SELF-CONSISTENT SIMULATION OF THE CSR EFFECT

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Abstract

When a microbunch with high charge traverses a curved trajectory, the curvature-induced bunch self-interaction, by way of coherent synchrotron radiation (CSR) and space-charge forces, may cause serious emittance degradation. In this paper, we present a self-consistent simulation for the study of the impact of CSR on beam optics. The dynamics of the bunch under the influence of the CSR forces is simulated using macroparticles, where the CSR force in turn depends on the history of bunch dynamics in accordance with causality. The simulation is benchmarked with analytical results obtained for a rigid-line bunch. Here we present the algorithm used in the simulation, along with the simulation results obtained for bending systems in the Jefferson Lab (JLab) free-electron-laser (FEL) lattice.

1 INTRODUCTION

When a short bunch with high charge is transported through a magnetic bending system, curvature-induced coherent synchrotron radiation and space charge forces set a wakefield across the bunch, inducing energy spread and causing emittance growth. This phenomenon has raised considerable concern in the design of free-electron-laser drivers containing bunch-compression chicanes and recirculation arcs. Circumventing this deleterious effect demands a thorough understanding of the physics involved as well as computational tools for the prediction of the CSR effect in lattice designs.

Earlier analyses of the CSR-induced wakefield were largely based on the rigid-line-charge model. By studying the longitudinal CSR wakefield for a periodic circular orbit as well as a transient trajectory both in free space and with shielding [1-5], these analytical works help us to understand the mechanism of the curvature-induced bunch self-interaction. In reality, however, the bunch has finite transverse size and its dynamics responds to the CSR interaction. So, a self-consistent simulation is needed to study the actual dynamical system. The feedback of the CSR-induced wakefields on bunch emittance was first simulated in DESY[3], where the wakefields were computed from the rigid-line-charge model using designed bunch dynamics, and its effect on the bunch emittance degradation was consequently obtained. This scheme works when the CSR-induced change in the bunch dynamics is small.

In this paper we present a self-consistent simulation for the CSR effect on beam dynamics in free space. The dynamics of the bunch, under the influence of the CSR wakefield, is simulated by the motion of a set of macroparticles. This CSR wakefield in turn depends on the history of bunch dynamics in accordance with causality. The simulation is benchmarked against analytical results obtained for a rigid-line bunch. The simulation algorithm is presented here together with the simulation results for bending systems in the Jefferson Lab FEL lattice. The extension of the simulation from free space to the case with shielding by two parallel plates should be straightforward with the image charge method.

2 SIMULATION ALGORITHM

2.1 General Problem

First we outline the problem to be solved in the simulation. The dynamics for an electron in the bunch is governed by

$$d(\gamma m_e \dot{\boldsymbol{v}})/dt = e(\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B}), \tag{1}$$

with $\boldsymbol{\beta} = \boldsymbol{v}/c$, $\boldsymbol{E} = \boldsymbol{E}^{\text{ext}} + \boldsymbol{E}^{\text{self}}$, and $\boldsymbol{B} = \boldsymbol{B}^{\text{ext}} + \boldsymbol{B}^{\text{self}}$. Here $\boldsymbol{E}^{\text{ext}}$ and $\boldsymbol{B}^{\text{ext}}$ are the external designed electromagnetic (EM) fields, and $\boldsymbol{E}^{\text{self}}$ and $\boldsymbol{B}^{\text{self}}$ are the EM fields from bunch self-interaction, which in turn depends on the history of the bunch charge distribution ρ and current density $\boldsymbol{J}$ via the scalar and vector potentials ϕ and $\boldsymbol{A}$:

$$\boldsymbol{E}^{\text{self}} = -\nabla\phi - \partial\boldsymbol{A}/c\partial t, \qquad \boldsymbol{B}^{\text{self}} = \nabla \times \boldsymbol{A}, \tag{2}$$

with

$$\phi(\boldsymbol{r}, t) = \int \frac{d\boldsymbol{r'}}{|\boldsymbol{r} - \boldsymbol{r'}|}\rho(\boldsymbol{r'}, t'), \tag{3}$$

$$\boldsymbol{A}(\boldsymbol{r}, t) = \frac{1}{c}\int \frac{d\boldsymbol{r'}}{|\boldsymbol{r} - \boldsymbol{r'}|}\boldsymbol{J}(\boldsymbol{r'}, t'). \tag{4}$$

Here the retarded time t' is defined as

$$t' = t - |\boldsymbol{r} - \boldsymbol{r'}|/c. \tag{5}$$

For an ultrarelativistic bunch on a circular orbit, the self EM fields are dominated by CSR fields.

2.2 Macroparticle Model

A straightforward way to simulate the bunch self-interaction is to use a macroparticle model. Since CSR mainly couples the bunch length effect into the bend plane (typically horizontal) dynamics, at this stage we only simulate the longitudinal-horizontal dynamics in the $z = 0$ plane (z stands for vertical offset from design orbit), with the bunch distribution simulated by a set of round Gaussian discs in this longitudinal-horizontal plane with zero vertical extent. An example configuration is illustrated in Fig. 1.

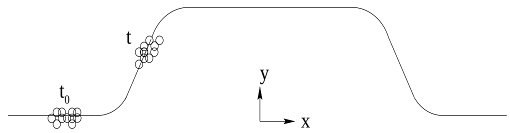

Figure 1: Macroparticles in longitudinal-horizontal plane.

The single macroparticle density distribution at $\boldsymbol{r} = \{x, y\}$ at time t is given by

$$n_m(\boldsymbol{r} - \boldsymbol{r}_0(t)) = \frac{1}{2\pi\sigma_m^2} e^{-\frac{(x - x_0(t))^2 + (y - y_0(t))^2}{2\sigma_m^2}}, \quad (6)$$

where $\boldsymbol{r}_0(t) = \{x_0(t), y_0(t)\}$ is the centroid of the macroparticle at time t, and σ_m is its rms size. Knowing the centroid location $\boldsymbol{r}_0^{(i)}(t)$ and velocity $\boldsymbol{\beta}_0^{(i)}(t)$ for each macroparticle, one can construct the charge distribution $\rho(\boldsymbol{r}, t)$ and the current density $\boldsymbol{J}(\boldsymbol{r}, t)$ of the whole bunch:

$$\rho(\boldsymbol{r}, t) = q \sum_{i=1}^{N} n_m(\boldsymbol{r} - \boldsymbol{r}_0^{(i)}(t)), \quad (7)$$

$$\boldsymbol{J}(\boldsymbol{r}, t) = q \sum_{i=1}^{N} \boldsymbol{\beta}_0^{(i)}(t) n_m(\boldsymbol{r} - \boldsymbol{r}_0^{(i)}(t)), \quad (8)$$

with i the index of the macroparticle and N the total number of macroparticle in a bunch. For a bunch with total charge Q, the charge per macroparticle is $q = Q/N$.

It can be shown that if the centroids in Eq. (7) have a Gaussian distribution with bunch length σ_s, and each macroparticle has a Gaussian distribution as in Eq. (6), then the effective bunch length simulated by Eq. (7) is $\sigma_{\text{eff}} = \sqrt{\sigma_s^2 + \sigma_m^2}$. In general, one should choose $\sigma_m \ll \sigma_s$ in order to have $\sigma_{\text{eff}} \simeq \sigma_s$. The number of macroparticle N should be chosen to ensure the overlap of macroparticles for the suppression of shot noise.

2.3 CSR Wakefields in Macroparticle Model

The computation of the CSR wakefield $\boldsymbol{E}^{\text{self}}$ and $\boldsymbol{B}^{\text{self}}$ in Eq. (1) is the core of the simulation. After applying the macroparticle model as described in Eqs. (7) and (8) to ϕ and $\boldsymbol{A}$ in Eqs. (3) and (4), one differentiates over the potentials and gets from Eq. (2) the CSR wakefield on $(\boldsymbol{r}, t)$ generated by the whole bunch, which is the superposition of contributions from individual macroparticles:

$$\boldsymbol{E}^{\text{self}}(\boldsymbol{r}, t) = \sum_{i=1}^{N} \boldsymbol{E}^{(i)}(\boldsymbol{r}, t), \quad B_z^{\text{self}}(\boldsymbol{r}, t) = \sum_{i=1}^{N} B_z^{(i)}(\boldsymbol{r}, t), \quad (9)$$

where the single-particle-generated CSR wakefields are 2D integrals over the area surrounding the source macroparticle's previous path:

$$\boldsymbol{E}^{(i)}(\boldsymbol{r}, t) = q \int \frac{d^2\boldsymbol{r'}}{|\boldsymbol{r} - \boldsymbol{r'}|} n_m(\boldsymbol{r'} - \boldsymbol{r}_0^{(i)}(t')) \cdot \left\{ \frac{\boldsymbol{r'} - \boldsymbol{r}_0^{(i)}(t')}{\sigma_m} - \frac{\dot{\boldsymbol{\beta}}_0^{(i)}(t')}{c} \sigma_m - \boldsymbol{\beta}_0^{(i)}(t') \left[\frac{\boldsymbol{r'} - \boldsymbol{r}_0^{(i)}(t')}{\sigma_m} \cdot \boldsymbol{\beta}_0^{(i)}(t') \right] \right\}, \quad (10)$$

$$B_z^{(i)}(\boldsymbol{r}, t) = q \int \frac{d^2\boldsymbol{r'}}{|\boldsymbol{r} - \boldsymbol{r'}|} n_m(\boldsymbol{r'} - \boldsymbol{r}_0^{(i)}(t')) \cdot \left[\boldsymbol{\beta}_0^{(i)}(t') \times \frac{\boldsymbol{r'} - \boldsymbol{r}_0^{(i)}(t')}{\sigma_m} \right], \quad (11)$$

with t' given in Eq. (5). To evaluate the integrand of the above integrals at $\boldsymbol{r'}$, one needs to interpolate the phase parameters $\boldsymbol{r}_0^{(i)}$ and $\boldsymbol{\beta}_0^{(i)}$ for the centroid at retarded time t', using the history of the phase parameters at discrete timesteps $t^{(k)}$ obtained from the leap-frog scheme in Sec. 2.4.

The 2D integral representation of single-macroparticle-generated CSR wakefields can be understood from the retardation nature of radiation: if we divide the ith macroparticle into grids, the wakefield observed at $(\boldsymbol{r}, t)$ is the sum of fields emitted by each grid at its own retarded location and time. In our simulation the integrals in Eqs. (10) and (11) are numerically implemented by applying the 2D Simpson's Rule, in which we divide a neighborhood of the previous path of the bunch into a grid, and sum up the integrand on the grids using appropriate coefficients. This procedure converges fast with respect to the number of grid points. The singularities in Eqs. (10) and (11) are taken care of by a singularity removal technique.

2.4 Leap-Frog Scheme

With the EM field obtained in Sec. 2.3, we integrate the equation of motion in Eq. (1) numerically using a leap-frog scheme. In 2D cylindrical coordinates, Eq. (1) is written as

$$\begin{cases} (\gamma m_e \dot{r})/dt - \gamma m_e r\dot{\theta}^2 = e(E_r + r\dot{\theta}B_z/c) \\ d(\gamma m_e r\dot{\theta})/dt + \gamma m_e \dot{r}\dot{\theta} = e(E_\theta - \dot{r}B_z/c) \end{cases}. \quad (12)$$

Consider a circular orbit with design radius r_0 and design energy $\gamma_0 m_e c^2$. For a macroparticle with its centroid at (r, θ) and energy $\gamma m_e c^2$, we let $s = r_0\theta$ be its longitudinal coordinate and $x = r - r_0$ be its horizontal offset from the design orbit. With k denoting the index of the timestep, and Δt the increment of time at each timestep, the discrete form of Eq. (12) in a leap-frog scheme for the centroid of the macroparticle is then

$$[(\gamma\beta_x)_{k+1} - (\gamma\beta_x)_k]/c\Delta t = [\tilde{E}_x - \beta_s(\gamma\beta_s/r - \gamma_0\beta_0/r_0 + \tilde{B}_z)]_{k+\frac{1}{2}}, \quad (13)$$

$$[(\gamma\beta_s)_{k+1} - (\gamma\beta_s)_k]/c\Delta t = [\tilde{E}_s + \beta_x(\gamma\beta_s/r - \gamma_0\beta_0/r_0 + \tilde{B}_z)]_{k+\frac{1}{2}}. \quad (14)$$

Here $(\beta_x, \beta_s) = (\dot{r}, r\dot{\theta})/c$, $\gamma^2 = (\gamma\beta_x)^2 + (\gamma\beta_s)^2 + 1$, and $\tilde{E}_{x,s} = (e/m_e c^2)E_{x,s}^{\text{self}}$, $\tilde{B}_z = (e/m_e c^2)B_z^{\text{self}}$. Since

$(\gamma\beta_{x,s})_{k+1/2}$ on the right-hand side of Eqs. (13) and (14) are defined as

$$(\gamma\beta_{x,s})_{k+\frac{1}{2}} = [(\gamma\beta_{x,s})_k + (\gamma\beta_{x,s})_{k+1}]/2, \qquad (15)$$

Eqs. (13) and (14) are two coupled nonlinear equations to be solved simultaneously in order to obtain the reduced momentum $(\gamma\beta)_{x,s}$ at each timestep. Consequently, the longitudinal-horizontal coordinates for the macroparticle centroids at the next half timestep yield

$$\begin{cases} x_{k+\frac{3}{2}} = x_{k+\frac{1}{2}} + (\beta_x)_{k+1}\, c\Delta t \\ s_{k+\frac{3}{2}} = s_{k+\frac{1}{2}} + \left(\dfrac{r_0\beta_s}{r}\right)_{k+1} c\Delta t \end{cases} . \qquad (16)$$

By setting $r_0 \to \infty$, the above equations naturally reduce to equations in Cartesian coordinates, which are used for the straight sections of a design orbit. Notice that from Eqs. (13), (14), and (15), it is straightforward to derive the energy equation:

$$(\gamma_{k+1} - \gamma_k)/c\Delta t = (\tilde{\boldsymbol{E}} \cdot \boldsymbol{\beta})_{k+1/2}. \qquad (17)$$

This implicit energy conservation is the reason we chose the leap-frog scheme for the numerical integration of Eq. (1) in our simulation. Note that at each timestep, only the centroids' dynamics gets advanced, and the macroparticles move translationally without rotation about their centroids. Therefore the speed of light is not exceeded in this simulation.

3 SIMULATION RESULTS

3.1 Benchmark

First, we turn off the CSR force by setting $E^{\rm self}_{x,s} = B^{\rm self}_z = 0$ in Eqs. (13) and (14) and make sure the single particle dynamics agrees with results from other optics codes, such as DIMAD in the ultrarelativistic case. Next, we benchmark the simulated CSR wakefield with earlier analytical results for a simple beamline: straight-bend-straight, using LCLS bend parameters[7]: $\theta = 11.4$ deg, $r_0 = 25.13$ m, $\sigma_s = 50\,\mu$m and $Q = 1$ nC. To imitate the rigid-line Gaussian bunch used in the analysis, we let the macroparticle centroids have a Gaussian distribution along the design orbit, and keep the bunch rigid by not responding to the CSR interaction. The simulation results of the CSR wakefield and their comparison with analysis are shown in Fig. 2 for the bunch at steady state. It is clear that the simulation results agree perfectly with the analytical result obtained for the effective bunch length $\sigma_{\rm eff}$, which gets closer to the analytical result for actual bunch length σ_s as σ_m gets smaller. Fig. 2 also shows that the radial field $E_x - \beta B_z$ is an order of magnitude smaller than the longitudinal field, as was observed in Ref. [3, 6]. The transient longitudinal wakefield at the entrance of a bend also agrees well with analysis.

3.2 Results for JLab FEL Chicane

The goal of our simulation is to compute the CSR induced emittance growth generated from the bending systems in the Jefferson Lab FEL lattice. Of particular interest is the effect of the optical chicane in front of the wiggler. To proceed, we first generate a macroparticle phase space distribution using the design beam parameters upstream of the optical chicane obtained from PARMELA. Then the CSR wakefield is computed at each timestep, which is used to advance the macroparticle dynamics to the next timestep. The transient radiation interaction after the bunch exits from a bend is also computed. Due to the short drift length between magnets for the JLab FEL chicane, the simulation shows nontrivial coupling between adjacent bends. With all these effects included, our simulation yields a 15% emittance growth in the horizontal phase space as a result of the CSR effect in the chicane. This is to be compared with the JLab FEL experiment. Recently the JLab FEL successfully lased at 150 W infrared laser power. More measurements of CSR effect will be carried out in the near future for the comparison of experiment and simulation.

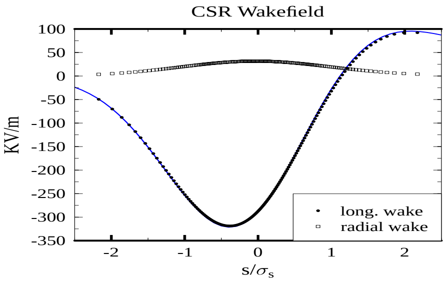

Figure 2: CSR wakefield on the bunch at steady state; the dots are the simulated wakefields with the rms of the macroparticle centroid distribution being σ_s, and the solid curve is the analytical longitudinal wakefield for the effective bunch length $\sigma_s^{\rm eff}$. Here $\sigma_m/\sigma_s = 0.4$.

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4 REFERENCES

[1] Y. S. Derbenev, J. Rossbach, E. L. Saldin, and V. D. Shiltsev, DESY Report No. TESLA-FEL-95-05, 1995.

[2] B. Murphy, S. Krinsky, and R. L. Gluckstern, BNL-63090, (1996).

[3] M. Dohlus, T. Limberg, Proc. 1996 FEL Conference, Rome, Italy, 26-31 August 1996. NL-63090, (1996).

[4] E. L. Saldin, E. A. Schneidmiller, and M. V. Yurkov, DESY-TESLA-FEL-96-14, (1996).

[5] R. Li, C. L. Bohn, and J. J. Bisognano, Proc. SPIE, San Diego, (1997).

[6] Y. Derbenev, V. Shiltsev, Fermilab-TM-1974, May 1996.

[7] Private communication with P. Emma of SLAC, (1998).