CORRECTION OF VERTICAL DISPERSION AND BETATRON COUPLING FOR SOLEIL STORAGE RING

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Abstract

We study the effectiveness of various correction strategies which minimize different combinations of concerned quantities like vertical dispersion, coupling excitation amplitude, closed orbit, cross talk closed orbit. The objective is to calculate statistically the resulting vertical beam size and to determine the strengths as well as the locations of dipolar correctors and skew quadrupoles.

1 INTRODUCTION

The SOLEIL project [1] is designed with a very small beam emittance to produce very high brilliance beams of synchrotron radiation in the VUV and soft X-ray regions. This requires relatively strong focusing, and magnet errors will therefore introduce large distortions in closed orbit (C.O.) as well as residual dispersion and large betatron coupling. To achieve the SOLEIL performances we need to control the coupling value in a range from few 0.1 % to about 10 %. We have studied the correction of the spurious vertical dispersion and betatron coupling from a statistical point of view, using the BETA-LNS code [2]. The first approach is based on a complete analytical treatment [3], in addition to confirm this method, we compared the results with a statistical analysis applied to sets of random errors. The random error standard deviation used by the both method, are listed in table 1.

Imperfections	1 r.m.s
Δz, Δx quadrupole	1.10^{-4} m
Δz, Δx sextupole	1.10^{-4} m
Δz, Δs dipole	5.10^{-4} m
$\Delta B/B$ dipole	1.10^{-3}
$\Delta\varphi_s$ quadrupole	2.10^{-4} rad
$\Delta\varphi_s$ dipole	2.10^{-4} rad

Table 1 : Main imperfections used

The correction schemes are :

- Correction of the vertical spurious dispersion with dipolar or skew quadrupole correctors,
- Correction of both vertical spurious dispersion and betatron coupling with skew quadrupole correctors.

2 VERTICAL EMITTANCE CORRECTIONS

After the closed orbit (C.O.) correction, the main contribution to the residual vertical dispersion are :

- vertical misalignment of sextupoles
- the kick induced by the vertical C.O. in the sextupoles
- tilted quadrupoles
- the kick induced by the vertical C.O. in the quadrupoles

The other contribution coming from magnet errors are small and also corrected with C.O. correction. Fortunately, contributions of C.O. in both element sextupoles and quadrupoles tend to cancel each other as we reduce the local chromaticity.

The mean vertical emittance induced by the vertical dispersion is then of 1.5 10^{-11} π.m.rad. With a horizontal emittance of 3. 10^{-9} π.m.rad, the mean resulting coupling is $<K^{\Delta D}>= 0.50$ % [4].

To reduce this emittance we tried two methods of vertical dispersion correction :

- correction of both C.O. and residual dispersion with the same 8 dipolar correctors per cell on the BPM
- correction first of the C.O. with 8 dipolar correctors following by the correction of the residual dispersion with a set of 12 skew quadrupole correctors per cell on the same BPM. The skew correctors are located in sextupoles

The latter one has the advantage to set the two sequential stage correction uncoupled since the C.O., at the first order, do not depend on the presence of skew quadrupoles. Results of the two correction schemes are shown on the figure 1.

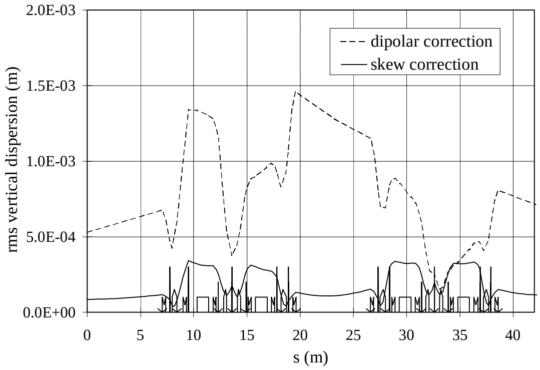

Figure 1 : One standard deviation of the corrected spurious vertical dispersion along a half period.

We reduce a mean coupling induce by the spurious vertical dispersion to 0.03 % using 8 dipolar correctors and to 0.008 % using 12 skew correctors per cell. In this latter case the total resulting mean coupling is reduced from 1.9 % to 0.52 % witch is in quite good agreement compared to the statistical mean coupling of 0.68 %.

From a theoretical point of view, the correction of the dispersion on the BPM doesn't give the minimum value of the emittance. One have to minimize the ΔH_z function on each dipole, however, the resulting corrected dispersion on BPM (or emittance) is already far beyond the measurement possibility.

In figure 2, we plot the corrected r.m.s spurious vertical dispersion in both cases statistical and analytical with 12 skew correctors per cell. The discrepancy between both statistical and analytical results are mainly due to the different kick modeling induced by C.O. in quadrupoles on the vertical dispersion.

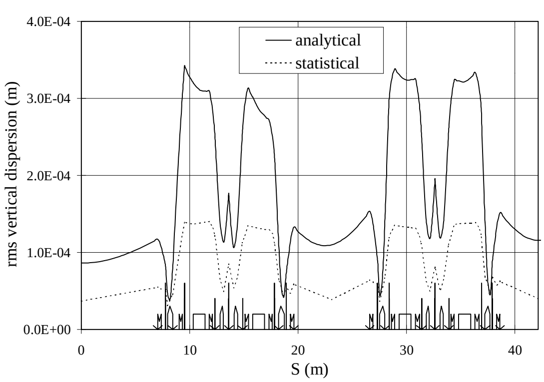

Figure 2 : One standard deviation of the corrected vertical dispersion along a half period with 12 skew correctors per period.

In order to reduce the complete emittance, we applied the last scheme to minimize a weighted function of both vertical dispersion and betatron coupling.

Has it shown in a companion paper [4], the betatron coupling contribution to the vertical emittance has different sources :

- a local one which depend on the cell location
- a global (as the dispersion) which is generated in the dipole by the synchrotron radiation.
- a last global contribution which is a mixed term between the residual dispersion and betatron coupling.

Looking to the analytical formulation, the best option to minimize the betatron coupling consist of minimizing the two functions : $|A|^2$ and $|B|^2$ in each dipole. Since these two functions are independent and composed of two quadratics and also independent terms, we need at least four skew quadrupoles to set them to zero in one point along the cells.

In a practical point of view, we will minimize this two functions at each BPM. This is equivalent of minimizing the cross matrix [5,6].

Comparison will be made with the minimization of the cross talk matrix using also the skew correctors. The cross talk matrix is given by :

$$\frac{\partial x_i}{\partial F_j^z} = \sum_{\text{skew corr}} c_{it}^x N_t \frac{\partial z_t}{\partial F_j^z} + 2\sum_{\text{quad}} c_{ik}^x \delta\Phi_k K_k \frac{\partial z_k}{\partial F_j^z}$$
$$- 2\sum_{\text{sext}} H_l c_{il}^x \left(x_l \frac{\partial x_l}{\partial F_j^z} - z_l \frac{\partial z_l}{\partial F_j^z}\right)$$
$$\frac{\partial z_i}{\partial F_j^x} = \sum_{\text{skew corr}} c_{it}^z N_t \frac{\partial x_t}{\partial F_j^x} + 2\sum_{\text{quad}} c_{ik}^z \delta\Phi_k K_k \frac{\partial x_k}{\partial F_j^x}$$
$$+ 2\sum_{\text{sext}} H_l c_{il}^z \left(x_l \frac{\partial z_l}{\partial F_j^x} + z_l \frac{\partial x_l}{\partial F_j^x}\right)$$

with the following simplification :

$$\frac{\partial x_m}{\partial F_j^z} \approx 0 \quad , \frac{\partial x_m}{\partial F_j^x} \approx c_{mj}^x \quad , m = t, k, l$$
$$\frac{\partial z_m}{\partial F_j^x} \approx 0 \quad , \frac{\partial z_m}{\partial F_j^z} \approx c_{mj}^z \quad , m = t, k, l$$

where X and Z are the orbit, F the dipolar correctors, N the skew correctors, K the quadrupoles , H the sextupoles and $\delta\Phi$ the quadrupole longitudinal rotation error. The indices i and j stand respectively for the BPM and the dipolar correctors. The c_{iq}^y are the usual sensitivity matrix for the orbit given by :

$$c_{iq}^y = \frac{1}{2\sin(\pi\upsilon_y)} \sqrt{\beta_{yi}\beta_{yq}} \cos(\pi\upsilon_y - |\varphi_{yi} - \varphi_{yq}|) \quad y = x, z$$

We search the set of skew correctors which minimize the quantity S, using the eigen values and vectors decomposition algorithm [7] :

$$S = \sum_i \sum_j \left(\frac{\partial x_i}{\partial F_j^z}\right)^2 + \sum_i \sum_j \left(\frac{\partial z_i}{\partial F_j^x}\right)^2$$

and we perform a statistical treatment of the results.

We plot in the figure 3 the resulting optimized mean coupling in both case, analytical and statistical, along the structure. Both are in quite good agreement and we reach a minimum between 2 and 3 10^{-4}. We reduce then by a factor of about 100 the mean coupling. In addition the statistical method allows us to plot the distribution of the mean coupling at the entrance of the structure , as it is shown in the figure 4. In terms of probability, two times the mean coupling enclose about 90 % of the cases. The residual spurious vertical dispersion is also plotted in the figure 5.

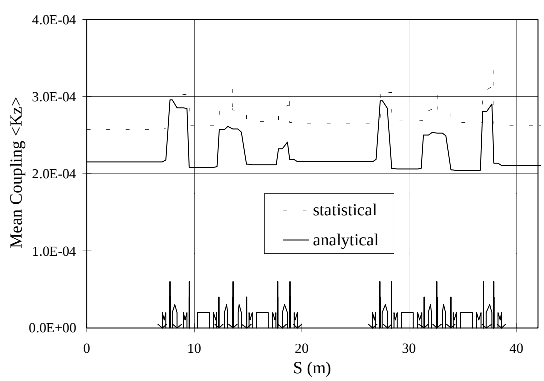

Figure 3 : Corrected mean vertical coupling with 12 skew correctors along a half period.

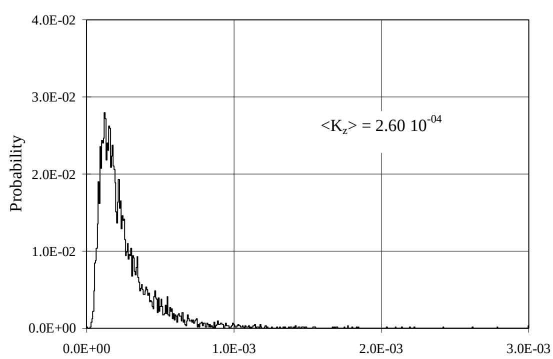

Figure 4 : Distribution function of the vertical coupling obtained by a statistical computation.

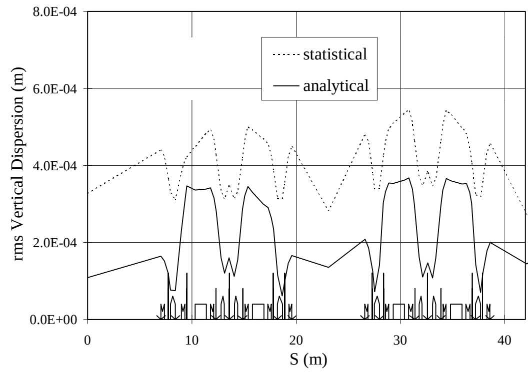

Figure 5 : One standard deviation of the resulting residual vertical dispersion with 12 skew correctors along a half period.

In summarize, we list in table 2 the different mean coupling obtained using the 4 available different correction schemes.

The r.m.s skew corrector strength needed for the correction, in all cases, is about 1.5 10^{-03} m^{-1}.

The comparison of both method are in quite good agreement.

Correction	Mean coupling	
	Analytical	Statistical
C.O.	1.92 %	1.97 %
C.O. + Dz	0.52 %	0.68 %
C.O. + Coupling	0.169 %	0.093 %
C.O. + Dz + Coupling	0.022 %	0.026 %

Table 2 : Results for different correction schemes**.**

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