FILLING AND BEAM LOADING IN TESLA SUPERSTRUCTURES *

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Abstract

The new design of the accelerating structures for TESLA, the so-called superstructures composed out of several multicell cavities, is studied. Here a superstructure is composed out of four seven-cell superconducting resonators, e.g. in total 28 coupled cells. A central question is the filling characteristic of the superstructure since each is fed by one coupler only. The filling and the beam loading of the superstructure is studied in this paper with a model taking into account the lowest 28 monopole modes. Their calculation is done with MAFIA. The large number of cells and the fact that the eigenvalues are clustered puts some difficulty to the mode computation which is carefully carried out to get reliable results. The input coupler is simulated by a driven current loop. This model leads to a coupled set of differential equations which is set up and solved in time.

1 INTRODUCTION

The effective accelerating gradient of future linear colliders should be as high as possible in order to reduce investment costs. For given gradient limits of the accelerating structures only shortening of passive elements allows for a higher effective gradient and a shorter overall length. Therefore a structure of 4 coupled 7-cell cavities has been proposed by Sekutowicz [1]. This also will give the advantage of less expensive construction of cryomodules, a much smaller number of input couplers and a simplified tuning. Each of this so-called *superstructures* has to have only one input coupler, which leads to the question of a proper filling and refilling of the complete structure with field energy within the given temporal limits. This question has been studied theoretically with an approach based upon eigenmode expansion (compare for example [2], [3], [4]). The eigenmodes of the superstructure have been calculated with MAFIA [5] in rotational symmetry. The transient fields were expanded in the set of the first 28 eigenmodes, i.e. the fundamental passband of the 4×7-cell structure. The coupling to a transmitter was modelled as a short on-axis wire with a driving current in the left beam pipe near the first cavity. The transmitter resistance leads to a much higher damping than wall losses (the external Q is much lower then the unloaded one), therefore the latter ones were neglected. This resistance furthermore induces a mode-to-mode coupling. The beam is calculated as a sequence of equidistant Dirac pulses with infinite stiffness. According to this model the transient fields during build-up and a sequence of bunch transversals were calculated in a semi-analytic manner (sect. 3). Details and results of the eigenmode determination with MAFIA are described in sec. 2.

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2 CALCULATION OF EIGENMODES

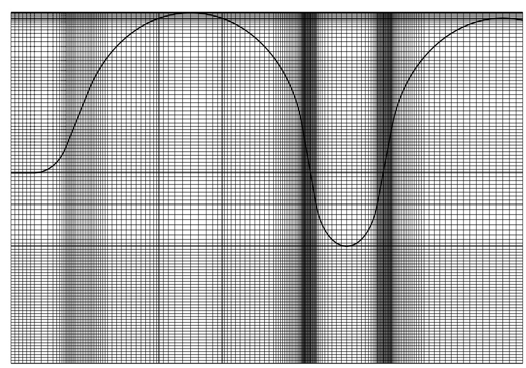

Figure 1: Detail of grid and cavity cross section

For the eigenmode calculation a 2×7-cell structure with an additional $\lambda/2$ beam pipe at the left hand side has been discretized. The calculation was performed in two subsequent runs with electric and magnetic boundary conditions at the right border, thus exploiting the symmetry of the full 4×7-structure with respect to this plane. The geometry data were provided by J. Sekutowicz, DESY [6]. The maximal radius of the outermost left cell was increased from $r_{max} = 104.935$ mm to 105.044 mm in order to improve the flatness of the accelerating π-0-mode. A special procedure was applied to set up the cartesian grid which is used by MAFIA. For every radial line, this procedure calculates all z-coordinates of intersections with the cavity boundary and places z-lines at this positions. This avoids corners in the material shape and reduces the number of mesh lines needed for a good approximation of the geometry (Fig. 1). The accelerating mode was found at 1.300640924 GHz; its profile is the steady state case of fig. 3. The maximal field in the outermost cells showed to be 4.4 % lower than the overall maximum. Some further improvement of the field flatness may be achieved by slight variation of r_{max}.

3 TRANSIENT FIELDS

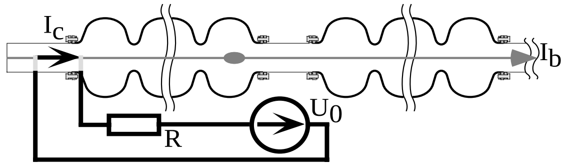

Figure 2: Cavity chain with transmitter (U_0, R) driven current I_c and beam I_b

The time dependent fields are expanded in the set of the $n = 28$ (normalized) modes $\vec{E}_\nu$ of the fundamental passband. The truncation of higher TM_0-type modes is justified by the large frequency gap to the following passband that arises only above 2 GHz.

$$\vec{E}(\vec{r},t) = \sum_{\nu=1}^{n} a_\nu(t)\vec{E}_\nu(\vec{r}) \tag{1}$$

The cavity chain is excited by the beam current I_b and a transmitter driven current I_c. The latter one is located near the beginning of the first cell and - for reasons of simplicity - on axis. This has the same effect as a peripheral input coupler, assuming a common factor between field amplitudes of both places of excitation. The external transmitter has an ideal source with voltage U_0 and a resistance R. This resistance is the only damping within the system which also induces a mode coupling. The total current distribution is expanded in the electrical eigenfields. Applying (1), Maxwells Equations, orthonormality of the eigenmodes and Kirchhoffs law for the external circuit yields a system of coupled and damped oscillators:

$$\begin{aligned}\left(\omega_\nu^2 + \frac{\partial^2}{\partial t^2}\right) a_\nu + \frac{1}{R}\sum K_\nu K_m \dot{a}_m &= \\ = \quad -\frac{\partial}{\partial t}\left(c_\nu + \frac{K_\nu}{R}U_0(t)\right) &=: -i\omega_\nu s_\nu \end{aligned} \tag{2}$$

$$\text{with} \quad K_\nu = -\int_{\text{curr.path}} \vec{E}_\nu(\vec{r})\,\mathrm{d}\vec{r} \quad , \tag{3}$$

$$c_\nu = \int_{V_R} \vec{j}_{\text{beam}} \cdot \vec{E}_\nu(\vec{r})\,\mathrm{d}V \quad . \tag{4}$$

Introducing variables $b = \frac{1}{i\omega}\dot{a}$ this is transformed to a first order system with twice the dimension:

$$\begin{pmatrix} \dot{a}_1 \\ \dot{b}_1 \\ \vdots \\ \dot{a}_n \\ \dot{b}_n \end{pmatrix} - \begin{pmatrix} 0 & i\omega_1 & \dots 0 & 0 \\ i\omega_1 & -\frac{K_1^2}{R} & \dots 0 & -\frac{K_1 K_n \omega_n}{R\omega_1} \\ \vdots & \vdots & \ddots \vdots & \vdots \\ 0 & 0 & \dots 0 & i\omega_n \\ 0 & -\frac{K_n K_1 \omega_1}{R\omega_n} & \dots i\omega_n & -\frac{K_n^2}{R} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \\ \vdots \\ a_n \\ b_n \end{pmatrix} = \begin{pmatrix} 0 \\ s_1 \\ \vdots \\ 0 \\ s_n \end{pmatrix}$$

or

$$\dot{\vec{v}} - \underline{\underline{M}}\vec{v} = \vec{s} \quad . \tag{5}$$

The solution $\vec{v}$ is composed of eigenvectors $\vec{v}_j$ with time dependent factors $u(t)e^{\lambda_j t}$, λ_j being the corresponding eigenvalues of the coefficient matrix:

$$\vec{v}(t) = \sum_{j=1}^{n} u_j(t)e^{\lambda_j t}\vec{v}_j \quad . \tag{6}$$

Cramers rule gives:

$$u_j(t) = \frac{\det(\underline{\underline{v}}_j)}{\det(\underline{\underline{v}})} \quad , \tag{7}$$

$$\underline{\underline{v}} = (\vec{v}_1, \dots, \vec{v}_n), \quad \underline{\underline{v}}_j = (\vec{v}_1, \dots, \vec{w}_j, \dots, \vec{v}_n) \quad . \tag{8}$$

and $\vec{w}_j$ results by integration from the inhomogenity $\vec{s}$:

$$\vec{w}_j = \int_{\tau=0}^{t} e^{-\lambda_j \tau}\vec{s}(\tau)\,\mathrm{d}\tau \quad . \tag{9}$$

Except moments with a bunch being inside the cavity $\vec{w}_j$ is found analytically, as long as the exchange integrals of field and current are known. In case of the transmitter current these integrals equal K_ν (already used when setting up the matrix) resp.

$$\int_{\tau=0}^{L_{\text{Res}}/c_0} e^{-\lambda_j \tau} E_{z,\nu}(c_0\tau)\,\mathrm{d}\tau \tag{10}$$

in case of the beam current. They are calculated from the MAFIA field profiles. The procedure is described in more detail in [7].

In the TESLA scheme 1130 bunches of $5.7267 \cdot 10^{-9}$ As charge are foreseen following each other in a distance of 919 rf periods. The injection of the first bunch happens at rf period 760336 (584.6 μsec) at half the unloaded steady state voltage. From this an external $Q = 3.446120 \cdot 10^6$ follows. This data allows to calculate the transmitter resistance, which of course is only valid for a certain coupling, defined by length and position of the current path. For the calculations in Fig. (3) a (slightly to low) Q of $3.433810 \cdot 10^6$ was found from the eigenvalue. This results in a voltage decrease of about 0.55 % in the first, 0.37 % in the 7th and 0.8 % in the last cell (comp. Tab 1) from the first to the last bunch.

E_z/(MV/m)	cell 1	cell 7	cell 28
bunch 1	47.352	48.009	47.426
bunch 1130	47.089	47.831	47.044

Tab 1: Voltage decrease

The voltage was calibrated to be 25 MV/m inside the resonators at first injection, equal to 80.6756 MV in total. This total gradient was plotted for each bunch with an improved Q (Fig. (4)). The remaining deviations appear as jitter of $< \pm 1000$ V.

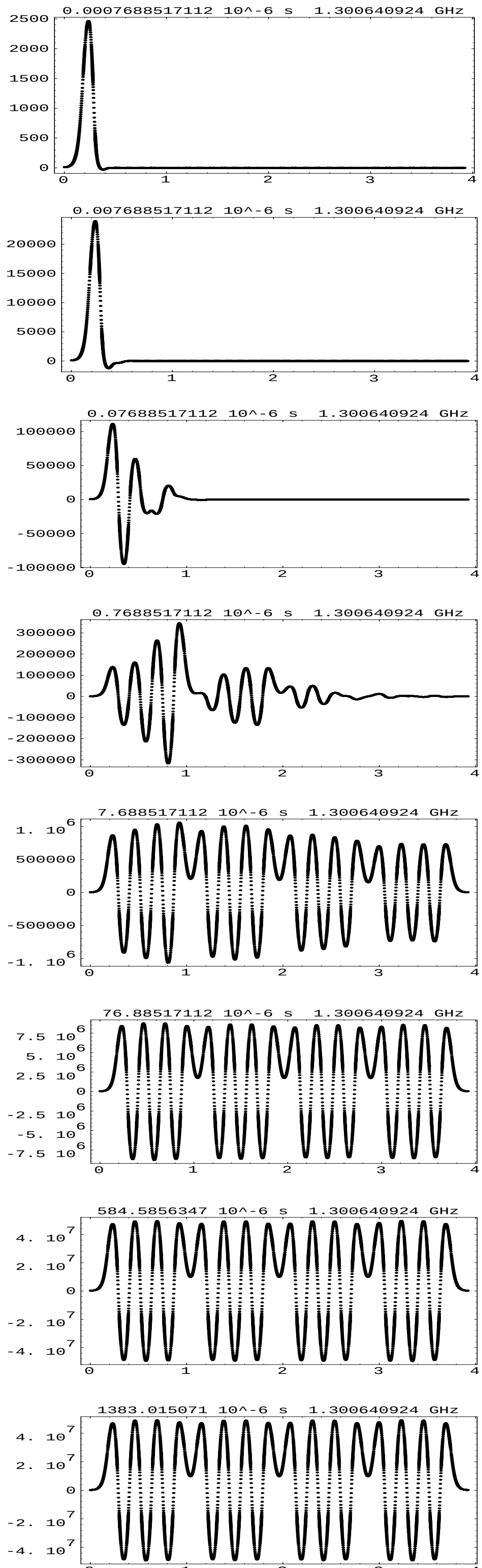

Figure 3: Sequence of field profiles ($E_z/(\mathrm{V/m})$ vs. z/m) at increasing time (10^0 to 10^5 rf periods; first, last bunch).

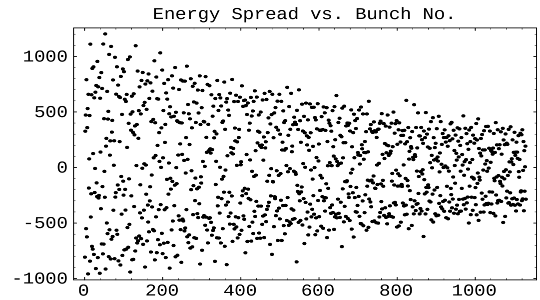

Figure 4: Deviation of integrated accelerating field from average value of 80.6756 MV vs. bunch number.

4 CONCLUSIONS

The model applied proved to be an appropriate tool to describe the transient fields in beam- and transmitter-driven cavities of high unloaded Q-values. In spite of the large problem size MAFIA calculations necessary to set up the analytical system can be done with reasonable effort giving reliable results. The most difficult aspect is to control the field flatness of the accelerating mode which is very sensitive to geometrical variations. The simulations indicate that filling and refilling of the proposed superstructure can be completed within the given time structure - even at the cavity end opposed to the coupler.

5 ACKNOWLEDGEMENT

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6 REFERENCES

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