

LONGITUDINAL RELAXATION OSCILLATIONS BEYOND THE THRESHOLD OF HOM INDUCED INSTABILITIES*

C.Limborg, J.Sebek, SSRL/SLAC,

PO Box 4349, MS 69, Stanford, CA, 94309-0219, USA

Abstract

Observations, measurements, and analysis of longitudinal oscillations that exhibit singular perturbations are presented. While the standard perturbation technique adequately describes instability thresholds, it fails to describe the behavior of the electron beam beyond threshold. In the presence of narrow band impedance, at threshold current, an instability develops in which the amplitude of the synchrotron sidebands undergoes a sawtooth oscillation. Since the impedance can be accurately controlled, and since both the beam current and energy can be varied, the damping time and the growth time of the instability, and the amplitude and phase of the non-linear driving term can be changed. The mechanism is explained analytically. Results from simulations give insights to details that are difficult to measure experimentally and to derive analytically.

1 INTRODUCTION

The perturbation theory of the onset of longitudinal instabilities due to long range wakefields has long been known. Open questions remain concerning the motion of the beam beyond threshold, as little experimental data exists which shows the detailed dynamics in this regime. While characterizing the HOMs of the SPEAR RF cavities [1], we repeatedly produced low frequency relaxation oscillations. Although this type of oscillation has been previously reported [2,3] and studied [4,5,6], the proposed models do not adequately describe our data. A characterization in frequency, amplitude, growth time, and damping time as a function of current, impedance, and energy was performed. Those results gave good clues to develop an analytical model of the oscillations. The dynamics of the instability was confirmed by tracking code simulations.

2 EXPERIMENTAL RESULTS

The features of the relaxation oscillation are most evident for the strongest narrow-band impedance of the system. Details of the phenomenon were first found on the fundamental mode of the idle RF cavity. The generalization to other HOMs was later confirmed experimentally. The longitudinal breathing of the bunch

results from the competition between the impedance induced driving force and the damping process. The driving term of the oscillation depends on the beam current and on the impedance of the HOM. The damping of the oscillation comes primarily from radiation, which increases as a function of beam energy. The spread of synchrotron frequency can also contribute to the damping for intense single bunch currents.

A full characterization of the frequency and amplitude of the relaxation oscillations was made by automatically varying the three tunable machine parameters while acquiring the time structure of the synchrotron sideband power with a spectrum analyzer, tuned as a narrow-band receiver. Measurements were completed with the use of a dual-sweep streak camera.

Qualitative explanation of experimental data

A maximum current of 2 mA in a single bunch can be maintained in SPEAR run with the parameters below and the fundamental impedance at its maximum value.

Table 1 SPEAR Experimental Parameters

E (GeV)	f_s (kHz)	L (m)	f_{rf} (MHz)	τ_{damp} ms
2.3	28.5	234	358.5	9.3

At the edges of the instability, the beam synchrotron amplitude does not execute any relaxation oscillation, but saturates at a constant value. The relaxation process begins at a slightly higher impedance. The period of this oscillation always exceeds a damping time, so the frequency is less than 100 Hz. The rise time is always exponential, but the decay time is not. When the spectral line starts on the higher edge of the resonance, as the synchrotron frequency shifts during growth, it sees a stronger impedance. The radiation damping then will be less effective, and so the damping time increases. (A quantitative analysis of this quadratic decay, typical of a diffusion process, is in preparation.) This justifies the asymmetry observed on the damping time, and therefore on the frequency while sweeping through the resonance [1]. The instantaneous synchrotron frequency decreases as much as 13% at the peak of the relaxation cycle, as predicted by the pendulum equation.

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Streak camera observations

Some important information about the mechanism was found with a double sweep streak camera. The synchroscan unit was locked to the 3rd sub-harmonic of the RF frequency. Every 3rd bucket could be seen, i.e. every 3rd turn was acquired in our single bunch experiment.

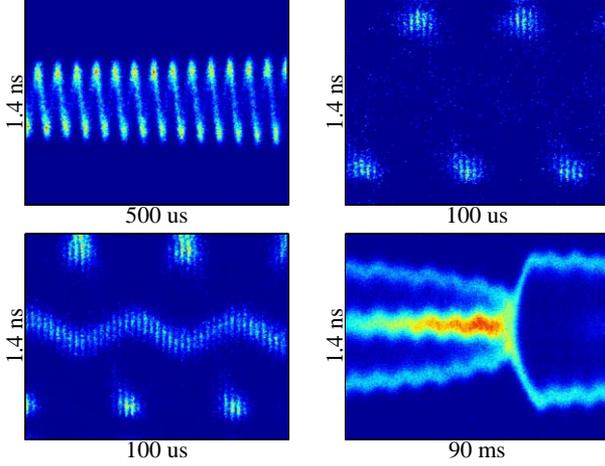


Figure 1: Single bunch streak camera images

The most striking characteristic of these data is that the bunch executes very large amplitude oscillations and remains a macro-particle during its growth. When the damping starts, the macro-particle starts to break-up. A large portion stays on the macro-particle orbit while a second sub-bunch forms near the center, π out of phase with what remains of the macro-particle. The entire bunch collapses toward the center, and the sequence starts again (Figure 1).

3 THEORETICAL MODEL

The bunch behaves as a single macro-particle during its growth. A standard driven oscillator [7,8] is used to represent the macro-particle motion. The driving force is given by the classical resonator-induced wakefield:

$$c = 2 \frac{\alpha_r}{\cos \phi_r} \frac{\omega_s^2}{\cos x_s} \frac{R_s I}{V_{rf}} \int_{-\infty}^t x(u) \exp(-\alpha_r(t-u)) \cos(\omega_r(t-u) + \phi_r) du \quad \text{Eq. (1)}$$

The integral is a causal function that approximates the discrete sum of bunch impulses and includes the slow decay of the excited field. The variables, t and u , are continuous approximations of the discrete times:

$$t = n T_0 + \tau \cos(\omega_s t_{\text{part}}); \quad u = k T_0 + \tau \cos(\omega_s t_{\text{source}}).$$

Eq. (1) combined with the pendulum equation leads to a system of two coupled second order equations, one of which is non-linear.

$$\begin{cases} \ddot{x} + 2\alpha_p \dot{x} + \omega_s^2 (\sin x - \sin x_s) / \cos x_s = c \\ \ddot{c} + 2\alpha_r \dot{c} + \omega_{r0}^2 c = 2\alpha_r \frac{\omega_s^2}{\cos x_s} \frac{R_s I}{V_{rf}} \dot{x} \end{cases} \quad \text{Eq. (2)}$$

with x : Phase excursion; α_p : Radiation damping rate

ω_s : 2π Synchrotron frequency; ω_r : 2π RF frequency
 R_s, ω_r, Q : Resonator parameters and $\omega_r = \omega_{r0} \cos \phi_r$
 $\alpha_r = \omega_r / 2Q$: Resonator damping rate

The model assumes that the amplitude and frequency of the synchrotron motion are slowly varying with respect to the synchrotron period and the short range wakefield effects can be neglected.

Since the excursion in phase is so large, perturbation expansion is insufficient to describe observations. In particular, the non-linear terms of the external RF wave, the phase of the source, and the phase dependent terms of higher order in the wake must be kept. For multiple particles, these equations still hold but c is now driven by the sum of all particles.

For a multiparticle system, c is generated only by the first derivative of the Center of Mass (CoM). Furthermore, each particle has c as its only driving term. In a linear system, the CoM is one of the modes of the system. The other modes, orthogonal to the CoM mode, do not contribute to the driving term and damp. For a non-linear system, the situation is slightly different in that the frequencies of the particles depend on their amplitudes. Now the CoM may not be an eigenmode of the system. For small non-linearities, however, the frequency difference is small enough and the wake force is large enough that the CoM is still a good approximation to an eigenmode and the above argument still holds.

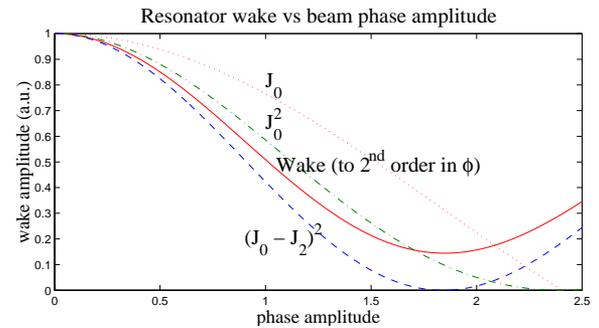


Figure 2: Wake vs. phase amplitude

The driving term decreases at higher amplitudes as the bunch rides higher on the HOM wave. Mathematically, this is seen in the reduction of the Bessel function coefficients $\approx J_0(\omega_r \hat{\tau})^2$ of the driving term. As the phase amplitude increases, the driving term can decrease by one order of magnitude, to a value equal to the damping “force”. The energy fluctuations of individual particles are now sufficiently large with respect to the perturbing voltage that they can go out of phase with respect to the CoM, and no longer contribute to the driving force. The out of phase particles slip further until they become π out of phase with the macro-particle. Now they form a “ π -mode” with part of the macro-

particle. Since this “ π -mode” does not have any driving term, it damps, bringing all of the beam back to the center of the RF bucket, where the bunch can recollect into a macro-particle and start its next cycle. The bifurcation has occurred because the characteristic of the bunch changes from a single particle to a multiparticle system.

4 SIMULATIONS

A multiparticle tracking code has been modified from the standard short range wake multiparticle model to include a long range HOM driving term, \mathbf{d} .

$$\mathbf{d}_{\text{new}} = \mathbf{d}_{\text{old}} \exp(-\alpha_r T_0) \cos(\omega_r T_0) + \dot{\mathbf{x}}_{\text{c.m.,old}} A_n$$

$$\dot{\mathbf{x}}_{\text{new}} = \dot{\mathbf{x}}_{\text{old}} - \frac{\omega_s^2 T_0}{\cos x_s} (\sin(x_{\text{old}} + x_s) - \sin x_s - F_{\text{s.w.}}(x))$$

$$- 2 \left(\alpha_p T_0 \dot{x}_{\text{old}} + r \sqrt{\alpha_p} \right) + d_{\text{new}} \omega_s T_0^2 A_n \frac{2\alpha_r}{\cos \phi_r} \frac{R_s I}{h V_{\text{rf}} \cos x_s}$$

$$x_{\text{new}} = x_{\text{old}} + T_0 \dot{x}_{\text{new}}$$

with $F_{\text{s.w.}}$: short term Wake Force in appropriate units, computed with distribution from the previous turn

$\dot{\mathbf{x}}_{\text{c.m.}}$: Center of mass velocity; r fluctuations
 T_0 : Revolution period; A_n : normalization

An important assumption here is that the synchrotron motion is much slower than the revolution period.

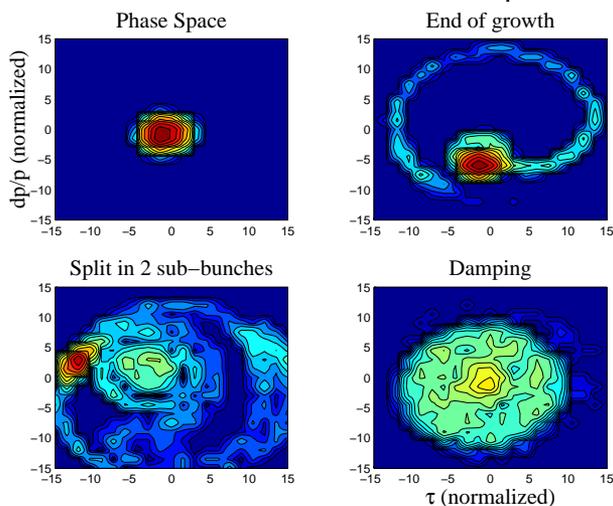


Figure 3 Simulations Intensity in logarithmic scale

The HOM is excited by the CoM mode of the beam. This excitation is the impulse train of the passing particle; the field decays over many turns. In each turn, the response to the current impulse is added to the existing field. The results of this code are in good agreement with experimental observations and the analytical mechanism described above. Using the measured machine and HOM parameters, relaxation oscillations with amplitudes and frequencies comparable to those measured on SPEAR can be reproduced. Growth time, amplitude, and frequency increase with the product of current and impedance.

5 GENERALIZATION TO HOMS

The amplitude of the oscillations of the HOMs is given by the $\omega_r \hat{\tau}$ value at which the damping term balances the driving term. This driving term essentially follows the product of the current with the curve represented in Figure 2. The maximum amplitude $\hat{\tau}$ is such that $\omega_r \hat{\tau} \approx 1.5$. Accordingly, $\hat{\tau}$ is smaller for higher resonant frequencies. Streak camera measurements showed a ratio of 2.5:1 in maximum phase amplitude between the fundamental ($h=280$) and the HOM at $h=751$. A strong HOM is typically one to two orders of magnitude weaker than the fundamental. Much higher current is needed to generate the relaxation oscillation. Furthermore, the wider frequency distribution for an intense bunch prevents it from forming distinct common and π modes, destroying the regularity and amplitude of the relaxation oscillation. In this case the effect of the short term wake cannot be neglected.

CONCLUSION

Extensive measurements, simulations and theoretical analysis were carried out to understand the mechanism of longitudinal coupled-bunch instabilities beyond threshold. The fundamental behavior of these instabilities is now understood. A more complete discussion of experiments and analysis will be presented in a subsequent paper.

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REFERENCES

- [1] C. Limborg, J. Sebek “Characterization of RF cavities”, these proceedings
- [2] G. Rakowsky, “Coherent Synchrotron Relaxation Oscillation in an Electron Storage Ring”, IEEE Trans. Nuc Sci NS-32, No. 5 (1985) 2377
- [3] A. Wrulich, et al “Observation of Multibunch Instabilities in ELETTRA”, EPAC96
- [4] T. Suzuki & K. Yokoya, “Simplified Criteria for Bunched Beam Instability due to RF cavities”, Nucl. Inst. & Meth. 203 (1982) 45-55 P45
- [5] S. Krinsky “Saturation of a longitudinal instability due to Nonlinearity of the wake field”, IEEE Trans. Sci Vol NS-32, No. 5, (1985) 2320
- [6] R. Nagaoka et al “Modelling of low frequency longitudinal coupled bunch oscillation” EPAC96
- [7] A. Hofmann, “Beam Instabilities” CAS Lectures CERN 95-06
- [8] A. Chao, “Physics of Collective Beam Instabilities in High Energy Accelerators” Wiley Interscience 1993