HIGH FREQUENCY PROPERTIES OF WAKE FIELDS IN TESLA CAVITIES *

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Abstract

The FEL operation in the TESLA linear collider project requires very short, intense bunches. In the cavities these bunches excite very high frequency electromagnetic fields. There are severe concerns, that these fields will remain inside the structure for a long time, break up the Cooper pairs and finally lead to the quench of super conductivity. The calculations of transient dynamics of wake fields of very short bunches show, how much of the energy is vanishing through the beam pipes immediately and how much energy is staying in the cavity for long time.

1 INTRODUCTION

While passing a cavity a bunch creates electromagnetic fields. A fraction of these fields is staying in the cavity for a very long time (captured modes). After several reflections, another part is leaving the cavity and the rest is chasing the bunch. In a time this field will catch the bunch and take its kinetic energy. The time or the distance, where the bunch is caught, is inversely proportional to the bunch length. It can be very long for a very short bunch. It is possible to imagine, that the spectrum of this part contains mainly high frequency modes. This effect was firstly investigated for PETRA cavities [1]. Now we give the analyses for the TESLA cavity, as it is very important from the point of feasibility of short bunch acceleration in super conducting cavities. To give quantitative values, we study the energy distribution of the wake field in a single cell cavity with connected tubes and in multi-cell cavities. For wake field computations MAFIA [2] and the code, designed for short bunch calculations in TESLA cavities [3] are used.

2 WAKE FIELD ENERGY

To find real fields, acting on particles, it is necessary to split the full field E_{full} in the wake field E_{wake}, that really acts on the bunch particles and the "self" field E_{bunch} , that is moving everywhere together with the bunch, but does not interact with particles (in the relativistic case).

$$E_{wake} = E_{full} - E_{bunch} \tag{1}$$

The energy distribution of the wake field, following the bunch, can be described by the longitudinal energy density $\Lambda(s)$, that is the transverse integral of the energy density at a distance s from the center of the bunch

$$\Lambda(s) = \frac{\epsilon_0}{2} \int [E^2_{wake}(s) + H^2_{wake}(s)] d\varphi r dr \tag{2}$$

When the bunch comes out of the cavity, the integral of this density $\Lambda(s)$ along the bunch way, shows the energy $T(s_0)$, that is following the bunch at a distance s_0 behind

$$T(s_0) = \int_{-\infty}^{s_0} \Lambda(s) ds \tag{3}$$

This integral is approaching the loss factor K_{loss}, when $s_0 \longrightarrow \infty$

$$K_{loss} = \int_{-\infty}^{\infty} \Lambda(s) ds = T(\infty) \tag{4}$$

3 SINGLE CELL CAVITY

While decreasing the bunch length, the loss factor is increasing, and more and more high order modes are excited in a cell. On Fig. 1 the loss factor in one cell of the TESLA cavity is shown over the bunch length, together with the energy integral $T(s_0)$.

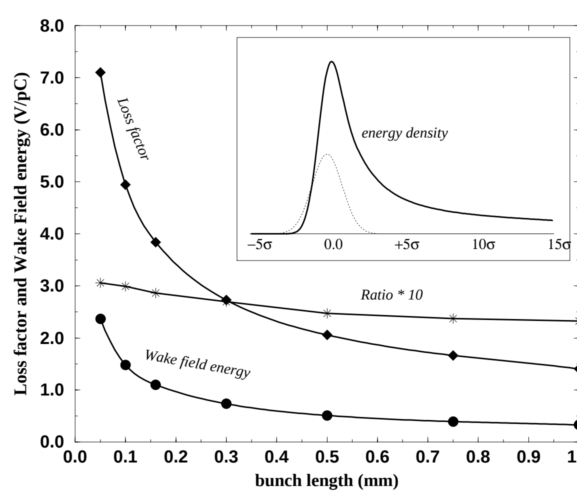

Figure 1: Loss factor and the wake field energy, following the bunch in the tube, after a single cell of the TESLA cavity over the bunch length. Ratio of field energy to the loss factor is multiplied by 10 (to use the same scale).

In this figure, inside the box, the energy density $\Lambda(s)$ in the tube and the bunch charge distribution are also shown.

The energy density has a slowly vanishing tail after the bunch. The energy of the following field is defined as $T(s_0 = 15\sigma)$.

* Work supported in part by DESY, Hamburg
† On leave of absence from BINP, Novosibirsk

The ratio of the energy integral to the loss factor is slightly growing up with the bunch length decreasing, coming to the value of 30%. So, one third of the ”excited energy” immediately leaves the cell with the bunch.

How far will the wake field follow the bunch in the tube? One can predict, that at least, up to the distance L, where the field ”catches” the bunch

$$L = \frac{a^2}{2\sigma} \tag{5}$$

where a is the radius of the tube, σ is the bunch length. For the regular cell of the TESLA cavity and for the bunch length $\sigma = 0.5$ mm, this distance $L = 1.225$ m, is equal to the length of the 9-cell cavity. For the bunch of $\sigma = 50\mu$, the distance is more than one accelerating cryomodule length.

To investigate the behavior of the field inside the cavity, the magnetic fields on the surface of the cavity and in the tubes are monitored in time. These fields are shown on the Fig. 2.

One can see the periodical time structure of very short pulses. The period is equal to the doubled time of passing the cavity.

So, in the cavity a bunch excites short pulses of high frequency modes, that are traveling mainly in the longitudinal direction with reflections of the tubes. After a time of 2 ns very short pulses disappear and approximately 4 ns later, high frequency modes leave the cavity.

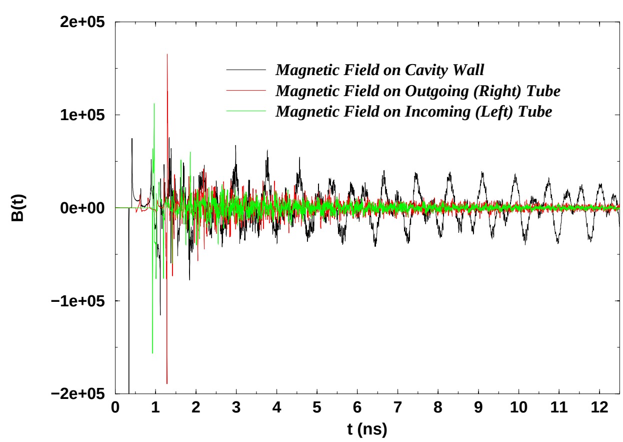

Figure 2: Magnetic field, monitored on different position on the cavity and tube surface

Several captured modes stay in the cavity for a long time. These modes are inside the 10 GHz band, as can be clearly seen from the Fourier transform of the time signal (Fig. 3).

4 MULTI-CELL CAVITY

As the wake field follows the bunch for a long time, in a multi-cell cavity this field is increased with the number of cells, that the bunch meets on a distance L (eq. 5). Results of the computation of the energy integral $T(15\sigma)$ along the

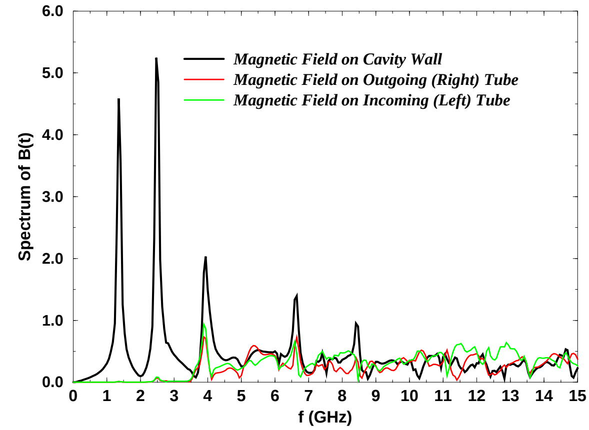

Figure 3: Amplitudes of Fourier transform of magnetic field, monitored on different position on the cavity and tube surface

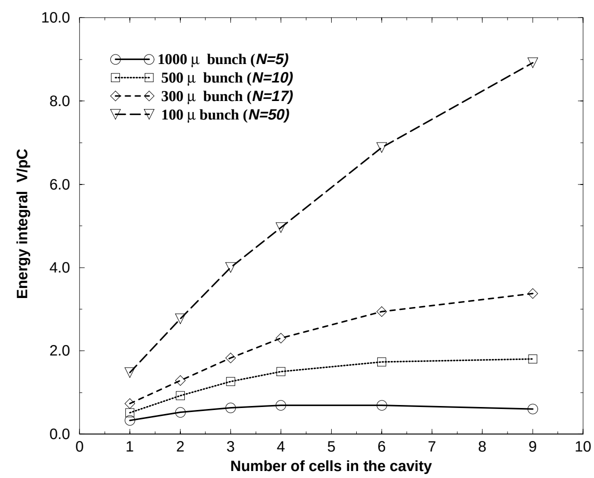

Figure 4: Wake field energy, following the bunch in the TESLA cavity, for bunchlength of 1 mm, 0.5 mm and 0.1 mm.

TESLA cavity are shown on Fig. 4. It can be seen, that the energy of the following field is linearly growing up in first cells. The number of cells, where the field approaches the asymptotic solution, is determined by the ”catch up” length L and period of structure D

$$N = L/D = \frac{a^2}{2\sigma D} \tag{6}$$

Values for N for the TESLA cavity (9-cell) are shown in Fig. 4 for different bunch length.

5 DIRECTED AND REFLECTED FIELDS IN TESLA CAVITY

To study the reflected and the directed fields in the TESLA cavity, the model of excitation of one cell was used. As the full field is separated (eq. 1) and we do calculations only for the wake field, then on the surface of the ”excited” cavity the wake field has to take the value of the bunch field with negative sign in accordance with boundary conditions.

The geometry of the ”excited” cell (N2) and the cells around (N1, N3) are shown in Fig. 5. Cell N2 is excited by the bunch of 0.5 mm length. The energy in the cells is calculated and presented in time. When the bunch leaves cell N2 the wake field energy is going with it and excites cell N3; then coming to the end of cell N3, one part of the energy goes to the next cell and the energy in cell N3 is going down. At the same time, the field, reflected from the iris between cell N2 and N3, is crossing cell N2 and coming to cell N1. We can estimate the reflected and the directed coefficients compare the cells energies. At first, the transmitted energy is around 30 %, in the next cell it is increasing to 50 %. Later high frequency modes are traveling along the structure almost without reflections.

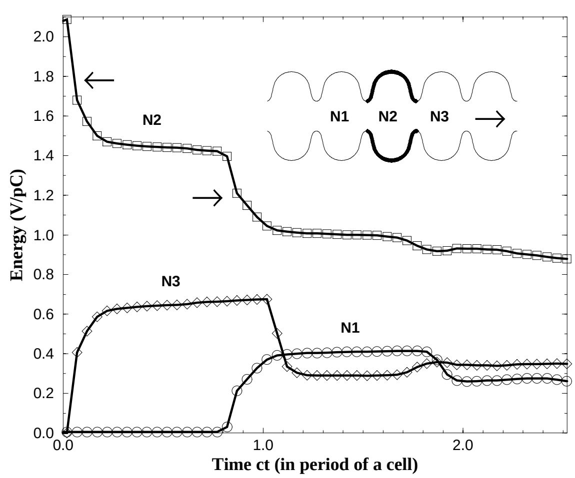

Figure 5: Field energy in cells of TESLA cavity.

Corresponding picture of electric force lines of wake fields in the time of two periods is shown on Fig. 6. The

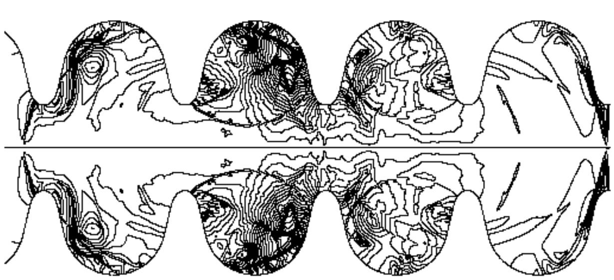

Figure 6: Electric force lines in TESLA cavity, after ”excitation” of one cell

structure of reflected and transmitted fields is clearly seen. If we know the transmission coefficient of the energy Π, then we can estimate the time dependence of the energy in the ”excited” cell by

$$\mathcal{E}(t) = \mathcal{E}_0(1-\Pi)^{ct/D} = \mathcal{E}_0 e^{-t/\tau} \qquad (7)$$

and find the energy attenuation time τ of high frequency modes in one cell

$$\tau = -\frac{D}{c\ln{(1-\Pi)}} \qquad (8)$$

Taking Π=0.3 we get τ = 1 ns. This is in good agreement with the field attenuation time (Fig. 2).

6 TRANSFORMATION OF WAKE FIELDS

Coming to the periodic solution, wake fields experience a strong transformation. The amplitude goes down and the shape changes. The wake fields of 50 μ bunch in cavities with different number of the cells are normalized per one cell and are shown in Fig. 7. The wake amplitude become

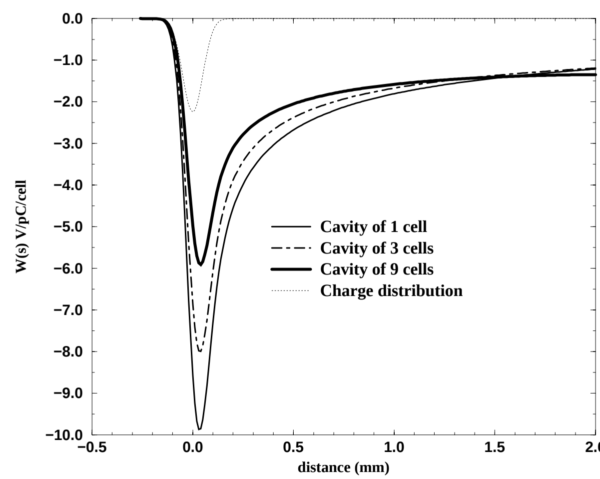

Figure 7: Wakes of a 50 μ bunch in cavities with a different number of cells

twice as small after one TESLA cavity. In the next cavities of cryomodule, the wake field changes mainly the shape, approaching the integral of the charge distribution.

7 REFERENCES

[1] T.Weiland, “Parasitic Energy Loss due to Cavities in PETRA”, Internal Report,DESY M-81/04, April 1981.

[2] The MAFIA collaboration ”User Guide”, CST GmbH, Darmstadt, Germany

[3] A.N.Novokhatski and A.Mosnier, “Short Bunch Wake Potentials for a Chain of TESLA Cavities”, DAPNIA/SEA-96-08, 1996.