

ANALYSIS OF THE MULTIPURPOSE SEXTUPOLE OF SOLEIL USING HALBACH'S PERTURBATION THEORY AND THE OPERA-2D CODE*

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Abstract

The SOLEIL storage ring sextupole is a multipurpose magnet. In addition to its primary function as a sextupole, it provides horizontal and vertical dipolar fields and skew quadrupolar field using auxiliary sets of coils. In this paper, we will discuss the application of Halbach's perturbation theory to calculate the required dipolar excitation. We will also show the effectiveness of this theory in determining the extra multipolar components introduced by the additional fields and their strengths. Finally, we will show the good agreement between these results and those provided by the OPERA code.

1. INTRODUCTION

In a storage ring, the use of sextupoles is essential both for correcting chromaticity and reducing nonlinearities. There are 112 sextupoles provided for the storage ring of the SOLEIL project. In order to save space, the dipolar correctors for closed orbit correction and skew quadrupoles for betatron coupling correction are planned to be inserted as auxiliary coils inside the sextupole. The aim of this paper is to present the results of the analysis of this multipurpose sextupole using an analytical approach based on a perturbation theory for iron-dominated magnets developed by K. Halbach [1] and to compare these results with those obtained using the OPERA-2D code [2].

Three auxiliary coils (vertical and horizontal steering and skew quadrupole) will create additional fields superimposed to the main sextupole field. This can induce a loss of sextupole symmetry and introduce further multipoles. Systematic multipoles of the sextupole can to a great extent be reduced by changes in design details, however the multipole values due to the auxiliary coils are fixed and cannot be improved. If their effect on dynamic aperture is unacceptable, the only alternative is to remove them from the sextupole. Halbach's perturbation theory is a good tool to investigate this kind of problem as already pointed out by S. Marks [3].

Fig. 1 shows a cross sectional view of the SOLEIL storage ring sextupole and Table 1 gives its main characteristics.

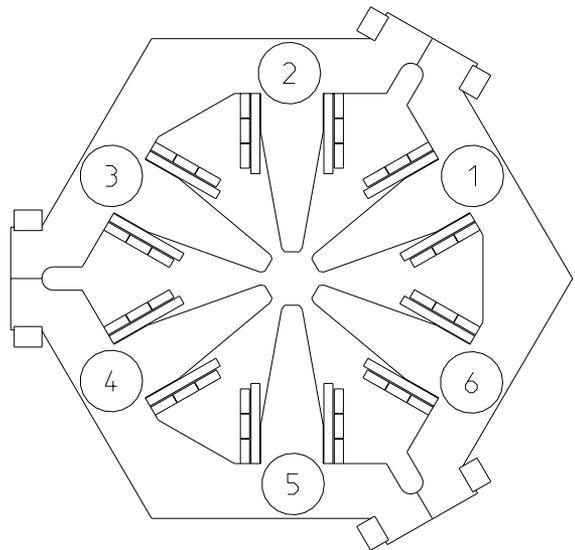


Fig. 1. Sextupole cross section.

In order to avoid mechanical interference with 1° and 4° beamlines, the yoke width had to be minimized on one side in the median plane. For this purpose, the yoke presents a notch in the median plane. Initially this was repeated 6 times to maintain a 6th-order symmetry, but calculations showed that it was possible to suppress half of the notches with a neglectable effect. This reduction to 3rd-order symmetry increases mechanical stiffness. Then, each elementary steel plate has two poles whose sizes permit installation of main and correcting coils.

A supplementary mechanical constraint is the vacuum chamber which imposes a 19 mm minimal distance between two poles. This will indeed constitute a limitation for the 30-pole component optimization.

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Table 1. sextupole main characteristics.

Max. field strength	320 T/m ²
Bore diameter / Yoke length	78 10 ⁻³ m / 0.16 m
Good field region	30 10 ⁻³ m
Sextupole ampere-turns	5 069
Horizontal steering	0.0275 T
Vertical steering	0.0275 T
Skew quadrupole	32 10 ⁻² T/m

2. OUTLINE OF HALBACH'S PERTURBATION THEORY

The first order perturbation effects in iron-dominated 2-D symmetrical magnets are expressed in terms of generation or changes in multipole coefficients [1].

For a symmetric 2N-pole magnet, the potential is :

$$F(z) = \sum C_N(2m+1) z^{N(2m+1)} \quad (1)$$

Where C_n are the multipole coefficients. The term C_N is the fundamental and the odd multiples are the allowed harmonics. The comparison between the field components H_n and H_N is written as a function of these coefficients :

$$\left| \frac{H_n}{H_N} \right| = \frac{n}{N} \frac{C_n}{C_N} z^{n-N} \quad (2)$$

For a particular perturbation, the theory provides the sensitivity of the coefficients $\Delta C_n(o)$ for a reference pole centered in the horizontal axis. Numerical values are given in Halbach's paper as normalised coefficients $\frac{n}{N} \frac{\Delta C_n(o)}{i\epsilon}$ where ϵ is the strength of the perturbation [1].

The effect of this same perturbation applied to a pole whose symmetry axis is rotated by α is described by $\Delta C_n(\alpha)$ and is obtained as follows :

$$\Delta C_n(\alpha) = \Delta C_n(o) \sum_j e^{-in\alpha_j} \quad (3)$$

The equation (2) can then be re-written as :

$$\left| \frac{H_n}{H_N} \right| = i\epsilon \left(\frac{n}{N} \frac{\Delta C_n(o)}{i\epsilon} \right) \sum_j e^{-in\alpha_j} \quad (4)$$

3. HORIZONTAL DIPOLAR CORRECTION

This requires a vertical dipolar field, which can be obtained by using the six dipolar coils shown in Fig. 1. The coils corresponding to the poles 1, 2, 3 are of opposite polarity to those of 4, 5, and 6. From symmetry, we will give the same current I_1 to the coils 1, 3, 4 and 6 and the same current I_2 to the coils 2 and 5. As expected, these two dipolar excitations will create two unwanted sextupolar components and modify the main sextupolar excitation. These two sextupoles are respectively

proportional to the pole angular positions as pointed out by the equation (4) :

$$I_1 \alpha \sum_{j=1}^4 e^{-3i\alpha_j} = \left(e^{-3i\frac{\pi}{6}} - e^{+3i\frac{\pi}{6}} \right) + \left(e^{-3i\frac{5\pi}{6}} - e^{+3i\frac{5\pi}{6}} \right) = -4i \quad (5)$$

$$I_2 \alpha \sum_{j=1}^2 e^{-3i\alpha_j} = \left(e^{-3i\frac{\pi}{2}} - e^{+3i\frac{\pi}{2}} \right) = +2i$$

Therefore it appears that in order to eliminate the sextupolar components we must apply a current I_2 twice as high as I_1 .

The required values for ampere-turns can be calculated with the use of Halbach's perturbation coefficients. For $n = 1$ (dipole) and $N = 3$ (sextupole), the value of the normalised coefficient is : $\frac{1}{3} \frac{\Delta C_1(o)}{i\epsilon} = 0.0979$ [1]. The

factor $\sum_{j=1}^6 e^{-i\alpha_j}$ being equal to $-6i$, we can deduce the

value of the strength of the perturbation ϵ for the maximum value of the dipole :

$$\epsilon = \frac{H_1/H_3}{6 \times 0.0979} = 0.0961 \quad (6)$$

where $H_1 = 0.0275$ T and $H_3 = 0.487$ T.

The sextupole excitation being $NI \approx 5\,069$ A.t, the ampere-turns required for dipolar horizontal correction are then :

$$(NI_1) = 0.0961 \times 5\,069 = 487 \text{ A.t} \quad (7)$$

$$(NI_2) = 2 \times 487 = 975 \text{ A.t}$$

The same values were independently predicted by OPERA-2D calculations.

Other multipoles generated can not be compensated and their effect on beam dynamics must be taken into account. Table 2 sums up their relative strengths calculated at $z = 30$ mm, using both perturbation theory and OPERA-2D code.

Table 2. Multipoles generated by horizontal dipolar correction.

n	5	7	11
H_n/H_N (theory)	$2.4 \cdot 10^{-2}$	$2.7 \cdot 10^{-3}$	$-6.9 \cdot 10^{-4}$
H_n/H_N (OPERA-2D)	$2.6 \cdot 10^{-2}$	$3.3 \cdot 10^{-3}$	$-5.4 \cdot 10^{-4}$

The agreement between the two calculations is very good.

4. VERTICAL DIPOLAR CORRECTION

In this case a horizontal dipolar field is needed. As shown in Fig. 1, this can be obtained by using four dipolar coils driven by the same current I_3 . The coils of the poles 1 and 6 are of opposite polarity to those of 3 and 4. Coils 2 and 5 are not used in this case.

Using the same calculations as before, we determine the corresponding value for the perturbation strength, $\epsilon = 0.167$. The ampere-turns are then :

$$NI_3 = 0.167 \times 5\,069 = 850 \text{ A.t.}$$

The strengths of the multipoles generated by this horizontal dipolar field are summarized in Table 3.

Table 3. Multipoles generated by vertical dipolar correction.

n	5	7	11
H_n/H_N (theory)	$-2.4 \cdot 10^{-2}$	$2.7 \cdot 10^{-3}$	$6.9 \cdot 10^{-4}$
H_n/H_N (OPERA-2D)	$-2.5 \cdot 10^{-2}$	$2.8 \cdot 10^{-3}$	$7.6 \cdot 10^{-4}$

Once again the agreement is very good.

5. SKEW QUADRUPOLE

We can produce a skew quadrupole by using two coils on poles 2 and 5 with the same polarity, driven by the same current I_4 . In this case $n=2$ and $N=3$, the corresponding normalised perturbation coefficient is 0.156. The maximum value required for the skew quadrupole was determined to be 0.32 T/m which corresponds to a peak field of 0.0125 T. The factor $\sum_{j=1}^2 e^{-2i\alpha_j}$ is equal to $2 \cos \pi = -2$. Therefore the value of

the perturbation strength is $\epsilon = 0.082$. The needed ampere-turns are then $NI_4 = 0.082 \times 5\,069 = 416 \text{ A.t.}$

The non-zero values of the multipoles generated by this field are given in Table 4.

Table 4. Multipoles generated by skew quadrupole.

n	4	8	10
H_n/H_N (theory)	$1.7 \cdot 10^{-2}$	$-4.7 \cdot 10^{-4}$	$-2.4 \cdot 10^{-4}$
H_n/H_N (OPERA-2D)	$1.7 \cdot 10^{-2}$	$-4.8 \cdot 10^{-4}$	$-2.6 \cdot 10^{-4}$

The agreement is as good as in previous cases.

6. POLE DESIGN/FEM CALCULATIONS WITH OPERA 2D CODE

Because of the presence of dipolar and skew quadrupolar correctors, two 2D models were used : a half sextupole and an entire one.

Poles were widened as much as possible to limit saturation. OPERA-2D results in Tables 2, 3, 4 are obtained using finite permeability curves, although the perturbation theory assumes $\mu = \infty$. The closeness of figures confirms that overall saturation is limited.

A 3-facet pole shape was chosen, as studies with cubic shape plus shims proved not to bring much improvement. In addition, it would be too complicated in the case of a post-assembly machining of poles.

Optimization of systematic sextupolar components was performed by varying the position of pole extremities (Fig. 2). Point (x_0, y_0) controls the 18-pole component while point (x_1, y_1) controls the 30-pole component. Nevertheless, they are not totally independent and 30-pole reduction is limited by the finite width of the pole.

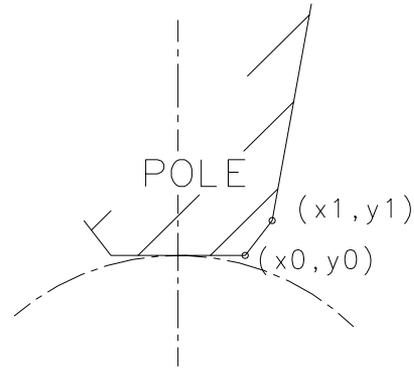


Fig. 2. Pole profile.

It must be said that a C-shaped yoke could have been chosen [3]. Nevertheless, such a design modifies sextupolar field by introducing new multipolar harmonics. Concerning horizontal steering, the C-shape introduces an equal perturbation on each pole : thus, theoretical A-turns balance (NI in poles 1, 3, 4, 6 and $2NI$ in poles 2, 5) is altered and consequently imposes an adjustment of A-turns in poles 2 and 5, either by means of power supplies or by the number of turns per coil. For the sake of standardization of coils and power supplies, this solution was not adopted.

CONCLUSION

Horizontal and vertical dipolar corrections in addition to skew quadrupole field are incorporated in the sextupole magnet as auxiliary coils. The use of Halbach's perturbation theory has allowed to decouple the four magnetic fields and evaluate the strength of the multipole components. The results are in full agreement with OPERA-2D calculations.

REFERENCES

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