LINEAR AND NONLINEAR EVOLUTION OF LONGITUDINAL INSTABILITIES IN THE ESR

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Abstract

We have investigated the dynamics of longitudinal instabilities of coasting beams in the ESR for several working conditions determined by different values of the RF cavity eigenfrequency. The experimental data resulting from our measurements have shown a good agreement with theoretical predictions for the linear phase of the instability and with numerical simulations carried out using the particle-in-cell code PATRIC. Finally, a nonlinear cold fluid model is proposed to explain typical beam late stage evolutions such as wave steepening and generation of higher harmonics. The predictions of this model are also compared with observed data and with simulations.

1 INTRODUCTION

Coherent instabilities of particle beams in accelerators and storage rings, which are produced by the action of self-induced electromagnetic fields on the beam particles, have recently become object of new attention as a consequence of the development of the research on high-current machines operating below transition energy for several applications (e.g., for heavy ion fusion [1]).

Insofar the beam longitudinal dynamics is analyzed, one might show that a high particle density in a circular machine is able to cause various self-field effects associated with the longitudinal motion of particles. In particular, there exist conditions under which longitudinal density fluctuations in a coasting beam get amplified because of the interaction with the fields excited in the surroundings [2]. These conditions can be expressed via the longitudinal coupling impedance that models the environment in which the beam propagates [3].

Experimentally the unstable longitudinal motion of the circulating particles can be detected by a pick up as a modulation of the beam current. In the measurements we carried out at the ESR [4, 5], longitudinal self-bunching and further nonlinear evolution have been clearly observed for well-defined impedances by means of a precise cavity eigenfrequency control. A very good agreement between the predictions of the linear theory and the early phase of the observed instability has been found to exist for the working conditions we operated in. In this paper we also discuss that wave steepening and generation of higher order harmonics can be explained with a nonlinear fluid model.

2 MEASUREMENT PROCESS AND RESULTS

2.1 Data acquisition and analysis

The total longitudinal impedance acting on an ESR beam consists mainly of the space charge reactance and a narrow-band cavity contribution [6]. Thus, the longitudinally unstable evolution of a C^{+6} beam ($I_0 = 0.3$ mA, $f_0 = 1.886633$ MHz) has been systematically explored at the ESR by varying the coupling impedance of the cavity, which depends only on the difference between beam revolution frequency and cavity eigenfrequency ($\Delta f = f_0 - f_{\rm cav}$), and observing the longitudinal pick up signal in several different situations. For each measurement the beam, after being injected and then cooled down to an equilibrium momentum spread $(\Delta p/p)_{\rm FWHM} = 1.1 \cdot 10^{-5}$, was proven to be stable in a situation of strongly detuned cavity ($|\Delta f|_{\rm in} \approx 70$ kHz), in spite of the high current, exceeding some 4 times the Keil-Schnell limit (the space charge impedance alone, about -i$700\ \Omega$ per harmonic, cannot destabilize the beam [3]). Then, the eigenfrequency of the ESR cavity was tuned much closer to the revolution frequency of the beam ($\Delta f_{\rm fin}$ spanning between -32 and 18 kHz through the different measurements) by a linear frequency ramp within 15 ms. Starting from 30 ms before the eigenfrequency ramp, the beam current signal from a longitudinal beam monitor was sampled at 2 MHz and stored over 1 s. Due to the very small energy spread of the beam and to the choice of an appropriate sampling frequency, this signal was effectively undersampled with no loss of information: the bunch shape could still be reconstructed by taking a series of 17.64 samples from different turns, and high accuracy zooms were made possible in the off-line analysis by interleaving the samples from n subsequent series [4].

For the interpretation of the measurements we plot the cavity detuning curve on the stability diagram in the complex plane (Fig. 1). When the cavity eigenfrequency comes sufficiently close to f_0, the impedance working point in the longitudinal stability diagram ends up far outside the stability boundary and the beam grows unstable.

In order to control precisely the ESR cavity eigenfrequency, a small external RF voltage (300 V) was present at the cavity gap. Its effects, which are not taken into account in the linear theory, may be anyway studied via numerical simulations, where the contribution of an external electric force could be easily added besides the self-induced fields.

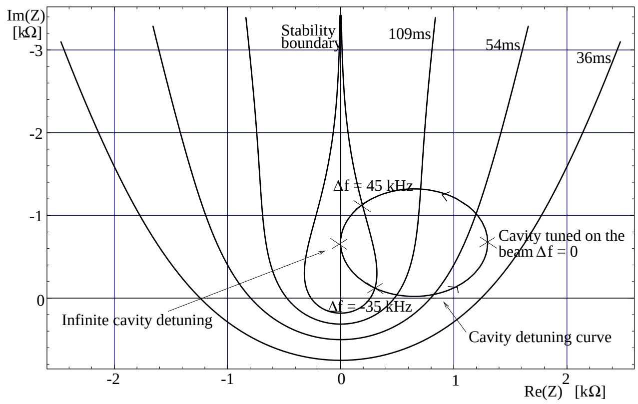

Figure 1: Stability diagram, rise time trajectories and cavity detuning curve (Δf spanning from $-\infty$ to $+\infty$).

2.2 *Interpretation of the measured data*

The analytical theory predicts that due to the positive resistive cavity impedance the longitudinal instability arises on the slow wave, which corresponds to the plasma wave running backwards in the beam frame. This may be seen in Figs. 2a and 2b. In Fig. 2a the beam modulation also breaks and splits up into a higher harmonic order oscillation from $t = 200$ ms.

We studied the unstable modulation signal from different measurements corresponding to different working conditions. At the early stage of the instability we observed mostly symmetric sinusoidal signals, whereas in the nonlinear region asymmetric bunch shapes occurred. The wave front steepening is a common feature of waves in the nonlinear regime [7] (see also next section).

The rise times from the linear phase of the instability were calculated by performing an exponential fit through several beam current first harmonic signals analyzed at different time points of the beam evolution. Then, these estimated rise times have been confronted with the theoretically predicted ones and with those evaluated from the numerical simulations carried out with the PIC code PATRIC. In Fig. 3 theoretical, simulated and measured rise times are plotted on the same graph: the results from PATRIC, along with the measurements, show a reduction in the rise times caused by the residual RF voltage in the vicinity of the resonance condition $|\Delta f|_{\rm fin} = 0$.

3 FLUID MODEL FOR THE NONLINEAR EVOLUTION

A simple nonlinear cold fluid model of a coasting beam is used to explain the generation of higher order harmonics and steepening of line charge density profiles observed during the development of longitudinal instabilities in the ESR. Starting from the Vlasov equation for the beam longitudinal dynamics and after its integration over the velocity space with the method of the distribution function momenta, one obtains in the cold fluid limit the following set of differential equations:

$$\begin{cases} \frac{\partial \lambda}{\partial t} + \frac{\partial}{\partial s}(\lambda U) = 0 \\ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial s} = \frac{q U_0 \eta}{2\pi r_0 p_0} \phi(s,t) \end{cases} \quad (1)$$

Here $\lambda(s,t)$ and $U(s,t)$ represent the beam charge line density and its mean velocity along the machine circumference. The electric driving term - at the right hand side of the second equation - using the narrow-band storage ring impedance formalism [8], becomes:

$$\phi(s,t) = U_0 \sum_n \dot{Z}(n\omega_0)\lambda_n(t) \mathrm{e}^{-ink_0 s} + \phi_{\rm ext}(s,t) \quad (2)$$

For the ESR, the impedance $\dot{Z}(\omega)$ is mainly made up of the space charge impedance and the cavity contribution. Since the nonlinear convective terms have been taken into account in our model, the numerical solution of Eqs. (1) allows us to follow the beam evolution up to its late nonlinear phase. One should keep in mind, at any rate, that the predictions coming from such a model are correct only insofar as the effects of the kinetic pressure may be neglected in the momentum conservation equation, and then

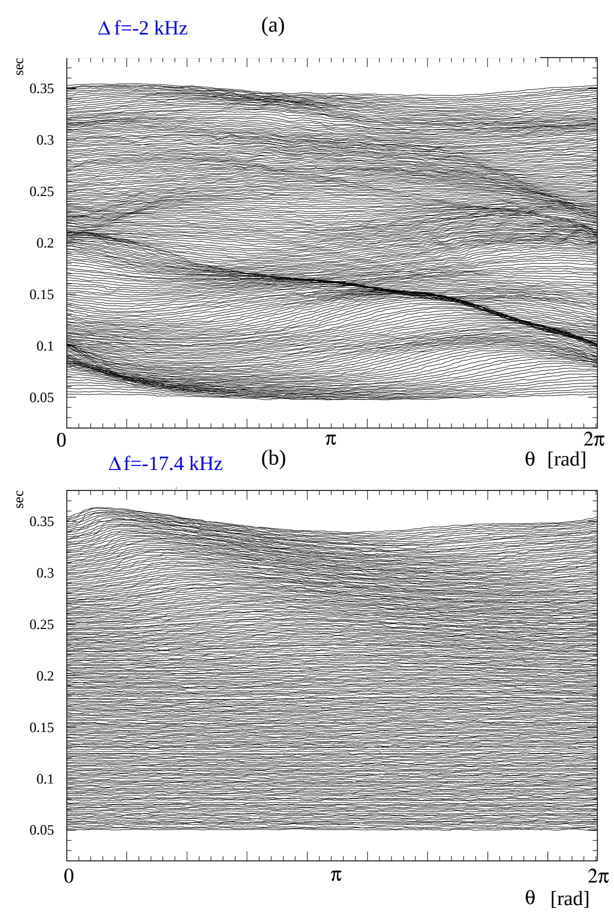

Figure 2: Waterfall diagrams. Here the line density traces along the ring are plotted over one another at several instants in the interval $50 - 350$ ms.

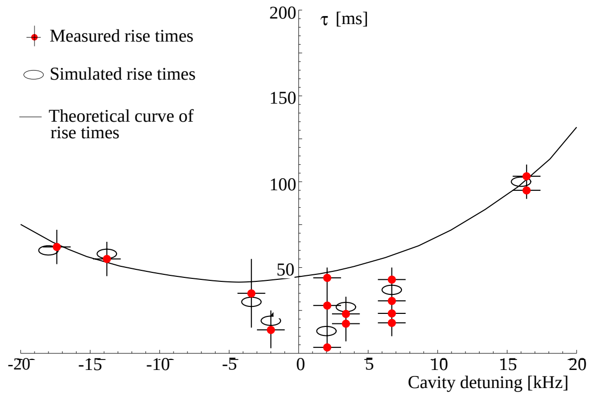

Figure 3: Rise times of the longitudinal instability: measured, theoretical, simulated.

this system of fluid equations is closed at the first order. The beam current first harmonic grows exponentially in the early phase, in agreement with the predictions of the linear theory. Similarly to what we observed in simulations as well as in measurements, higher order harmonics are produced later on, after the instabililty has gone through its linear phase and the first harmonic has become high enough to significantly drive their growth. If we compare the shapes of the line density profiles obtained by using the fluid model with those measured at the ESR before the saturation occurs and with those simulated with the PATRIC code at the same time instants in the instability development ($\Delta f = -17.4$ kHz) an excellent agreement is found out to exist (Fig. 4). Unfortunately the saturation of the instability doesn't occur in the cold fluid evolution: the line density modulation would keep going up as long as the beam is not completely bunched. Nevertheless, by having retained the nonlinear convective terms in the equations of motion we have shown that wave steepening and higher order harmonics generation are purely fluid effects having no connection with the actual kinetic structure of the beam.

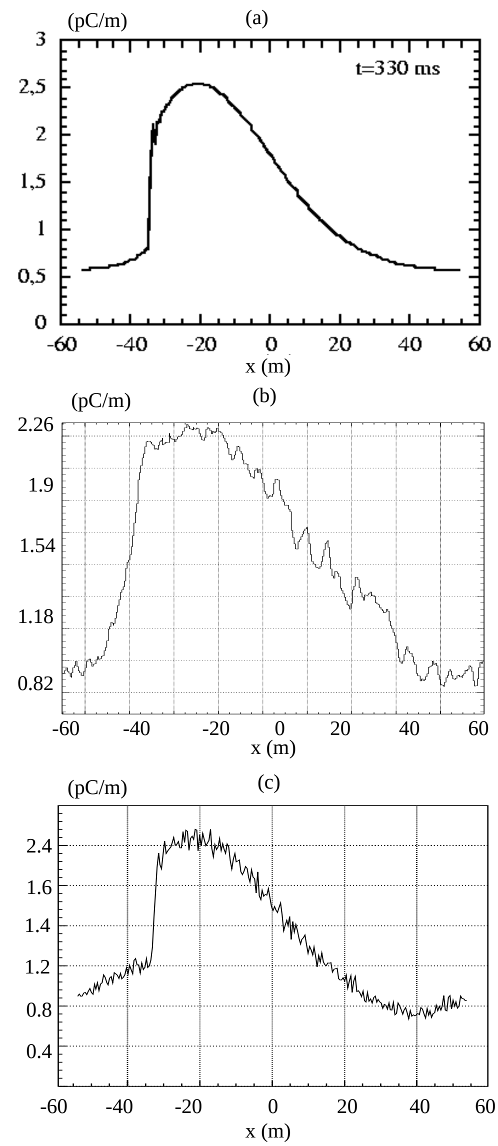

Figure 4: Distribution of the line charge density during the wave steepening phase: (a) cold fluid model; (b) ESR measurements; (c) PATRIC simulation.

4 CONCLUSIONS AND OUTLOOK

The instability rise times measured at the ESR in different operating conditions satisfactorily fit to the ones expected from linear theory and to the simulated ones. The presence of a residual RF gap voltage can significantly influence the instability rise times if $|\Delta f|_{\rm fin}$ is very close to 0.
Wave steepening and generation of higher order harmonics, which are observed late in the instability evolution, are nonlinear fluid effects. Saturation and energy spread dynamics need to be explained by futher studies.

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