EMITTANCE GROWTH OF A BEAM OWING TO COULOMB INTERACTION OF CHARGED PARTICLES MOVING THROUGH A DRIFT PATH

V.M. Khoruzhiy, M. Dreval
Kharkov Institute of Physics and Technology, Kharkov, 310108, Ukraine

Abstract

We theoretically investigated a drift of charged particle beam with current I, *normalized emittance* E_n, *Coulomb potential* U, *radius* R, *initial "temperature"* T_0 *and particle charge* q. *We obtained the analitical expression for a beam emittance growth* $\frac{E_{nk}}{E_{n0}} = \sqrt{1+\frac{qU}{2T_0\gamma^2}}$. *Another form of the expression is* $\frac{E_{nk}}{E_{n0}} = \sqrt{1+\alpha I}$ *for homogeneous density of a beam. The characteristic length of a drift for a beam emittance growth is* $\Delta z_0 = \frac{R\beta\gamma}{2}\sqrt{\frac{m_0c^2\gamma}{2qU}}$. *Emittance growth of an intensive beam is considerable value for* $\Delta z > \Delta z_0$.

As is known, Coulomb interaction of particles results in growth of a beam emittance [1]. Coulomb potential of a beam produce a reduction in longitudinal energy of particles and equal increase in transversal energy, that is in a final result in increase of a beam emittance. Real experimental installations always have spaces of drift, in which, as will be shown below, can take place an irreversible increase of an emittance for intensive charged particle beams (for example, electron or ion beams). The purpose of the given report is determine analytical expression for emittance beam growth and characteristic length for one.We shall consider a continuous cylindrical beam with initial W_0, current I, normalized emittance, $E_í$, radius R and

Coulomb potential U. Under coiiisionless process each charge particle moving in a longitudinal direction with W_0 simultaneously one occurs resulting influence from other beam particles as Coulomb transversal force

$$F = qE_{rc}.$$

In linear approximation transversal Coulomb field has a view

$$E_{rc} = G(r)r,$$

where G- gradient of an electrical field. Coulomb potential U (r) is connected to Coulomb field by a usual ratio

$$E_{rc} = -\frac{\partial U(r)}{\partial r}.$$

A value of Coulomb potential

$$U=U(0)-U(R)$$

is not depended on radius R and emittance $E_í$ of a given beam.. Increasing of transversal effective temperature of particles occurs as a result of such collective interaction of charge particles. An effective temperature of a beam have a view [2]

$$T_{0,ë} = \frac{E_{í\,0.ê}\, m_0c^2}{4R^2\gamma}, \qquad (1)$$

where m_0c^2 - rest energy, γ - relativistic factor. The indexes 0, k correspond (here and further) before and after change of value because of Coulomb interaction of particles. Owing to Coulomb interaction of particles redistribution of potential in a beam occurs, that results in reduction of longitudinal velocity of particles. The relative change of longitudinal velocity

$$\frac{\Delta v}{v} = \frac{qU}{2W_0}$$

(here $\frac{\Delta v}{v}$ <<1}) If a beam is a conservative system in a drift space, then total energy of beam particles is constant value. Longitudinal energy variation (maximum on a axis of a beam) of necessity lead to increase in transversal energy of particle. A maximum reduction in longitudinal energy $\Delta W_\parallel$ and maximum increase in transversal energy $\Delta W_\perp$ coincide

$$-\Delta W_\parallel = \Delta W_\perp = qU.$$

Transversal move Hamiltonian H of beam particles in absence of external forces is move integral [1]

$$H = p_\perp^2(r)/2m_0\gamma + qU(r)/\gamma^2 = const$$

where $p_\perp$ - a transversal impulse of particle. For r=0 and r=R we determined

$$p_\perp^2(R)/2m_0\gamma = p^2_\perp(0)/2m_0\gamma + qU/\gamma^2 \qquad (2)$$

where U=U(0)-U(R) -- Coulomb potential of a beam.
From (2) taking into account (1) we determined

$$T_{\hat{e}} = T_0 + qU / 2\gamma^2 \qquad (3)$$

Below the deviation of longitudinal particle energy $W = W_0 - 2(T_{\hat{e}} - T_0)$ is neglected, i.e. we supposed that

$$(W_0 - W)/W << 1 .$$

It is also supposed, that radius of beam R remains constant value. From (3) taking into account (1) we received

$$\frac{E_{í\hat{e}}}{E_{í0}} = \sqrt{1 + \frac{qU}{2T_0\gamma^2}} \qquad (4)$$

Let $E_{í\hat{e}} = E_{í0} + \Delta E_í$, where $\Delta E_í$ variation of normalized emittance because of Coulomb interaction of particles, then from (4)

$$\frac{E_{í\hat{e}}}{E_{í0}} = \sqrt{1 + \frac{qU}{2T_0\gamma^2}} - 1 \qquad (5)$$

As a example, we consider a proton beam with parameters W_0=100KeV (β=0.0146), I=200mA, $E_{í0}$=0.3 cm*mrad, R=0.3cm. Coulomb potential for a cylindrical beam with uniform density have a view [1]

$$U = \frac{I}{4\pi\varepsilon_0 \beta c} \qquad (6)$$

where *I*- - current of a beam, ,[I]=A , β- - normalized longitudinal velocity, c- velocity of light ([c] = m/sec), $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m. In our case *U*=411 V. From (1) we determined initial temperature T_0=235eV. Then we determined from (3) final value of temperature $T_{\hat{e}}$=440eV, then from (1) we determined $E_{í\hat{e}}$ =0.41cm*mrad. From (4) and (6) follow that the emittance increasing of a beam because of Coulomb interaction of particles is connected to a current of a beam by a ratio.

$$\frac{E_{í\hat{e}}}{E_{í0}} = \sqrt{1 + \alpha I} \qquad (7)$$

here $\alpha = \dfrac{q}{8\pi\varepsilon_0 \beta c T_0 \gamma^2}$

We'd like to notice, that according to a CERN's report minimum output emittance of a accelerator is proportional to a current of a beam in a power 1 / 3 or 1 / 2 [3]. We evaluated characteristic time Δt_0 and length Δz_0 for variation of emittance. $\Delta E_í$ .. Coulomb field has a view

$$E_{rc} = \frac{2U}{R^2} r$$

for linear approximation, then the equation of tranversal movement has a following view

$$\frac{d^2 r}{dt^2} - k_1{}^2 r = 0 \qquad (8)$$

where $k_1{}^2 = \dfrac{c}{R\gamma}\sqrt{\dfrac{2qU}{m_0 c^2 \gamma}}$.

Solution of a equation (8) is $r = r_0 \exp(k_1 t)$, where, r_0 - initial value of a particle radius. At r=R wereceived $\Delta t = \dfrac{1}{k_1}\ln(\dfrac{R}{r_0})$. For particles inside a tubularflow, limited by $r_0 = R/\sqrt{e}$ and r=R (e=2.72) in which is contained 60 percent all of particles under uniform density of a beam, we derived

$$\Delta t_0 = \frac{R\gamma}{2c}\sqrt{\frac{m_0 c^2 \gamma}{2qU}}$$

$$\Delta z_0 = \frac{R\gamma\beta}{2}\sqrt{\frac{m_0 c^2 \gamma}{2qU}} \qquad (9)$$

For above example we determined Δt_0=5.3nsec and Δz_0=2.3cm. The expression (9) actually defines a characteristic drift length of a charged particle beam, excess over one leads to a growth of a beam emittance. This effect is essential at low beam energies. Expressions (4,5) permit to determine emittance growth of a beam owing Coulomb interaction of charged particles in a drift path, the expression (7) determines emittance growth depending on a beam current and expression (9) determines characteristic length for emittance growth .

REFERENCES

[1]. Kapchiskiy I.M. The theory of linear resonance accelerators. Moscow, Publishing House "Energoizdat", 1982

[2]. Lawson J.D, The physics of charged-particle beams. Clarendon Press, Oxford, 1977 (in Russian: Publishing House "Mir", Moscow, 1980, p.206 "Mir" (1980)).

[3]. Smith et al In: Proceedings of the 1968 proton linear accelerator conference, Brookhaven, p.433.