INCREASING THE LIFE TIME OF SR SOURCES BY RF PHASE MODULATION

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1 INTRODUCTION

The proposed lattice for a third generation synchrotron light source, ASTRIDII [1], is design to provide a low electron emittance of 5 nm at 1.4 GeV and 1 nm at 0.6 GeV. In such low emittance SR sources a key concern is the decrease in life time due to enhanced scattering, the Touschek effect. In principle an improvement in the beam life time can be obtained by increasing the emittance coupling [2] or by increasing the longitudinal bunch dimension, however, both methods affect the whole bunch and have an adverse effect on the light quality [3].

To address these problems we have developed a method of RF field phase modulation. The nature of this method is based on the parametrization of the radiation damping decrement in the longitudinal plane. The RF phase modulation results in a particle redistribution in the longitudinal plane. It is leading to a lower density in the bunch core and consequently a lower probability of the intrabeam scattering. The equilibrium longitudinal emittance is determined by the balance of the quantum excitation and the radiation damping processes. We modify this balance for the growth of the phase area, occupied by the particles in the central part of the longitudinal plane. We show that phase modulation with a frequency three times that of the synchrotron oscillation excites the core growth with an increment greater than the decrement of the radiation damping process. The model is compared to measurements made at the ASTRIDI storage ring, which gave a factor of two increase in life time.

2 PARAMETRIZATION OF RADIATION DAMPING DECREMENT

For a beam particle in a circular accelerator, when the RF phase is modulated with an amplitude ψ_m and a frequency ω_m, the equations of the motion are given by

$$\frac{dW}{dt} = eU\sin[\varphi + \psi_m\cos(\omega_m t + \theta)] - \tilde{W}_r , \qquad \frac{d\varphi}{dt} = -\frac{1}{2\pi}\frac{\alpha\omega_r h}{p_o R_o}(W - W_o) \qquad (1)$$

where U is the peak RF voltage, φ is the particle phase angle relative to the RF field zero with fixed phase, θ is the initial phase of the phase modulation, α is the momentum compaction factor, ω_r is the revolution frequency, h is the harmonic number, p_o is the momentum of the reference particle, R_o is the average radius of the orbit and $\tilde{W}_r$ is the radiation energy loss per turn. At this point, we introduce the concept of the reference particle, for which energy loss due to synchrotron radiation is exactly compensated by the energy gain:

$$\frac{dW_o}{dt} = eU\sin\varphi_o - \tilde{W}_{ro} = 0 , \qquad (2)$$

In reality such a particle does not exist, since the perturbation $\psi_m\cos\omega_m t$ has a coherent character. The particle with the minimum amplitude oscillation we shall call the quasi-synchronous particle and it's average phase, as the reference phase.

For an explanation of the parametrization effect of the radiation damping we will make some simplifications. First, the damping time is expected to be much longer than the period of synchrotron oscillation. This means that it does not affect the eigen frequency of the oscillator and we can first find the solution without damping and then determine the damping by integration of the equation motion. Secondly, since we are interested in the redistribution of density in the centre of the bunch we can consider the problem in the linear approximation and then investigate the contribution of non-linear terms.

Thus, the equation for a particle oscillation in the linear approximation without damping can be written in the form:

$$\ddot{\varphi} + \Omega_s^2\varphi = -\psi_m\Omega_s^2 e^{i(\omega_m t + \theta)} , \qquad (3)$$

where $\Omega_s = \left(\frac{eU\alpha\omega_r h}{2\pi p_o R_o}\right)^{1/2}$ is the synchrotron frequency for the small amplitude oscillation. The solution of this equation for the non-resonant case ($\omega_m \neq \Omega_s$) can be represented through the sum of the homogeneous and inhomogeneous equation solutions:

$$\varphi(t) = \psi_i e^{i(\Omega_s t + \varphi_i)} + \psi_m \frac{\Omega_s^2}{\omega_m^2 - \Omega_s^2} e^{i(\omega_m t + \theta)} . \qquad (4)$$

The first of the summands describes an incoherent motion and it is defined by the initial phase φ_i and the oscillation amplitude of unperturbed motion ψ_i. The second is responsible for the coherent motion and describes the motion of the bunch as whole.

The energy deviation of a particle $\Delta W = W - W_o$ is proportional to $\frac{d\varphi}{dt}$ (see eq. 1) and it follows from equation (4), that it is modulated by the harmonic with frequency $\delta\omega = \Omega_s - \omega_m$:

$$W - W_o \propto \left[1 + \frac{\Omega_s^2}{\omega_m^2 - \Omega_s^2}\frac{\omega_m}{\Omega_s}\frac{\psi_m}{\psi_i} e^{i[(\omega_m - \Omega_s)t + \theta - \varphi_i]}\right]. \quad (5)$$

From this expression we can see that under fixed $\delta\omega$ the magnitude of the energy modulation is determined by the ratio between the coherent RF phase modulation amplitude ψ_m and the incoherent oscillation amplitude of the particle ψ_i. Figure 1 shows the phase trajectories of a particle with the different oscillation amplitudes at the

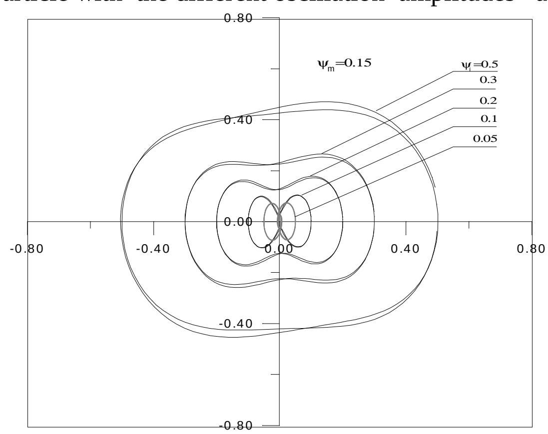

Figure 1. The phase trajectories of a particle with different amplitude of oscillation under the same RF phase modulation amplitude.

same phase modulation, derived by integration of the original equation (1). With a change of the amplitude ψ_i the shape of the curves varies as well.

Now, knowing the energy deviation, let us define the radiation damping for the case of the RF phase modulation. In the general case the radiation energy loss for a revolution time T of an electron, moving with the energy $\gamma = \gamma_o + \Delta\gamma$ in the magnetic field $B = B_o + \Delta B$ is determined by the well known formula:

$$\tilde{W}_r = \int_0^T P_o\left(1 + 2\frac{\Delta\gamma}{\gamma_o} + 2\frac{\Delta B}{B_o}\right)dt, \quad (6)$$

where $P_o = \frac{8\pi}{3}\gamma_o^2 B_o^2 r_e^2 \varepsilon_0 c$ is the radiation power loss of the reference particle with an energy γ_o in the magnetic field B_o, $r_e = e^2/4\pi\varepsilon_0 m_o c^2$ is the classical radius of an electron. Substituting in expression (6) the meaning of $\frac{\Delta\gamma}{\gamma}, \frac{\Delta B}{B}$, then averaging over the closed orbit and using the expression (5), we can represent the radiation loss by view:

$$\tilde{W}_r = \tilde{W}_{ro} + \frac{P_o}{\omega_r p_o R_o}\Delta W \cdot \overline{[2 - (1 - 2n)\eta]} \cdot \{1 + \chi\cos[(\omega_m - \Omega_s)t + \theta - \varphi_i]\}, \quad (7)$$

where $\chi = \frac{\Omega_s^2}{\omega_s^2 - \Omega_s^2}\frac{\omega_m}{\Omega_s}\frac{\psi_m}{\psi_i}$, $n = -\frac{R_o}{B_o}\cdot\frac{\partial B}{\partial R}$ and η is the dispersion function. Thus, the radiation losses are modulated by the harmonic with the frequency $\delta\omega = \omega_m - \Omega_s$ and the amplitude is inversely proportional to ψ_i. Now we can combine the equations (3), (7) and derive the second order equation for the phase oscillation with the modulated radiation damping:

$$\ddot{\varphi} + 2\xi_o\{1 + \chi\cos[(\omega_m - \Omega_s)t + \theta - \varphi_i]\}\dot{\varphi} + \Omega_s^2\varphi = -\psi_m\Omega_s^2\cos(\omega_m t + \theta), \quad (8)$$

where $\xi_o = \frac{P_o J_z}{2\omega_r p_o R_o}$ is the decrement of the radiation damping in the longitudinal plane without phase modulation and $J_z = [2 - (1 - 2n)\eta]$ is the damping partition function. Following the Bogoljubov and Mitropolsky method [4] the solution of equation (8) can be written in the form:

$$\varphi(t) = \psi_i e^{-\xi t}\cos(\Omega_s t + \varphi_i) + \psi_m\frac{\Omega_s^2}{\omega_m^2 - \Omega_s^2}\cos(\omega_m t + \theta), \quad (9)$$

and the decrement is determined by the first harmonic $e^{i\Omega_s t}$ of the expression between the curly brackets multiplied by $\dot{\varphi}(t)$ in equation (8):

$$\xi = 2\xi_o\frac{1}{2\pi}\int_0^{2\pi}[1 + \chi\cos(\omega_m - \Omega_s)t]\sin^2(\Omega_s t + \Theta)\,d(\Omega_s t), \quad (10)$$

where Θ is the total phase of a particle. In order to get the non-zero average action of parametrization to the radiation damping we need to set $\omega_m = 3\Omega_s$. In this case:

$$\xi = \xi_o\left(1 - \frac{\chi}{2}\cos 2\Theta\right). \quad (11)$$

Taking into account the spontaneous character of electron irradiation, the stationary distribution is determined by the balance of the quantum excitation and the radiation damping processes:

$$\sigma_E^2 = \frac{\dot{N}_{tot}\langle\varepsilon_{ph}^2\rangle}{4\xi_o\left(1 - \frac{\chi}{2}\cos 2\Theta\right)}, \quad (12)$$

where $\dot{N}$ is the total rate of the quantum emission, $\langle\varepsilon_{ph}^2\rangle$ is the mean square quantum energy. Since the phase 2Θ is changed with twice the periodicity the sign of the additional term in the decrement modifies the distribution in two directions, extending in one direction and compressing in another perpendicular to the first (see figure 1). Making the correct adjustment of the phase of the phase modulation we can determine the extension in the length of the bunch and the compression in the momentum spread. The degree of this redistribution depends on how the particle is far from centre of the bunch. Varying the RF phase modulation strength, we can change the phase area, where the distribution is modified.

The parametrization effect can be reached by the RF amplitude modulation as well and the nature is similar.

3 INFLUENCE OF NON-LINEAR RESONANCE ON REDISTRIBUTION OF DENSITY

The linear approximation of the synchrotron motion allows to find the global affect on the redistribution due to the parametrization. However, the non-linear resonance can play the significant role in the changing of the distribution directly [5] and through the parametrization of the decrement as well. From this point of view let us see the non-linear effect on the parametrization of the radiation damping decrement. Really the eigen solution for the oscillator in the field of the sinusoid shape wave contains all odd harmonics[4]:

$$\varphi(t) = \psi_i \cos\Psi - \frac{\psi_i^3}{192}\cos 3\Psi + \ldots, \qquad (13)$$

where $\Psi = \Omega_s t + \varphi_i$. It is easy to see after substituting this solution in the integral (10), that we get the resonance for all frequencies $\omega_m = 2\Omega_s + 2k$, where k=1,2,3,…, but not for $\omega_m = 2\Omega_s$. However, with the increase of the resonance number the required RF modulation amplitude has to be increased inversely proportionally to the coefficient of harmonic, what causes the harmful effect of build up of coherent oscillations. Therefore the parametrization is most strong at the third harmonic.

Now let us consider the redistribution due to the non-linear effect. Since we are interested in the redistribution of the density inside the bunch we should split the coherent component out and seek the solution for the phase ϕ relative to the centre of the bunch (the quasi-synchronous phase φ_s):

$$\frac{d^2\phi}{dt^2} + \Omega_s^2\{\sin[\phi + \varphi_s(t) + \psi_m \cos\omega_m t] - \sin[\varphi_s(t) + \psi_m\cos\omega_m t]\} = 0. \qquad (14)$$

In turn, the quasi-synchronous phase shows, how the whole bunch oscillates relative to the modulated RF phase with $\tilde{\varphi}_s = \varphi_s + \psi_m \cos\omega_m t$ and:

$$\frac{d^2\tilde{\varphi}_s}{dt^2} + \Omega_s^2 \sin\tilde{\varphi}_s = -\psi_m\omega_m^2\cos\omega_m t. \qquad (15)$$

This is the equation for the bunch oscillation with an external resonance. The resonance Hamiltonian in the above case, after the canonical transformation to the action-angle variables I, ϑ has the form [6]:

$$H_r(I,\vartheta) = I\cdot\Delta - \frac{I^2}{16} + \frac{\omega_m^2}{\Omega_s^2}\psi_m\sqrt{\frac{I}{2}}\cos\vartheta, \qquad (16)$$

where $\Delta = \delta\omega/\Omega_s$. In the simplest case, when $\Delta > 1$ and $\psi_m << 1$, the oscillation of the bunch is described by $\tilde{\varphi}_s(t) = \psi_m \dfrac{\omega_m^2}{\omega_m^2 - \Omega_s^2}\cos\omega_m t$. Substituting in equation (14) under $\omega_m = k\Omega_s$, we get the non-linear equation for a particle inside the bunch:

$$\ddot{\phi} + \Omega_s^2\sin\phi\left[J_o\left(\frac{k^2\psi_m}{k^2-1}\right) - 2J_2\left(\frac{k^2\psi_m}{k^2-1}\right)\cos 2k\Omega_s t + \ldots\right] = 0, \qquad (17)$$

From this equation we can see that the particles in the bunch experience the parametric non-linear resonance and the intensity of the resonance is mainly defined by the amplitude of the coherent oscillation. In principle all harmonics can excite the non-linear resonance, but the intensity goes down as $\dfrac{\psi_i^{k-1}}{2^{k-1}(k-1)!}$.The distribution for the parametric resonance is defined by Hamiltonian [6]:

$$H_r(I,\vartheta) = \frac{I\cdot\Delta}{2} - \frac{I^2}{16} + 2J_2\left(\frac{k^2\psi_m}{k^2-1}\right)\cdot\frac{I}{4}\cos\vartheta. \qquad (18)$$

4 EXPERIMENTAL RESULTS AND DISCUSSION

We have performed some crude experiments with RF phase modulation to investigate this phenomenon. The life time is increased up to three times at the frequency $\omega_m = 57$ kHz $(k=3)$ and an amplitude of the modulation $\approx 0.1 \div 0.2$ rad. The gain coefficient of the life time depends on the current. The maximum of the gain is reached at a current 120-150 mA and almost no gain below 40 mA. We studied the modulation of the phase at the harmonic number $k = 1,2,4$ and found also a significant life time increase. Obviously the first harmonic have to affect the redistribution, although the influence of the second and fourth ones is not in accordance with our understanding. In the future we intend to continue these investigations.

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