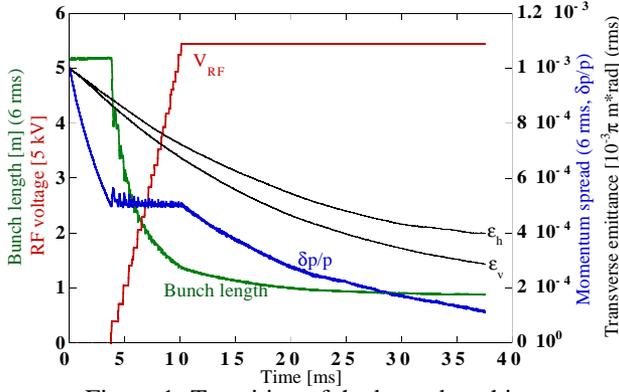


Figure 1: Transition of the beam bunching.

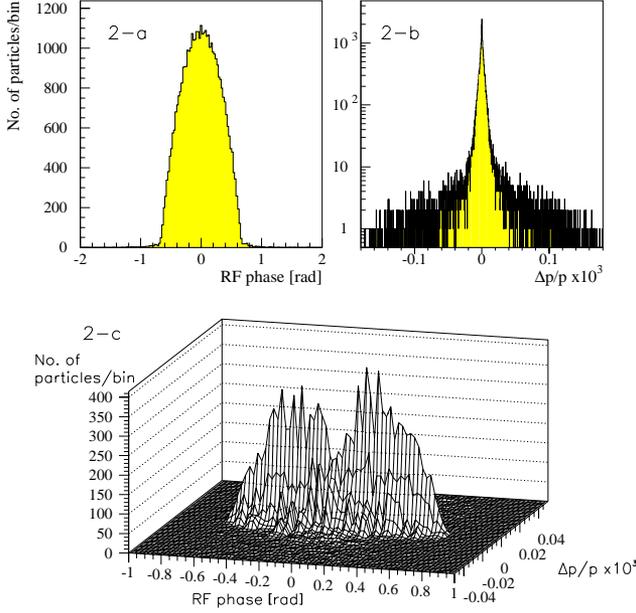


Figure 2: Longitudinal phase-space distribution.

estimated at -14.8 kV, the effective RF voltage seen by ions is 1.5 kV. The ratio of the ordinary RF voltage to the effective voltage is 11, which is considerably high compared with Ref[3].

## 2.2 Momentum spread

The momentum spread at 37 ms is  $0.12 \times 10^{-3}$  as  $6 \times \text{rms}$ . The momentum distribution shown in Figure 2-b has long tails on both sides, and can be represented by a broad Gaussian distribution with  $6 \times \text{rms}$  spread of  $0.18 \times 10^{-3}$  and a core distribution with  $6 \times \text{rms}$  spread of  $0.025 \times 10^{-3}$ , only 7% of the beam belonging to the broad distribution. The core distribution shown in Figure 2-c shows that the momentum spread is the largest at the center along the bunch.

## 2.3 Spectrum of the line density

The beam line density at 37 ms of the bunching has the spectrum shown in Figure 3. For the sake of displaying ap-

parently signals with the amplitude  $> 10^{-3}$ , signals with the amplitude  $\leq 10^{-3}$  are cut out in the Figure where the spectrum is normalized in such a way that the DC component is one. The spectrum has the following characteristics.

1) The main signals at  $\Omega = n\omega_0$  are derived from Fourier components of the parabolic-like-shaped bunch.

2) The signals displayed as 'V' in the Figure means that the line density at the frequency  $n\omega_0$  is modulated with the frequency  $\Delta\Omega = \Omega - n\omega_0$  and the amplitude of some  $10^{-3}$ . The dispersion relation applied to coasting beams shows that neglecting the momentum spread the disturbance with the frequency  $\Omega$  is created on the beam line density by self-induced fields[4]:

$$\Omega - n\omega_0 = \pm n\omega_0 \sqrt{\frac{i\epsilon\eta I}{2\pi\beta^2 E} \frac{q}{A} \frac{Z_{\parallel}}{n}}.$$

The equation shows  $\Delta\Omega = \pm n\omega_0 8.3 \times 10^{-5}$ , the peak current  $I_p = 23$  mA being substituted into  $I$  under the dominant space-charge impedance  $Z_{\parallel}/n = 1850i \Omega$ . On the other hand, the spectrum data show  $\Delta\Omega = \pm n\omega_0 6.8 \times 10^{-5}$  at  $n/h = 255$ . The disagreement is resolved when the average current  $2I_p/3$  within the parabolic-like-shaped bunch is substituted to  $I$ . The agreement suggests that the longitudinal behavior of the cooled bunched beam under the great distortion of RF wave form is similar to that of a coasting beam of the current of  $2I_p/3$ .

The incoherent synchrotron tune for faint beams is 0.016 under the RF voltage 16.3 kV. There is, however, no synchrotron satellite with the amplitude  $> 10^{-3}$  along the main signals. This suggests such a longitudinal behavior, too.

3) Over the frequency of  $n/h = 150$ , the signals expand on  $\Delta\Omega = \pm\omega_0$ , too, because the line density is modulated once per turn as the RF voltage is applied on the beam at a position along the ring.

4) There is no enhancement around  $\omega_r$ , or  $n/h = 23$ . This means that the broad-band impedance is not strong enough to induce instability under the dominant space-charge impedance and the strong electron cooling.

## 2.4 Longitudinal stability

The above suggestion of the similarity encourages one to discuss the stability of the bunched beam at 37 ms by using the Krook model and the dispersion relation applied to coasting cooled beams[5]. The stability diagram is shown in Figure 4, which was made by using the momentum distribution in Figure 2-b and parameters of the average current of  $2I_p/3$ , the cooling time of 5 ms for a faint coasting beam, and  $n = 2001$  at  $\omega_r$ . The diagram predicts that microwave instability is induced with the growth time of 2.7 ms through the broad-band impedance even if the beam is cooled by the electron cooling, and that the frequency shift is  $\Delta\Omega = \pm n\omega_0 6.7 \times 10^{-5}$ . The above simulation results do not support the prediction of the growth time.

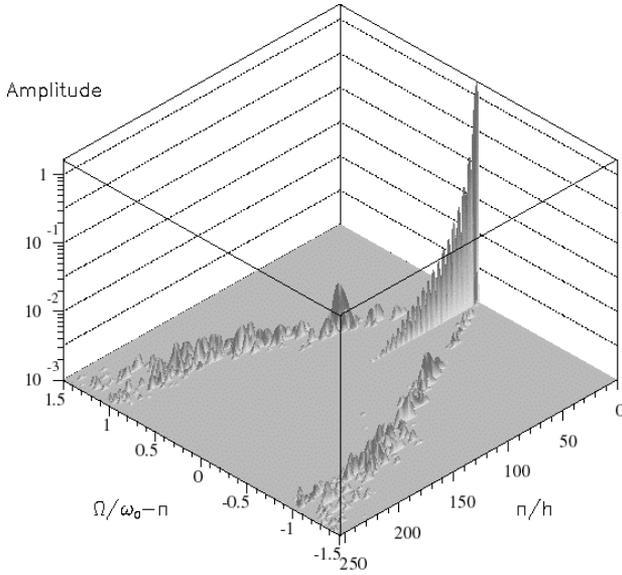


Figure 3: Spectrum of the line density during 100 turns.

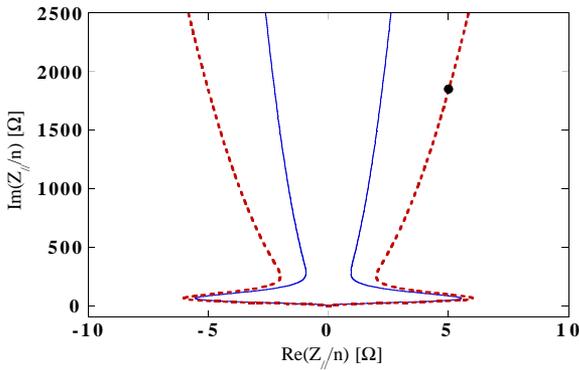


Figure 4: Stability diagram. The solid curve represents the stability limit under the electron cooling of the cooling time of 5 ms. The dashed curve represents the instability of the growth time of 2.7 ms under the cooling and at  $n = 2001$ . The closed circle represents the assumed impedance of the ring at  $n = 2001$ .

### 3 WEAKENING OF THE ELECTRON COOLING

Exposition of ion beams to a large-current electron beam of the cooler for a long time is not preferable, because the radiative electron capture limits the life time of ion beams. In the simulation, the electron current was reduced linearly from 5 A to 0.5 A for 0.8 ms after 34 ms of the bunching. Figure 5 shows the transition after 34 ms. In the simulation case without consideration of the broad-band impedance nor the resistive wall impedance, the momentum spread and the bunch length increase exponentially. On the other hand, in the case with the consideration the spread and the length increase faster in the beginning stage. The effects of broad-band impedance on the bunching process can be seen clearly at the beginning stage, but can less near

the equilibrium.

Near the equilibrium, or at 44 ms the peak current, the bunch length, and the momentum spread are 0.80, 1.8, and 4.8 times, respectively, compared with before weakening of the electron cooling. 26% of the bunch profile is approximately represented by a parabolic distribution of length of 0.44 m, and the rest by a broad Gaussian distribution. As the RF-like voltage induced through the profile is estimated at -16 kV, the effective RF voltage seen by ions is nearly 0 kV on the region except the Gaussian-shaped tails.

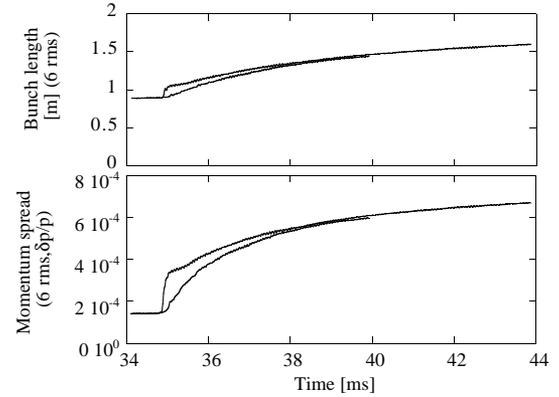


Figure 5: Transition on weakening of the electron cooling. The long curves are for the case with the broad-band impedance considered, and the short curves for the case without it.

## 4 CONCLUSION

By using the simulation of electron-cooling bunching of a  $U_{238}^{92+}$  ion beam, the following is remarked as conclusion. Near the equilibrium the broad-band impedance  $Z_{bb}/n = 5 \Omega$  has little effect on an electron-cooled bunched ion beam at 150 MeV/u under the dominant space-charge impedance  $Z_{sc}/n = 1600i \sim 1850i \Omega$  and the electron cooling of the cooling time of 5 ms or 50 ms. Just after the cooling is weakened, as effects of the broad-band impedance the bunch length and the momentum spread increase faster than in the simulation case without the broad-band impedance considered. The spectrum of the line density predicts that the cooled bunched beam behaviors like a coasting beam horizontally under the great distortion of the RF wave form. The dispersion relations applied to coasting beams predicts the same disturbance frequency of the line density of cooled bunched beams as the simulation does, but do not the same growth time as the simulation does.

## 5 REFERENCES

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