BOUNDARY INTEGRAL EQUATION APPROACH TO TIME DOMAIN CALCULATION OF ACCELERATOR ELECTROMAGNETIC FIELDS

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Abstract

This paper presents an accelerator wake fields simulation by a boundary integral equation method. It is shown that though there are some possibilities for formulation, available one is uniquely determined to specify the problem. As an example of numerical simulations by the boundary integral equation, transient electromagnetic fields produced by relativistic particles around a single disk structure are treated in this paper. And the result is compared with that of FIT method.

1 INTRODUCTION

A boundary integral equation approach to time domain calculations of accelerator electromagnetic fields gives us possibilities of analytical treatment of smooth trajectories of the relativistic charged particles and their Lienard-Wiechert fields. This is suitable for treatments of the cocherent synchrotron radiation and other phenomena of strong interactions between particles and fields. At the same time, it is however known that this approach has the following serious difficulties in practical use,

- often happened numerical instabilities
- too much storage memory & CPU time

These difficulties themselves are not only for wake fields calculations but also for general electromagnetic wave and still remain as one of biggest problems in electromagnetic fields simulation technology even now. [1-3] For spacial cases, those difficulties can be removed by individual devices depending on each case. For example, in thin wire scattering phenomena,a analytical evaluation of the integral equation is partially possible, it can remove noisy factor from the integral equation and produces stable time domain solutions.[4] We discuss wake fields around torus topology conductors in this paper. To introduce this assumption, concrete scheme can be uniquely determined.

2 FORMULATION

Governing equations of the wake fields are Maxwell's equations but here we use the inhomogeneous wave equation for the scalar and vector potentials because the wave equations match to the boundary integral formulation. One of the biggest problems in use of the potentials for numerical simulations is the boundary conditions. As to perfect conductors, the following conditions are possible, [5]

$$\phi = 0 \quad (1)$$

$$\mathbf{A} = 0 \quad (2)$$

these conditions are derived from a kind of gauge transformation, accordingly the boundary integral equation becomes as follows,

$$\phi = \phi_{LW} + \frac{1}{4\pi}\int \frac{dS'}{|\mathbf{x} - \mathbf{x}'|}\left.\frac{\partial \phi}{\partial n}\right|_{\tau} + \frac{\partial G}{\partial t} \quad (3)$$

$$\mathbf{A} = \mathbf{A}_{LW} + \frac{1}{4\pi}\int \frac{dS'}{|\mathbf{x} - \mathbf{x}'|}\left.\frac{\partial \mathbf{A}_t}{\partial n}\right|_{\tau} - \nabla G \quad (4)$$

where the unknowns, the normal derivatives of the potentials in the integrands are evaluated at the retarded time $= t - |\mathbf{x} - \mathbf{x}'|/c$ and then the unknowns at different times are independent each others. ϕ_{LW} and $\mathbf{A}_{LW}$ denote the Lienard-Wiechert potentials and G is a function which is generated at the gauge transformation and does not contribute field strength **E** and **B**. For the conditions (1) and (2), the electromagnetic field strengths directly related to the normal derivative of the potentials as follows,

$$\frac{\partial \phi}{\partial n} = -\mathbf{E} \bullet \mathbf{n} = \frac{\sigma}{\varepsilon_0} \quad (5)$$

$$\frac{\partial \mathbf{A}}{\partial n} \times \mathbf{n} = \mathbf{B} \times \mathbf{n} = \frac{\mathbf{K}}{\varepsilon_0 c^2} \times \mathbf{n} \quad (6)$$

where **n** is a unit normal vector of the surface, σ and **K** are the surface charge and current density. Therefore we have one more equation for the normal derivative of the potentials, the surface charge conservation law,

$$\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial n}\right) + c^2 \,\mathrm{div}\left(\frac{\partial \mathbf{A}_t}{\partial n}\right) = 0 \quad (7)$$

One possibility to delete the uncertain factor G in Eqs(3) and (4) is making field strength as follows,

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$$\mathbf{E} = \mathbf{E}_{LW} - \frac{1}{4\pi}\int \frac{dS'}{|\mathbf{x}-\mathbf{x}'|}\frac{\partial}{\partial t}\left(\frac{\partial \mathbf{A}_t}{\partial n}\right)\Bigg|_{\tau} + \frac{1}{4\pi}\int dS'\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\,\partial t}\right)\frac{\partial \phi}{\partial n}\Bigg|_{\tau} \tag{8}$$

This is just same equation as the standard MoM integral equation. For the axisymmetric system, the number of unknowns are two, absolute values of $\partial\phi/\partial n$ and $\partial \mathbf{A}_t/\partial n$ or σ and **K** for each nodes and we now have two equations (7) and (8). Therefore solving the time domain problem is basically possible. One however should carefully consider on the calculation of **K**. For the conductor which has torus topology, the above equations can not uniquely determine the surface current, because of arbitrariness of constant component (see Fig.1). This situation is included in the following fact. To do numerical simulation, we have to discretize the boundary integral equation (8) as follows,

$$\mathbf{E} = \mathbf{E}_{LW} + \sum_{j,k}[H'_{ijk}]\left[\frac{\partial \mathbf{A}_{tj}}{\partial n}\right]_{t-k\Delta t} \sum_{j,k}[G'_{ijk}]\left[\frac{\partial \phi_j}{\partial n}\right]_{t-k\Delta t} \tag{9}$$

then the arbitrariness of **K** appears as singularity of the matrix H in (9). Accordingly, we can calculate — to use the integral equation and **K** should be determined by the continuity equation (7). Then the arbitrariness appears again as the constant component of **K** or $\partial \mathbf{A}_t/\partial n$ because div-oparator deletes constant poloidal component of **K**. Here since we assumed that the material is the perfect conductor, the total poloidal component current should be zero because there are no magnetic field inside the material.

$$\oint_{poloidal} \frac{\partial \mathbf{A}_t}{\partial n}\, dl = 0 \tag{10}$$

To use this condition, the arbitrariness can be removed.

3 NUMERICAL SIMULATION

Discretized boundary integral equation has the structure of Fig.2 because of the property of the retarded time. Many matrices should be stored during the computing and too much memory is required for the computer. In this paper, we treat the transient electromagnetic fields produced by the charged particle around the single disk . Numerical model is shown in Fig.3. In this model, the number of the matrices is 74 and the size is 72x72. The bunch length and velocity are taken to be 3mm and 99.994 percents of the light velocity, respectively. Surface current behavior in time domain is shown in Fig.4. Simulation result by FIT method for the same parameters is shown Fig.5. The surface current behaviors are almost coincided each other but un-negligible differences still exist. Electric field profile in the cross-section at the time marked at Fig.6 and the wake potential is shown in Fig.7.

4 SUMMARY

This paper has presented an accelerator wake fields simulation by a boundary integral equation method. It has been shown that though there are some possibilities for formulation, available one is uniquely determined to specify the problem. As an example of numerical simulations by the boundary integral equation, transient electromagnetic fields produced by relativistic particles around a single disk structure have been treated in this paper. It has been shown that the surface current behaviors are almost coincided with that of FIT method but un-negligible differences still exist.

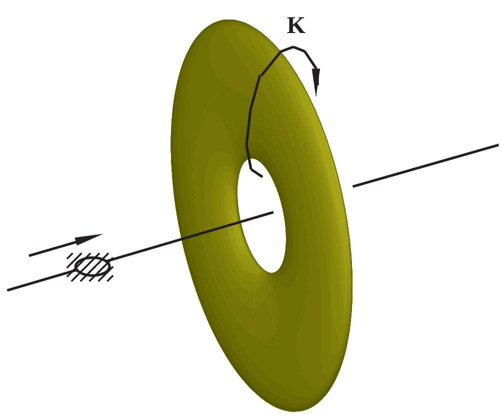

Fig.1 surface current on torus conductor

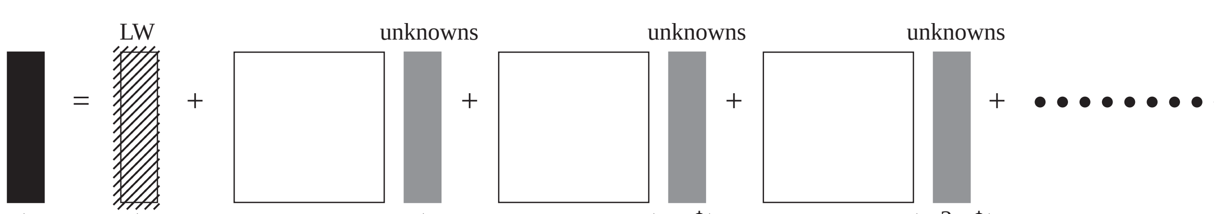

Fig.2 configuration of discretized boundary integaral equation

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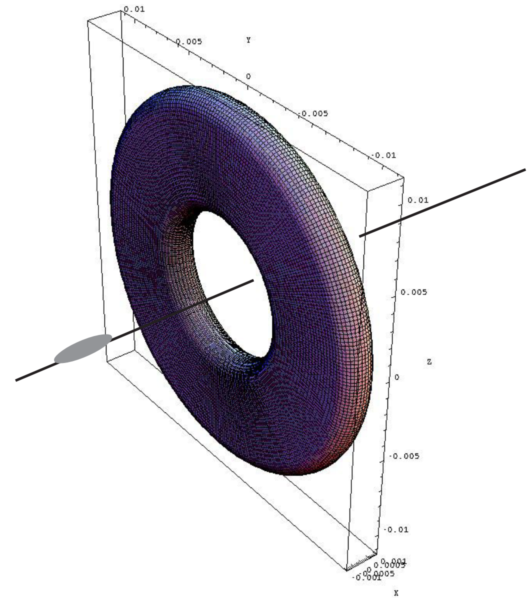

Fig.3 numerical model of single disk

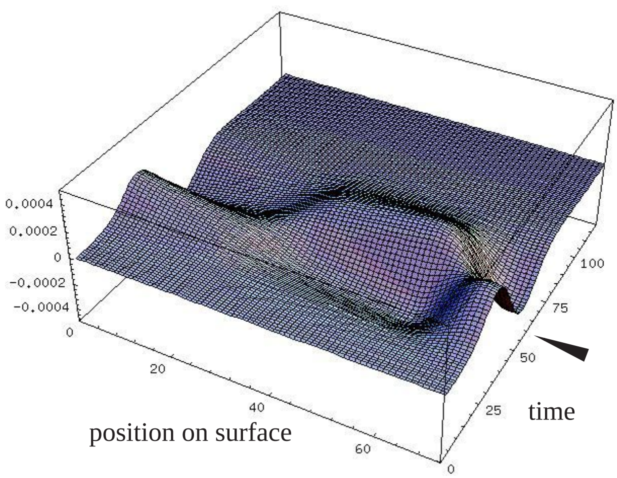

Fig.4 poloidal surface current on single disk (BEM)

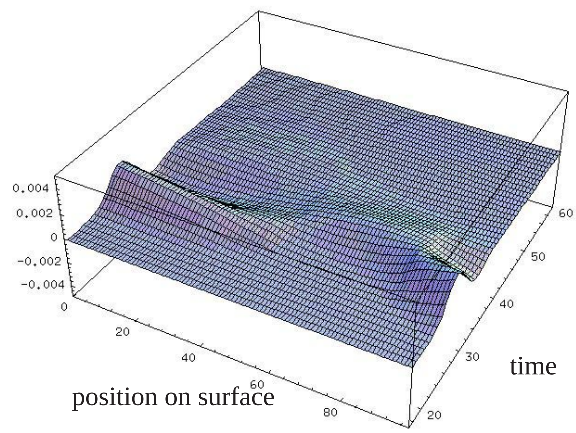

Fig.5 poloidal surface current on single disk (FIT)

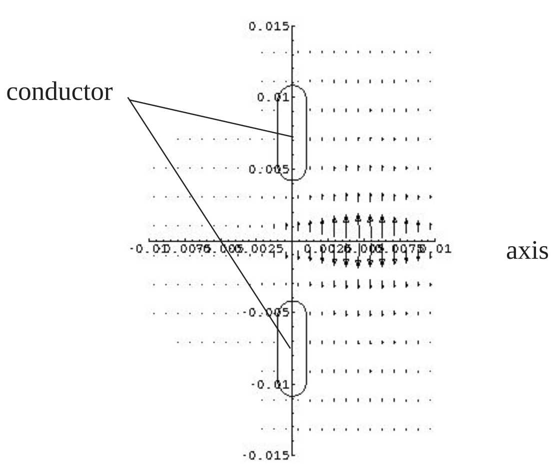

Fig.6 electric field profile

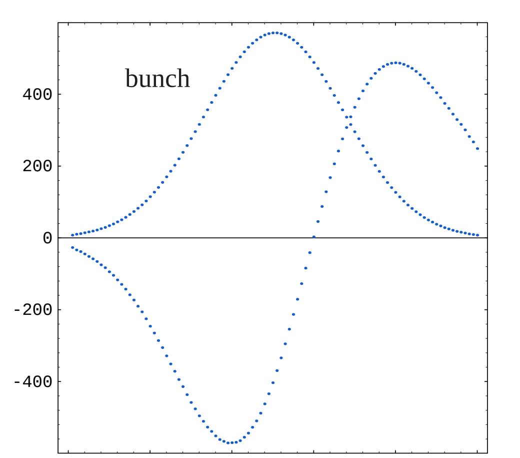

Fig.7 wake potential