MATHEMATICAL MODELS FOR ACCELERATING STRUCTURES OF SAFE ENERGETICAL INSTALLATION

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Abstract

Problems of obtaining optimal accelerating structures with quadrupole and alternate phase focusing are considered. It is supposed that RFQ and APF H-cavity may be used as basic structures of ion accelerator driver in the transported atomic energetical installation. The mathematical control model of accelerating cavities with RFQ and alternating phase focusing (APF) is suggested. On the basis of this model the software realizing the optimization process is developed. Optimization of RFQ and APF systems is considered. [1]

1 INTRODUCTION

During last years Linear Accelerators and Cyclotron Division of the Efremov Institute in St.Petersburg is working out ion linacs of a new generation for industry and medicine. St.Petersburg University takes part in development of mathematical providing for these researches. Contemporary rf ion linacs use frequencies diapason 400-500 MHz and produce intensive beams with small emittances and need special modelling codes for beam dynamics and accelerating structures geometry. The Efremov Institute use RFQ for particle acceleration up to 2 MeV and drift-tube linacs that works on π-mode for acceleration from 2 MeV up to 15 Mev and higher. DTL is not Alvarez type. Structure consists of separate cells each of them includes broad outer cylindrical rings. Inside of rings drift tubes are fastened on massive cross transversal holders (CTH-structure). This cavity uses alternate phase focusing or magnetic focusing of the beam. Electromagnetic field distribution for working type oscillation is according to H(TE) mode [1]. Both of RFQ and CTH structure types are named H-resonators in Russia.

The main purpose of the present report is to show the possibility of application of mathematical methods of control theory for optimal choice of accelerating and focusing structures [2]. These methods can be called constructive methods of control theory. They are based on using of analytical expression of variation of functional which describes the quality of the structure. The variation, which is also functional in the space of infinite dimension, can be approximated by the functional defined in the space of finite dimension, namely, gradient of the functional on the parameters by which the control function can be parametrized. After that, minimization of the quality functional can be fullfiled with the usual methods.

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2 CONTROL PROBLEM FORMULATION

In many cases the longitudinal motion can be assumed not to be depend on the transverse motion of the particles and the transverse forces acting on a particle are linear on transverse coordinates. Then the longitudinal and transverse coordinates can be considered separately and motion in the transverse phase space is described by linear model. In this case, we introduce macroparticle as such particle agregate that all particles in it have the same longitudinal phase coordinates but different transverse ones. The parameters of linear model for such agregate in the transverse phase space can be taken as transverse coordinates of the macroparticle. The particle interaction also can be include in this model under some simplifying assumptions one of which is uniform distribution of particles in the beam cross-section.

Then charge particles beam can be considered as a dynamical system describing by the equations

$$\begin{cases} dZ/dt = f_1(t,Z,U) + \int\limits_{M_{t,U}} f_2(Z,Z')\varrho(t,Z')\,dZ', \\ dX/dt = h_1(t,Z,X,U) + \\ \int\limits_{M_{t,U}} h_2(X,Z,X'(t,Z'),Z')\varrho(t,Z')\,dZ' \end{cases} \qquad (1)$$

where Z, X are vectors characterizing longitudinal and transverse motion correspondingly, $U = U(t)$ is control vector, $Z \in R^n, X \in R^m, U(t) \in K \subset R^l, t \in [0,T]$. The initial values of Z are supposed to fill some compact set $M_0 : Z(0) \in M_0 \subset R^n$ and $X(0) = X_0$. The image of the set M_0 at the mapping given by the system (1) is denoted by $M_{t,U}$. The integral terms describe the interaction between macroparticles, $\varrho(t,Z)$ is macroparticles density and satisfies to the equation

$$\frac{\partial\varrho}{\partial t} + \frac{\partial\varrho}{\partial Z}\cdot\frac{dZ}{dt} + \varrho\Big\{ div_Z f_1(t,Z,U) + \int_{M_{t,U}} div_Z f_2(Z,Z')\varrho(t,Z')\,dZ' \Big\} = 0,$$

which is partial integro-differential equation with characteristics described by the first equation of the system (1), with initial condition $\varrho(0,Z) = \varrho_0(Z),\ Z \in M_0$.

Let us formulate the control model for this case. The problem is to minimize the functional

$$I(U) = \int_0^T \int_{M_{t,U}} g(t, Z_t, X_t)\varrho(t, Z_t)\, dZ_t\, dt +$$

$$\int_{M_{T,U}} G(Z_T, X_T)\varrho(T, Z_T)\, dZ_T \qquad (2)$$

where g and G are some integrable on t, Z and differentiable on Z and X functions characterizing the quality of the beam, $Z_t \equiv Z(t)$, $X_t \equiv X(t, Z_t)$.

For such problem we can apply general approach (see [2]) taking into account that integral on some components of the phase vector, namely X, is reduced to the only value of the integrand. The method of optimization is based on the expression for functional variation

$$\delta I = -\int_0^T \int_{M_{t,U}} \Big\{ \Psi_X(t, Z_t)\Delta_U h_1(t, Z_t, X_t, U) +$$

$$\Psi_Z(t, Z_t)\Delta_U f_1(t, Z_t, U) \Big\} \varrho(t, z_t)\, dZ_t\, dt, \qquad (3)$$

$\Delta_U h_1 = h_1(t, Z, X, U(t) + \Delta U(t)) - h_1(t, Z, X, U(t))$, $\Delta_U f_1$ is expressed analogously. The auxiliary forms Ψ_X, Ψ_Z satisfy to following differential equations and terminal conditions:

$$\frac{d\Psi_X}{dt} = -\Psi_X \Big\{ \frac{\partial h_1}{\partial X} + \int_{M_{t,U}} \frac{\partial h_2}{\partial X} \varrho(t, Z')\, dZ' \Big\} -$$

$$\int_{M_{t,U}} \Psi_X(t, Z') \frac{\partial h_2}{\partial X'} \varrho(t, Z')\, dZ' + \frac{\partial g(t, Z, X)}{\partial X},$$

$$\frac{d\Psi_Z}{dt} = -\Psi_Z \Big\{ \frac{\partial f_1(t, Z, U)}{\partial Z} + \int_{M_{t,U}} \frac{\partial f_2}{\partial Z} \varrho(t, Z')\, dZ' \Big\}$$

$$- \int_{M_{t,U}} \Psi_Z(t, Z') \frac{\partial f_2(Z, Z')}{\partial Z'} \varrho(t, Z')\, dZ' -$$

$$\Psi_X \Big\{ \frac{\partial h_1}{\partial Z} + \int_{M_{t,U}} \frac{\partial h_2}{\partial Z} \varrho(t, Z')\, dZ' \Big\}$$

$$- \int_{M_{t,U}} \Psi_X(t, Z') \frac{\partial h_2}{\partial Z'} \varrho(t, Z')\, dZ' + \frac{\partial g(t, Z, X)}{\partial Z},$$

$$\Psi_Z(T, Z_T) = -\frac{\partial G(Z, X)}{\partial Z}\Big|_{\substack{Z = Z_T \\ X = X(T, Z_T)}},$$

$$\Psi_X(T, Z_T) = -\frac{\partial G(Z, X)}{\partial X}\Big|_{\substack{Z = Z_T \\ X = X(T, Z_T)}}.$$

3 APPLICATION FOR RFQ AND APF CHANNELS

The control model formulated above can be applied in the following important cases: RFQ channel and drift tubes channel with APF.

Suppose that the equations of particle transverse motion in RFQ channel can be written in the form:

$$d^2x/dt^2 = Q_x x, \quad d^2y/dt^2 = Q_y y$$

where

$$Q_{x,y} = \frac{eU_0}{m_0\gamma}\left(\pm\frac{\chi}{a^2} + \frac{k^2\Theta}{\pi}\right) \sin\eta \cos\varphi + Q_{self\ x,y}, \quad (4)$$

e and m_0 are charge and rest mass of the particles, U_0 is intervane voltage, $k = 2\pi/(L/\lambda)$, $\lambda = 2\pi c/\omega$ is the wavelength, L is modulation period of the electrodes, $\chi = 1 - 4\Theta I_0(ka)/\pi$, Θ is effectiveness of acceleration, η is electrodes modulation phase, and $Q_{self\ x,y}$ is coefficient accounting self field of the beam.

Suppose also that initially particles fill some ellipses in the planes x, x' and y, y' for each point in longitudinal phase space. Then at all subsequent instants particles fill some ellipses describing by symmetrical matrices: $B^{x,y}$: $X^*(B^x)^{-1}X \le 1$, $Y^*(B^y)^{-1}Y \le 1$ where $X = (x, x')^*$, $Y = (y, y')^*$. The elements of the inverse matrices satisfy the equations

$$\begin{cases} ds_{11}^{x,y}/dt = 2s_{12}^{x,y}, \\ ds_{12}^{x,y}/dt = Q_{x,y}s_{11}^{x,y} + s_{22}^{x,y}, \\ ds_{22}^{x,y}/dt = 2Q_{x,y}s_{12}^{x,y}. \end{cases} \qquad (5)$$

So, all particles in some point of longitudinal phase space can be described by these six variables which can be considered as transverse coordinates in (1). These variables have simple sense. For example, $\sqrt{s_{11}^{x,y}}$ are envelopes on x and y correspondingly.

Suppose also that the tranverse variables change so slowly along the beam axis that transverse field at some point with coordinate z can be regarded as creating by the beam with the same transverse particles distribution at all points along the beam axis as in the point under cosideration. It is usual assumption. For example, the Kapchinsky-Vladimirsky distribution is valid in its frames. Besides, assume that self field of particles at some point in longitudinal phase space is the same as the field of the beam with elliptical cross-section which semiaxes are equal to envelopes $\sqrt{s_{11}^{x,y}}$ and that expression for field outside the beam coincides with the one inside, i.e. linear on x and y. Under these assumptions we can take the self field term in (4) in the form

$$Q_{self\ x}x = \int_M h_{2\,x}(X, Z, X', Z')\varrho(Z')\, dz, \qquad (6)$$

$$h_{2\,x}(X,Z,X',Z') = \frac{e^2}{\pi\gamma^2\varepsilon_0 m_0} \frac{H(z-z')}{\sqrt{s'^x_{11}}(\sqrt{s'^x_{11}}+\sqrt{s'^y_{11}})}, \tag{7}$$

$H(\Delta z)$ is some smooth integrable function rapidly diminishing with increase of $|\Delta z|$, $\int H(z)\,dz = 1$. The term $Q_{self\ y}y$ can be expressed similarly.

It is more convinient to take the longitudinal coordinate z as independent variable t in (1) instead the time. In this case the phase φ and the reduce energy γ of a particle can be taken as longitudinal variables. The equations (5) should be transformed correspondingly being remained linear.

The equation of longitudinal dynamics can be written in the form

$$\begin{cases} d\varphi/d\zeta = 2\,\pi\,\gamma\,(\gamma^2-1)^{-1/2}, \\ d\gamma/d\zeta = \dfrac{2\,e\,U_0\lambda}{\pi\,m_0\,c^2} k\Theta\, \cos\eta\, \cos\varphi + F_z\,. \end{cases} \tag{8}$$

The term F_z is due to the longitudinal action of self field and determined on the base of the simplified expression for longitudinal component of the electric field in the channel:

$$E_z = \frac{2e}{\pi\varepsilon_0 a^2 J_1^2(j_{01}) sh\dfrac{j_{01}L}{2a}} \times$$

$$\int sh\left\{\frac{j_{01}}{a}(\frac{L}{2}-|z-z'|)\right\} sign(z-z')\varrho(z')\,dz'. \tag{9}$$

Here a channel apperture. The integral in (9) is taken over the period of modulation.

For modulation phase η we have additional equation

$$d\eta/d\zeta = 2\,\pi\,\gamma_s\,(\gamma_s^2-1)^{-1/2} - d\Phi_s/d\zeta$$

where Φ_s is phase of synchronous particle relative to phase of the space modulation.

The functions $u_1 = d\Phi_s/d\zeta$, $u_2 = \Theta$, $u_3 = \kappa/a^2$ can be taken as the components of the control vector. So, the dynamics equations in RFQ channel have form (1).

For drift tube channel with APF the dynamics equations are similar and also have the form (1).

4 CONCLUSION

So, in both cases we can use the mathematical control model described above. The wide choice of functional of the form (2) is possible. For example, we can take the functional (2) with functions g and G describing the capture of the particles in transverse and longitudinal motion correspondingly:

$$g(t,Z,X) = c_Z g_Z(Z) + c_X g_X(X).$$

Here c_Z, c_X are coefficients, g_Z and g_X are given by similar expressions, for example,

$$g_Z(Z) = \begin{cases} (z_i - z_{u\,i})^{p_i}, & z_i > z_{u\,i}, \\ 0, & z_i \in [z_{l\,i},\, z_{u\,i}], \\ (z_i - z_{l\,i})^{p_i}, & z_i < z_{l\,i}, \end{cases}$$

where p_i are some positive numbers. The function $G(Z,X)$ is expressed similarly.

The technique of optimization is based on approximation of components of control functions by functions depending on finite number of parameters. Then method of gradient descent can be applied [2]. Numerical realization included replacement of integration by summation and considering of systems (1) for discrete initial set instead of M_0 as it is described in [4]. Each point of the initial set has some weight equal to particle density in this point. The evolution of these points is described by the dynamics equation and their weights remain constant as the particle density is integral invariant.

For the solving of beam dynamics modelling and optimization problems original software environment has been developed. This software is based on the modern object oriented programming technique. According to this approach, the program is the set of components and objects which are necessary for some class of problems. The possibility of customizing of that components and methods is supposed.

The programs were tested on the known structures with the frequencies 148.5 MHz and 433 MHz. Results of modelling are satisfactory: close similarlity with known dynamics for these structures was obtained. During testing of the programs the output energy spread and transverse normalized emittance of the beam after optimization can be essentially decreased compared with ones before the optimization if their initial values were too great.

5 REFERENCES

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