# A PARTICLE-IN-CELL MODEL FOR SPACE CHARGE DYNAMICS IN RINGS*

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## Abstract

High power circular accelerators and storage rings have both stringent beam loss requirements and significant space charge forces. It is therefore important to study the space charge dynamics in rings. To represent the space charge forces of the beam for arbitrary distributions in particle tracking calculations, we have developed a particle-in-cell (PIC) model. This has been accomplished by including space charge forces in the injection and tracking code, ACCSIM. Because PIC calculations can become extremely time consuming, we use a fast Fourier transform (FFT) evaluation scheme for the space charge forces. Application of this scheme is made for the calculation of halo generation, emittance growth, and tune shifts in similar doublet and FODO lattice configurations. Also, comparison of the results and computer timing of the PIC model calculations is made with those of a parametrically matched particle core model (PCM) and with calculations having no space charge effects.

## 1 INTRODUCTION

Space-charge-induced emittance growth and halo generation are potential sources of beam loss in high intensity rings, such as the SNS. In such accelerators, uncontrolled losses to the walls as small as $10^{-4}$ would lead to activation (1), making maintenance difficult. For this reason it is important to study the effects of space charge on beam dynamics and halo generation in high intensity rings. Here, we apply a PIC formulation to address an important question: the impact of space charge effects on choice of lattice configuration.

We do this for one case of particular interest, the SNS accumulator ring, by comparing and contrasting the effects of transverse space charge forces on beam emittance growth and halo generation for three globally comparable lattice configurations: doublet, FODO, and uniform focusing, which is a mathematical construct. These lattices are fourfold symmetric, 221 m in circumference, and have bare tune values of $\nu_x = 5.85$ and $\nu_y = 5.70$. The beam parameters for these calculations are also similar to the SNS baseline design. Specifically, an $H^-$ beam of energy 1 Gev and maximum energy spread $\pm 9.4$ Mev is assumed. The number of beam particles, $3.08*10^{14}$ for a nonbunched

circulating beam, corresponds to $2.0*10^{14}$ particles for a bunched beam with bunching factor equal to 0.65. We use a coasting beam and neglect azimuthal variations in charge density. The rms beam emittances are chosen to be 100 mm-mrad in both the $x - x'$ and $y - y'$ planes. The tracking calculations are applied to the full coasting beam for a total of 1250 turns. In Ref. 2 results were presented that assume bunched beams, azimuthal charge density variation, and injection for beam accumulation.

In all calculations presented here K-V distributions of particles, representing the beam, are randomly initialized and then tracked. Both cases in which the beam is matched to the lattice and not matched are considered. For matched beam cases in the absence of collisions, the K-V distribution provides an equilibrium solution of the dynamic equations.

## 2 THE PARTICLE-IN-CELL MODEL

To study transverse space charge effects, a computational particle tracking approach is chosen. This is performed using a modified version of the injection and tracking code, ACCSIM (3). The details of the numerical models are described in Ref. 4, so only a brief description will be presented here. The particle dynamics can be summarized by Hill's equations for betatron oscillations:

$$x'' + K_x(s)x = F_x^{sc} + \frac{1}{\rho}\frac{\delta p}{p_0}$$

$$y'' + K_y(s)y = F_y^{sc}, \qquad (1)$$

where derivatives are taken with respect to the azimuthal lattice distance $s$; $K_x(s)$ and $K_y(s)$ represent the periodic linear focusing forces from the lattice magnets; $\frac{1}{\rho}\frac{\delta p}{p_0}$ is the dispersion term; and $F_x^{sc}(s)$ and $F_y^{sc}(s)$ are space charge forces. The Hill's equations are solved for linear transport and dispersion using the routines in ACCSIM. The nonlinear space charge terms are evaluated as kicks to the transverse momenta before and after the matrix transports so that the tracking integration scheme is second order symplectic.

The space charge force terms are calculated self consistently with the tracked particles representing line charges at the same azimuthal location. The self-

consistent PIC model calculates the space charge forces directly from the actual distribution of tracked macroparticles, according to the following expressions:

$$F_x^{sc}(x,y) = \frac{K}{N} \sum_{i=1}^{N} \frac{x - x_i}{(x - x_i)^2 + (y - y_i)^2 + \varepsilon^2}$$

$$F_y^{sc}(x,y) = \frac{K}{N} \sum_{i=1}^{N} \frac{y - y_i}{(x - x_i)^2 + (y - y_i)^2 + \varepsilon^2}, \qquad (2)$$

where the summation is over the macroparticles, $N$, $K$ is the perveance (6), and $\varepsilon$ is a numerical smoothing parameter designed to eliminate the singularities of short range macroparticle collisions. For computational speed, the actual computation is made using fast Fourier transform (FFT) techniques, and is discussed in detail in Ref. 4. Numerically, the computational results depend on the number of macroparticles, $N$, the FFT resolution, and the smoothing parameter, $\varepsilon$.

The numerical representation is described by a number of parameters. In our initial calculations, presented here, the values for these parameters were: 480, or 20/FODO cell, for the number of azimuths used in integration around the ring; $32 \times 32$ for the size of the FFT spatial grid; 7680 for the number of macroparticles, $N$; and $4.4 \times 10^{-3} m$ for the smoothing parameter, $\varepsilon$. Subsequent convergence studies refining the values of these parameters support these choices.

## 3   CALCULATED RESULTS

Self-consistent PIC calculations were carried out for matched K-V beams with each of the three lattices. After tracking for 1250 turns in the uniform focusing and FODO lattices, a mild spreading of the beam core is observed, but there is little or no halo generated. However, for the doublet lattice significant halo generation is observed in the $y - y'$ phase plane, accompanied by a modest beam core growth, comparable to the doublet and uniform focusing cases. Other quantities, including the actions and tunes of the particles, reflect this halo growth. Analysis of the tunes of the particles for the three lattices shows that the doublet tunes have the greatest spread, with the $y$ tune spread exceeding the $x$ tune spread. The average tune shifts for all three cases are comparable and close to the calculated Laslett tune shift of 0.105 for the K-V distribution in the uniform focusing case.

Consider now the time histories of the beams. For the uniform focusing and FODO lattices the rms emittances are almost constant, but for the doublet lattice there is an increase in the $y$ emittance of about 3%, occurring mostly between 400 and 800 turns, so the growth is observed in the $y$ motion in the doublet lattice. To gain a quantitative picture of the halo, we arbitrarily define

halo particles as those macroparticles with betatron oscillation amplitudes in either $x$ or $y$ at least 25% greater than the maximum betatron oscillation amplitude of the matched K-V beam in the corresponding direction. Figure 1 plots the fraction of halo particles versus turns for the three lattices. While the uniform focusing and FODO lattice beams display $\leq 0.1\%$ halo, the halo growth for the doublet lattice is $> 1\%$, and it occurs mostly between 400 and 800 turns.

The same cases were also calculated without space charge and with a particle core model (PCM). For all three lattices, these calculations reveal almost no beam core growth and essentially no halo generation. The main effect of the space charge forces in the PCM is to provide tune shifts, also in agreement with the Laslett tune shift. The relative computer timings of the calculations, carried out on various IBM RS6000 and DEC Alpha workstations were surprisingly favorable for the PIC model, which ran at basically the same speed as the PCM and four times slower than the calculations without space charge.

To clarify the differences between the PIC and PCM results, the calculations were carried out using a phase-averaged PIC model, identical to the full PIC model except for averaging the particle locations uniformly over betatron oscillation phase before binning to the FFT grid. This smooths and symmetrizes the space charge distribution by eliminating odd moments. The results were essentially those of the matched PCM. Although a numerical instability in the full PIC calculation is possible, the differences in these results suggest a space charge instability accessible only to the PIC model.

To examine the possibility of a numerical instability, PIC calculations using more macroparticles, finer FFT resolution, and reduced smoothing parameter values have been carried out, and they show the persistence of this behavior for the doublet lattice. For calculations differing only in the number of macroparticles: 7680, 30720, and 122880 macroparticles, the halo saturates between 1.0% and 1.5% of the beam by 800 turns. When the grid and smoothing parameters are varied, all cases again result in saturated halo fractions of between 1.0% and 1.5% of the beam by 800 turns. Beam diagnostic calculations are underway to determine the specific nature and source of the doublet instability.

Calculations for mismatched K-V beams have been carried out for all three lattices with the mismatch set by the PCM and the macroparticle distributions made consistent with the envelopes. The fraction of halo particles, when initial envelope radii are increased by 25% over their matched values, grows rapidly and saturates by 200 turns at 3% - 5% of the beam. The rms emittances behave similarly, displaying growths of about 8%. The behavior is similar for all three lattices. Figure 2 shows the beam cross sections at 1250 turns for the FODO lattice calculated with the PIC model, the PCM,

and without space charge. Again the PCM fails to obtain the emittance growth and halo generation that are observed with the PIC model, but the behavior of the tunes in the PIC and PCM calculations is similar to that found in the matched doublet case.

# 4 CONCLUSIONS

This is the initial paper in a detailed study of space-charge-induced halo and emittance growth in the SNS ring. Matched K-V beams were tracked for 1250 turns through three different lattices with parameters similar to the SNS. In all three lattices PIC model calculations yield some beam core spreading and, in the doublet lattice, significant halo creation. PCM and phase-averaged PIC calculations, together with an analysis of the $y$ particle tunes, show that it is likely that the observed behavior in the PIC calculations is a higher order space charge resonance, accessible to the PIC model only. The behavior of the doublet calculations persists throughout the parameter variation of convergence studies, diminishing the likelihood of a numerical instability.

In all these calculations, observed space charge tune shifts were found to be in good agreement with the calculated Laslett tune shift of 0.105, and because of the use of matched K-V distributions the tune spreads in the beam core were found to be small ( $< 0.05$ ).

Calculations with mismatched beams have been carried out, resulting in significant rapid emittance growth and halo generation for all three lattices. PCM calculations for all these cases show quiescent behavior, suggesting that the PIC calculations are seeing space charge instabilities of higher order than quadrupole.

## REFERENCES

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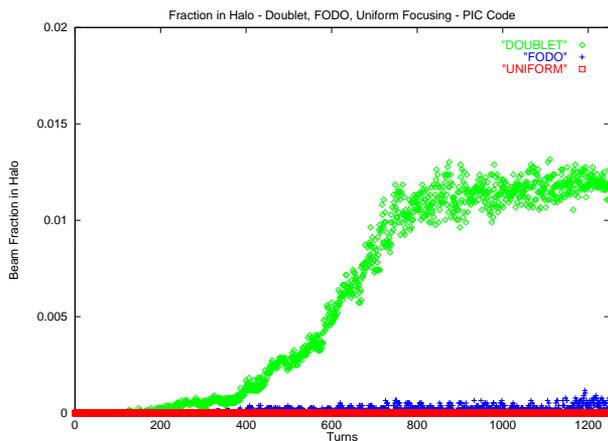
Figure 1: Fraction of beam in halo for the three lattices with matched beams.
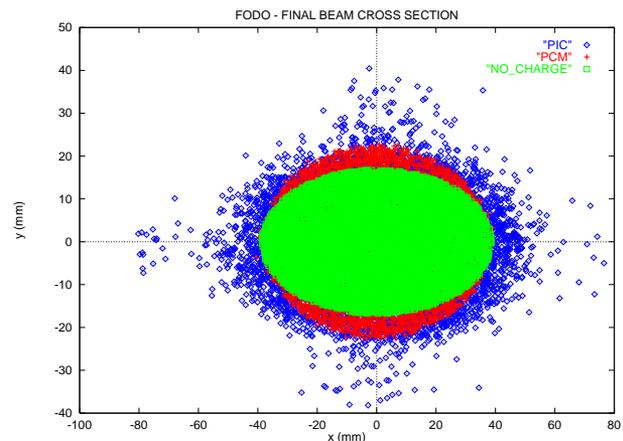


Figure 2: Beam cross section after 1250 turns for 25% beam radial mismatch in FODO lattice using PIC, PCM, and no space charge models.