

# A COMPARISON OF 4-ROD AND 4-VANE RFQ-FIELDS

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## Abstract

A 202.5 MHz, 665 keV, H<sup>-</sup> 4-rod Radio Frequency Quadrupole, designed by the Institut für Angewandte Physik at Frankfurt University, will replace the existing DC pre-injector on the ISIS Spallation Neutron Source at R.A.L. The 4-rod RFQ offers some advantages over 4-vane designs in terms of RF properties and ease of manufacture. However, the rod shaped electrodes give cells where the pole tips have constant centre of curvature rather than the constant transverse radius of curvature which is usual for pole tips of vane electrodes. In order to investigate the effects of this difference in geometry, new codes have been written to calculate a multipole expansion of the RFQ potential and simulate the beam dynamics in the resulting field. Results are presented comparing similar 4-rod and 4-vane designs. For the 4-vane design a comparison is made with the Los Alamos code PARMTEQM<sup>1</sup>.

## 1 RFQ POTENTIAL

Following the method of Kapchinski and Tepliakov (KT)[1] it is possible to derive an expression for the potential between the electrodes of a single RFQ cell [2]:

$$U(r, \theta, z) = \frac{V}{2} \sum_p A_{0(2p+1)} r^{2(2p+1)} \cos[2(2p+1)\theta] + \frac{V}{2} \sum_n \sum_m A_{mn} I_{2n}(mkr) \cos(2n\theta) \cos(mkz) \quad (1)$$

Where  $m+n = 2p+1$ .  $V$  is the potential difference between two electrodes,  $k = \pi/L$  and  $L$  is the length of the cell.  $I_{2n}$  is the modified bessel function of order  $2n$  and the  $A_{xx}$  are the multipole coefficients whose values depend on the pole tip geometry.

### 1.1 Two Term Potential

In order to approach RFQ design analytically, only the first term in each series of equation (1) is taken. This results in the two term potential function (TTF):

$$U(r, \theta, z) = \frac{V}{2} [A_{01} r^2 \cos(2\theta) + A_{10} I_0(kr) \cos(kz)] \quad (2)$$

<sup>1</sup> 'PARMTEQM' refers to the LANL RFQ design codes RFQUICK, PARI & PARMTEQM.

Given a cell with aperture ' $a$ ' and modulation ' $m$ ' then analytical expressions exist for  $A_{01}$  and  $A_{10}$ .  $A_{10}$  is often abbreviated to  $A$  and is called the acceleration efficiency. Another useful definition is the focusing force factor  $B$ :

$$B = \frac{q}{m_0 c^2} \lambda^2 A_{01} V \quad (3)$$

Design recipes from KT and others [1][3] allow values of  $A$  and  $B$  to be calculated for the desired beam dynamics in the RFQ.

## 2 POLE TIP GEOMETRY

Equation (2) describes equipotential surfaces with hyperbolic transverse sections. To generate the pure two term potential, electrodes would be required with this same hyperbolic shape. In practice, due to limits on the peak surface electric field and also to allow for ease of manufacture, the geometry of the electrode pole tip deviates from this ideal, often having a circular section. The two types of pole tip in common use are the vane type and the rod type as illustrated in Figure 1.

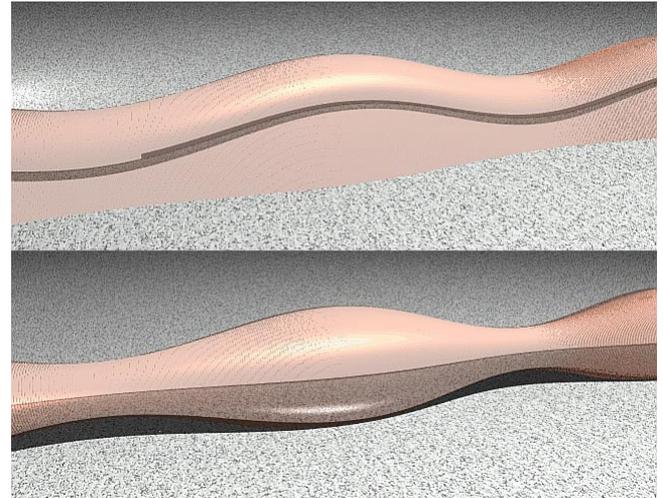


Figure 1: Approx. two periods of pole tip for a vane electrode (top) and a rod electrode (bottom).

The tip of the vane has the same transverse radius along the cell. By contrast, the rod's radius is a function of longitudinal position and modulation. The TTF is a poor approximation to these real electrode shapes so in order to more accurately calculate the resulting field additional terms are required in the potential function.

### 3 EIGHT TERM POTENTIAL

A program, RFQSIM, has been written to calculate the coefficients of a potential function using the eight lowest order terms of equation (1):

$$\begin{aligned}
 U(r, \theta, z) = & A_{01} r^2 \cos(2\theta) + A_{03} r^6 \cos(6\theta) + & (4) \\
 & A_{10} I_0(kr) \cos(kz) + A_{12} I_4(kr) \cos(4\theta) \cos(kz) + \\
 & A_{21} I_2(2kr) \cos(2\theta) \cos(2kz) + A_{30} I_0(3kr) \cos(3kz) + \\
 & A_{23} I_6(2kr) \cos(6\theta) \cos(2kz) + A_{32} I_4(3kr) \cos(4\theta) \cos(3kz)
 \end{aligned}$$

RFQSIM can in principle calculate the coefficients for any geometry for which the boundary can be defined. So far only constant radius vanes and rods have been investigated.

#### 3.1 Calculation of coefficients

The coefficients are calculated by a least mean squared (LMS) error fit to the boundary defined by the known electrode surface. For each point on the surface,  $U$ ,  $r$ ,  $\theta$  and  $z$  are known in equation (6) giving an expression with the coefficients as unknowns. The LMS fit is performed using 10000 surface points for each cell: 100 around the pole tip for each of 100 longitudinal positions. This is a fast method of calculating the coefficients as it doesn't rely on solutions found by over-relaxation. It is also flexible because there are no look-up tables for pre-defined geometries.

#### 3.2 Adjustment of cell parameters.

Given design values of  $A$  &  $B$  the problem is to find a cell geometry that achieves these values. Starting with values of  $a$  &  $m$  from the TTF, RFQSIM iteratively adjusts the cell parameters until  $A$  &  $B$  are within 1% of the design values. For vane electrodes the quantity  $\rho_0/r_0$  is held at the design value as the cell is adjusted. For rods the quantity  $\rho_0+r_0$  is preserved.  $r_0$  is the mid-cell aperture and  $\rho_0$  is the mid-cell transverse radius.

## 4 RESULTS

RFQSIM does time dependant tracking of macro-particles through the calculated field. A 3D, PPI space charge algorithm is used and multiple bunches are tracked. The results presented are for the ISIS RFQ design of Schempp[4] with  $\rho_0/r_0 = 0.83$ . Four 'designs' have been investigated.

Design 1: Coefficient calculation and tracking by PARMTEQM. Vane electrodes.

Design 2: Coefficient calculation by PARMTEQM. Tracking by RFQSIM. Vane electrodes.

Design 3: Coefficient calculation and tracking by RFQSIM. Vane electrodes.

Design 4: Coefficient calculation and tracking by RFQSIM. Rod electrodes.

Table 1 gives the transmission efficiency and final energy for each design with a beam current of 20 mA. Only accelerated beam is included.

Table 1: Transmission efficiency and final energy

Design	Transmission	Final Energy
1	92.7 %	665 keV
2	94.6 %	665 keV
3	89.9 %	665 keV
4	96.8 %	664 keV

When using the same coefficients (Designs 1 & 2), RFQSIM and PARMTEQM are in close agreement. The difference in transmission of ~2% can easily be explained by the different space charge algorithms and different criteria for lost particles. Figure 2 shows the output phase space distributions.

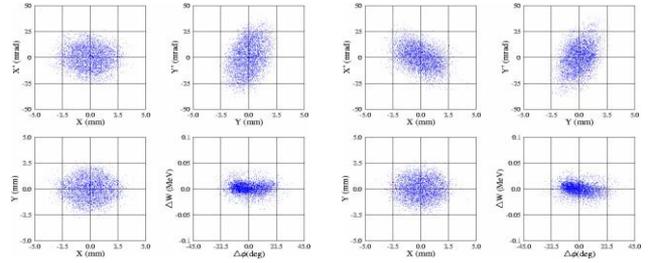


Figure 2: Output phase space distributions. Left - Design 1, Right - Design 2.

There is clearly quite good qualitative agreement between the two codes. As PARMTEQM is considered the 'industry standard' code this is a useful test of the beam dynamics part of RFQSIM.

When RFQSIM calculates the coefficients for vane electrodes (Design 3), the resulting design has a transmission ~5% less than the PARMTEQM design (2). Examination of the coefficients shows that Design 3 has a slightly lower value of  $B$  in the early cells of the RFQ as shown in Figure 3.

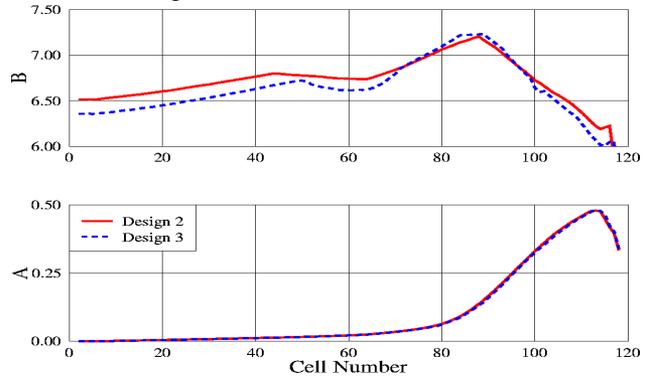


Figure 3: Values of  $A$  &  $B$  for Designs 2 & 3.

This slightly reduced focusing results in a larger beam radius in later cells and accounts for some of the additional beam loss. As the values of  $B$  in Design 3 are

very close to the design values this implies that the PARMTEQM coefficients give a value of  $B$  which is slightly too high in the early cells. Also, in later cells where the modulation is larger, RFQSIM calculates larger high order terms in the potential as can be seen from Figure 4. This may also account for additional loss.

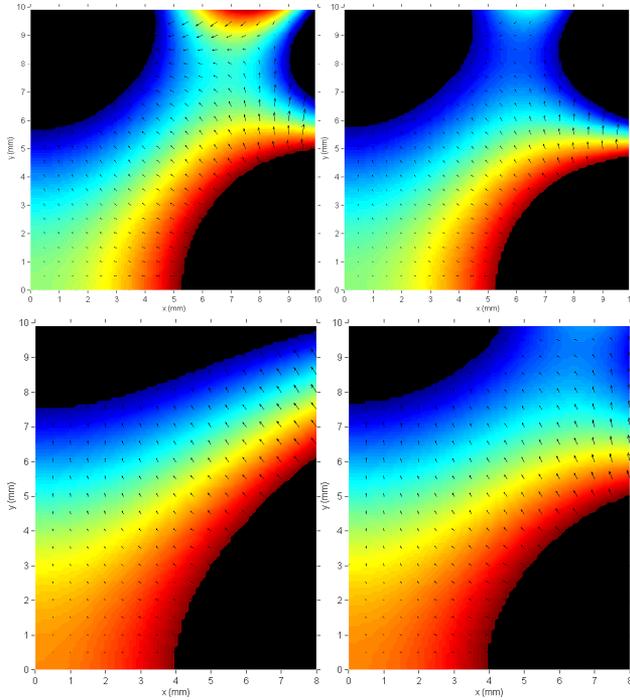


Figure 4: Electric potential and field in cell 40 (top) and cell 114 (bottom) for Design 2 (left) and Design 3 (right).

In Figure 4 the black regions are the electrodes ( $U \geq 1$ ) and the colours represent the potential from  $U=+1$  (red) to  $U=-1$  (blue). The plots are made at  $z=0$ , the entrance plane of the cell. The spurious ‘electrode’ in the top plots results from the truncated series of equation (4). In cell 40 where the modulation is modest, the potential plots are very similar. In cell 114 where the modulation is greater, the PARMTEQM coefficients give electrodes which are closer to the hyperbolic shape of the TTF. That these are vane electrodes can be seen from the plot at lower right where the pole tips have equal radius.

#### 4.1 Comparison with rods

Table 1 shows that Design 4, with rod electrodes, gives the highest transmission. This design has almost identical values of  $A$  and  $B$  to Design 3. One might expect that the more complex geometry of the rods would result in a potential with larger high order terms which in turn might be detrimental to performance. In practice however, this effect is balanced by more favourable cell parameters as shown in Figure 5. The aperture is increased by as much as 5% and the modulation is considerably decreased in the final cells.

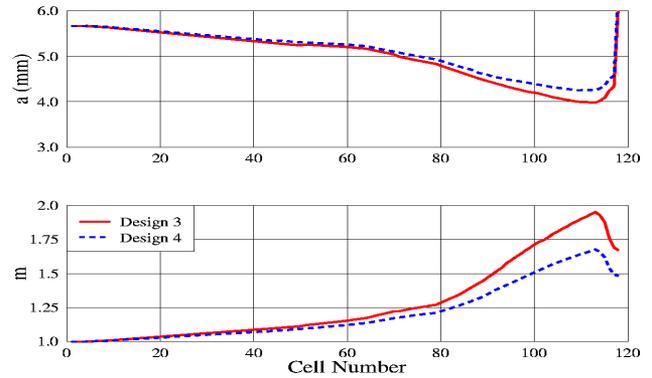


Figure 5: Values of  $a$  &  $m$  for Designs 3 & 4.

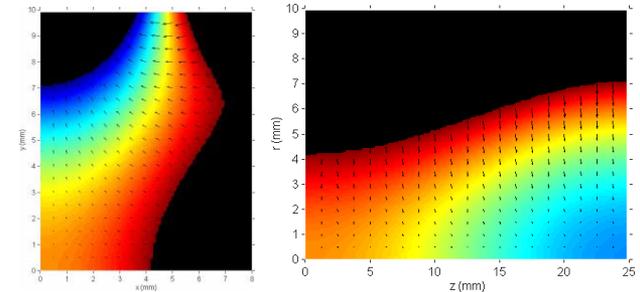


Figure 6: Electric potential and field in cell 114 of Design 4. Left - transverse, Right - longitudinal.

Figure 6 shows examples of the potential in Design 4. The longitudinal plot is at  $\theta=0$  and shows the horizontal electrode. It is clear from the transverse plot that the eight term potential is struggling to approximate the electrode shape away from the beam axis although the effect in the beam region appears to be small. It can be seen that the two pole tips have the same centre of curvature which is correct for rod electrodes.

## 5 CONCLUSIONS

For 4 vane designs, RFQSIM has performance comparable to PARMTEQM. For 4 rod RFQs, these results suggest that the same beam dynamical design can be achieved with a larger aperture and smaller modulation compared to the equivalent 4 vane RFQ. Code validation will form an important part of the ISIS RFQ test programme.

## REFERENCES

- [1] I. M. Kapchinski and V. A. Tepliakov, “Linear Ion Accelerator with Spatially Homogenous Strong Focusing”, *Prib. Tekh. Eksp.*, No 2, 19, 1970.
- [2] C. Biscari, “Computer Programs and Methods for the Design of High Intensity RFQs”, CERN/PS 85-67 (L1), CERN, 1985.
- [3] K. R. Crandall, R. H. Stokes and T. P. Wangler, “RF Quadrupole Beam Dynamics Design Studies”, *Proceedings of the LINAC conference*, Montauk, NY, 1979.
- [4] U. Bessler, A. Schempp, H. Vormann, A. Letchford, “A New High Duty Factor RFQ Injector for ISIS”, these proceedings.