PROGRESS ON INTENSE PROTON BEAM DYNAMICS AND HALO FORMATION

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Abstract

A major problem in the design of high intensity proton linacs is how to avoid particle losses. Losses cause activation of accelerator components and make unconstrained maintenance difficult. In a linac, losses occur radially due to the formation of beam halo. In addition, filamentation of the particle distribution in the longitudinal phase space can cause activation problems when injected into a ring.

In recent years, progress has been made in understanding halo production due to parametric resonances between single particles and the oscillating mismatched beam core. The mismatch of isotropic DC beams is described by 2 well known eigenmodes. For bunched beams 3 eigenmodes exist. The frequencies of these modes can be approximately expressed by the the full and zero current transverse and longitudinal tunes only. The knowledge of these eigenmodes allows identification of parametric resonance conditions for bunched beams. Correlations between halo production and parametric resonances have been verified by Monte Carlo simulations for bunched beam transfer lines and a high current linac.

1 INTRODUCTION

The major problem of the design of high current proton linacs is the loss of particles at higher energies. Particle loss leads to activation of accelerator components and reduces the flexibility of hands of maintenance. As a rule of thumb hands on maintenance is possible if the loss is less than 1 W/m. As losses occur radially only the transverse motion of particles is considered in, for example, a waste transmutation linac [1]. If a storage ring or a circular accelerator follows the linac, like for spallation sources, the longitudinal motion of particles has to be considered too [2].

We are still in the process of understanding and finding in detail the sources of particle loss and halo dynamics. The beam halo consists of a 'small' number of particles which oscillate around the bunch core. In recent years substantial progress has been achieved by identifying the parametric resonance conditions as a major source of halo production of DC beams. In the particle-core model particles outside the beam experience a nonlinear space charge force. This results in a single particle tune spread. Parametric resonances can occur between single particles tunes and the frequency of the oscillating mismatched beam core [3,4,5,6,7,8]. For realistic particle distributions with nonlinear space charge forces particles inside the core have a tune spread too. This one dimensional model describes the process of how particles can leave the beam core [9,10,11].

In this presentation the one dimensional parametric resonance model is generalized to bunched beams. Due to the two transverse and the longitudinal bunch dimension 3 eigenmodes exist for the mismatched envelopes.

The paper is organized in the following way. The three eigenmodes of bunched beams are derived in section 2 and verified numerically in section 3. Correlation between halo production and parametric resonances are verified by Monte Carlo simulation for a bunched beam transferline in section 4 and for a high current linac in section 5.

2 THE THREE ENVELOPE MODES OF MISMATCHED BUNCHED BEAMS

For the analytical approximation of the eigenfrequencies (modes) of the mismatched envelopes it is assumed that the beam is of ellipsoidal shape with uniform charge density. This results in linear space charge forces. In the rest frame of the bunch the bunch radii are denoted by a_x, a_y and a_z. If the bunch moves with the velocity v in longitudinal direction then the bunch length b in laboratory system is given by $b = a_z/\gamma$, where γ is the relativistic mass factor. The external forces for focusing and bunching are assumed to be linear and periodic in the longitudinal direction s with period length L. The envelope equations are given by

$$a_x'' + k_{xo}^2 a_x - \frac{IK_x}{a_y b} - \frac{\epsilon_t^2}{a_x^3} = 0,$$

$$a_y'' + k_{yo}^2 a_y - \frac{IK_y}{a_x b} - \frac{\epsilon_t^2}{a_y^3} = 0,$$

$$b'' + k_{zo}^2 b - \frac{IK_z}{a_x a_y} - \frac{\epsilon_z^2}{b^3} = 0.$$

Here k_{xo}, k_{yo} and k_{zo} are the external periodic force constants. K_x, K_y and K_z are proportional to the elliptical formfactors [12] and depend on the bunch dimensions too. I is the bunch current and ϵ_t and ϵ_z are the transverse and longitudinal emittances. This system of nonlinear coupled differential equations exhibits oscillating stable or unstable solutions. 'Matched' solutions have the same periodicity as the external focusing system and are denoted by a_{xo}, a_{yo} and b_o. For studying mismatched solutions it is appropriate to express the beam radii as a sum of the matched periodic solution and a mismatch

$$a_x = a_{xo} + \Delta a_x,\ a_y = a_{yo} + \Delta a_y,\ b = b_o + \Delta b.$$

Assuming small mismatches the equations can be linearized giving three coupled differential equations of Hill's type for the mismatches Δa_x, Δa_y and Δb. Applying now smooth approximation it is possible to solve the coupled envelope equations. One gets three eigenfrequencies, a pure transverse **quadrupolar mode**

$$\sigma_{env,Q} = 2\sigma_t$$

and a **high** and **low mode** which couple the transverse and longitudinal directions

$$\sigma^2_{env,H} = A + B,\ \sigma^2_{env,L} = A - B$$

with

$$A = \sigma^2_{to} + \sigma^2_t + \frac{1}{2}\sigma^2_{lo} + \frac{3}{2}\sigma^2_l$$

and

$$B = \sqrt{\left(\sigma^2_{to} + \sigma^2_t - \frac{1}{2}\sigma^2_{lo} - \frac{3}{2}\sigma^2_l\right)^2 + (\sigma^2_{to} - \sigma^2_t)(\sigma^2_{lo} - \sigma^2_l)}.$$

The mismatch modes are expressed by the full and zero current transverse and longitudinal tunes σ_t, σ_{to}, σ_l and σ_{lo}. The high and low mode have been investigated already [13]. The high and low mode correspond in some respect to the even and odd modes of an unisotropic DC beam. For isotropic DC beams the well known mode frequencies are given by

$$\sigma^2_{env,e} = 2\sigma^2_t + 2\sigma^2_{to},$$
$$\sigma^2_{env,o} = 3\sigma^2_t + \sigma^2_{to}.$$

The quadrupolar mode has the same properties as the odd mode of isotropic DC beams but with lower frequency.

In smooth approximation one gets in the case of the quadrupolar mode for the corresponding eigensolutions

$$\Delta a_x = -\Delta a_y \sim cos(\sigma_{env,Q} \cdot s/L), \Delta b = 0.$$

Here only a transverse mismatch is present and it is of opposite phase. In case of the high and low mode one has

$$\Delta a_x = \Delta a_y = g_{H/L}\Delta b \sim cos(\sigma_{env,H/L} \cdot s/L)$$

with the amplitude factors

$$g_{H/L} = \frac{\sigma^2_{to} - \sigma^2_t}{\sigma^2_{env,H/L} - 2(\sigma^2_{to} + \sigma^2_t)}.$$

g_H is always positive and g_L always negative. The high mode represents a pure 'breathing' of the ellipsoidal bunch. For the low mode the bunch breathes in transverse direction but the oscillation in longitudinal direction is of opposite phase. Any arbitrary mismatch can be expressed by superpositions of the three eigensolutions. The analytical formulas presented are derived by approximating the derivative of the formfactors K_x, K_y and K_z. This approximation is not valid for extensively elongated bunches where the mode-frequencies and the amplitude factors depend on the aspect ratio a_{xo}/b_o and a_{yo}/b_o [14].

3 NUMERICAL INVESTIGATIONS WITH LINEAR SPACE CHARGE FORCES

For periodic external focusing there exist stable and unstable solutions of the envelope equations. The unstable case can happen if one of the envelope tunes is near to 180^o. Stable and unstable situations have been studied numerically for a periodic bunched beam transportline. The geometry of the period is very similar to the first period of the coupled cavity linac (CCL) of the proposed European Spallation Source (ESS) [15,16]. Each period consists of two accelerating (bunching) cavities followed by a doublet for transverse focusing. The proton energy is 70 MeV and the beam current 214 mA.

3.1 Stable Case

For the stable case the transverse full current tune was set to 60^o per period. The matched radii for all three directions are shown in Fig. 1. Note that in longitudinal direction one has a weak focusing system giving an almost constant bunch length. For a small relative initial mismatch the envelope equations have been solved numerically for 20 periods and compared to the analytical approximation.

As an example two modes are shown in Fig. 2 and Fig. 3. There the relative mismatches $\Delta a_x/a_{xo}$, $\Delta a_y/a_{yo}$ and $\Delta b/b_o$ are plotted (top, middle, bottom) over 20 periods. The markers present the numerical solution of the nonlinear coupled envelope equations. The curves are cosine functions with the envelope tune σ_{env} as the only free parameter. The envelope tunes are adjusted to fit the numerical data. The resulting values for the envelope tunes are listed in Table 1 and compared to the theoretical values calculated by the formulas given above. Also listed are the tunes for this example and the two amplitude factors. Fig. 2 shows the 5% excitation of a quadrupolar mode. The horizontal and vertical oscillation have opposite phase and no excitation exists in the longitudinal direction. Fig. 3 shows the excitation of the high mode with 5% radial and 7.5% longitudinal excitation. For the high mode all three amplitudes are in phase.

Table 1: Parameters of the stable transport line

σ_t	σ_{to}	σ_L	σ_{lo}
60^o	87^o	72^o	90^o
$\sigma_{env,Q,the}$	$\sigma_{env,Q,fit}$	$\sigma_{env,H,the}$	$\sigma_{env,H,fit}$
120^o	130^o	167^o	168^o
$\sigma_{env,L,the}$	$\sigma_{env,L,fit}$	g_H	g_L
135^o	141^o	0.70	-0.94

3.2 Unstable Case

To test the unstable case the transverse tune σ_t was increased to 75^o. This gives an analytically calculated en-

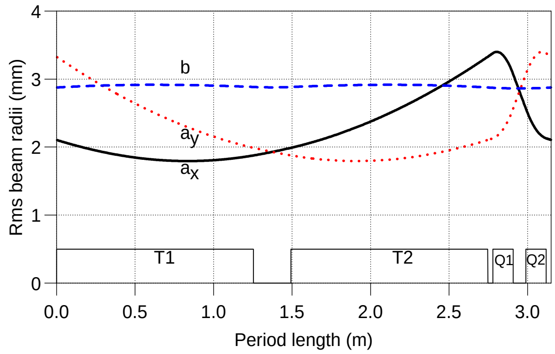

Figure 1: The matched beam radii along one period. T1, T2 bunching cavities, Q1, Q2 quadrupoles

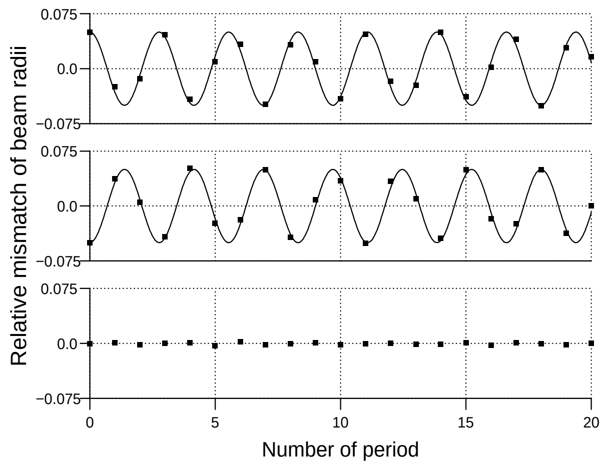

Figure 2: Excitation of the quadrupolar mode

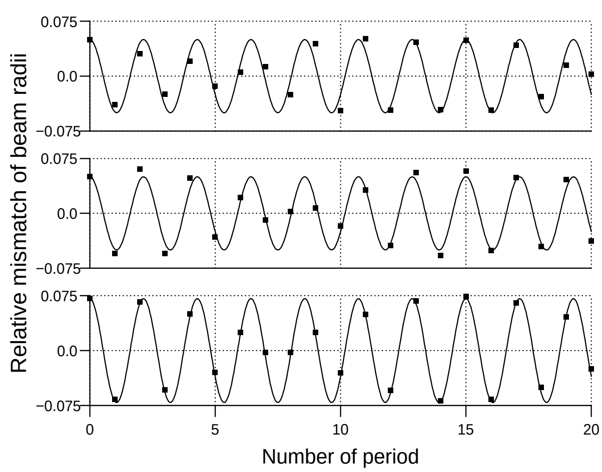

Figure 3: Excitation of the high mode

Table 2: Parameters of the unstable transport line

σ_t	σ_{to}	σ_l	σ_{lo}
75^o	103^o	69^o	90^o
$\sigma_{env,H,the}$	$\sigma_{env,H,fit}$	$\sigma_{env,Q,the}$	$\sigma_{env,L,the}$
188^o	192^o	150^o	141^o

velope tune for the high mode $\sigma_{env,H,the}$ of 188^o. The other two mode tunes are less than 180^o. All tunes are listed in Table. 2. Because $\sigma_{env,H,the}$ is nearby 180^o an unstable behaviour is expected. In Fig. 4 the excitation of this mode is shown by solving the envelope equations numerically with an initial mismatch of 5% radially and 2.5% longitudinally. The initial mismatch grows by a factor 10 after 20 periods. The curve corresponds to the function $e^{\alpha s/L} \cdot cos(\sigma_{env,H,fit} \cdot s/L)$. α is the growth rate and the fitted number is equal to 0.11 per period. Because of the nonlinearity of the envelope equations the oscillation frequency is amplitude dependent. Therefore it cannot be expected that the simple analytical curve works for larger mismatches.

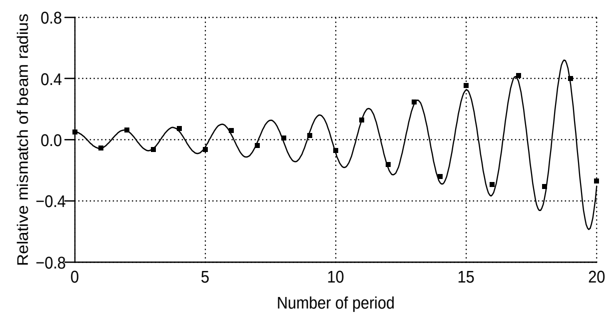

Figure 4: Relative mismatch in y-direction for the high mode of the unstable case

4 MONTE CARLO SIMULATIONS OF A STABLE TRANSPORT LINE

It is important to do multiparticle calculations of the bunched beam transfer line and compare the results with the model. There is an important difference. Due to phase space filling the multiparticle simulations have nonlinear space charge forces included. However the rms quantities are mainly determined by the linear space charge forces [17]. Therefore it is expected to see the above discussed mode excitation in the rms beam radii as long as the rms emittances are not changing significantly. In Fig. 5 the rms emittances are shown for the above discussed stable case with a rms matched 6d waterbag input distribution. The Monte Carlo simulations are done with 20 000 particles which interact fully in 3d. The results shown for 80 periods correspond to 20 plasma wavelengths for the bunched beam.

As an example in Fig. 6 the excitation of the high mode is shown. The dots are the values of the rms radii obtained by multiparticle calculation whereas the the solid line represents an analytical curve with the same frequency as in Fig. 3. The initial mismatch of this simulation is 20% radially and 30% longitudinally. As expected the excitation of the high mode is clearly visible.

Up to now the study has concentrated on the rms quantities which describe the core part of the bunch. Particles in the halo are strongly effected by nonlinear space charge

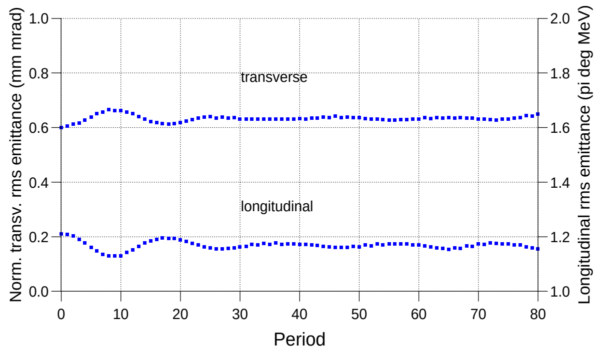

Figure 5: Rms emittances of the stable transferline

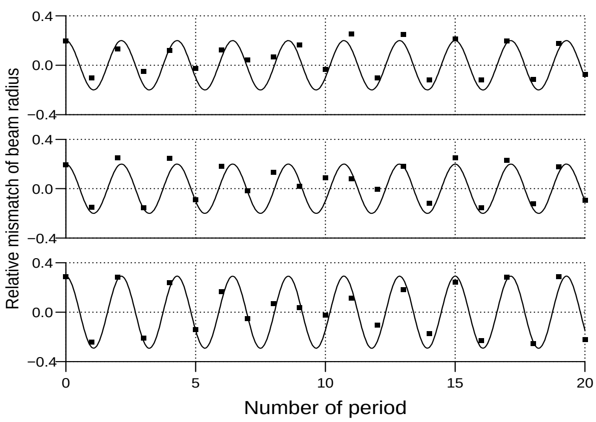

Figure 6: Excitation of the high mode. Dots are data from Monte Carlo simulation

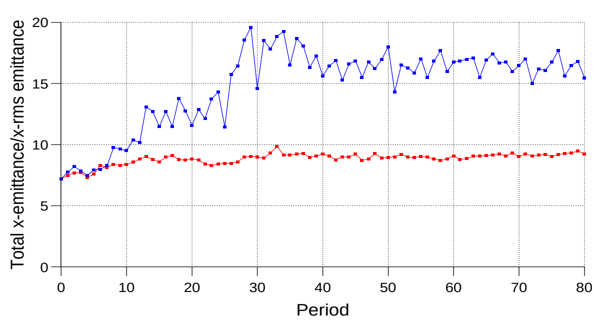

Figure 7: 99.9% total to rms emittance ratio for a matched (bottom) and a quadrupolar mode excited case (top)

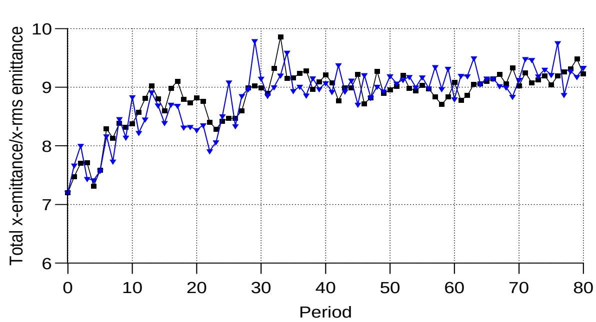

Figure 8: 99.9% total to rms emittance ratio for a matched (squares) and a high mode excited case (triangles). Please note the enlarged scale

forces resulting from the initial distribution. The tunes of the individual particles are distributed between the full current and zero current tune. Due to oscillation of the mismatched radii single particles can experience parametric resonances. Contrary to the one dimensional case different single particle tune spreads exist in the radial and longitudinal direction. Also the three different envelope tunes complicate the situation. The condition for exciting a parametric resonance either radially or longitudinally is given by

$$\frac{\sigma^p_{t,l}}{\sigma_{env}} = \frac{m}{n} = \frac{1}{2}, \frac{1}{3}, \ldots$$

with

$$\sigma_t \le \sigma^p_t \le \sigma_{to},$$

$$\sigma_l \le \sigma^p_l \le \sigma_{lo}$$

where σ_{env} is one of the three envelope tunes of the mismatched radii and $\sigma^p_{t,l}$ the single particle tune.

The low order resonances are the most dangerous ones. For the radial direction the 1/2 parametric resonance is always excited by the quadrupolar mode. The high or low mode can excite a parametric resonance either in the transverse or longitudinal direction. The frequency of the high mode should be limited below 180^o in order to avoid an envelope instability. As pointed out before a mismatch with equal amplitudes in radial and longitudinal directions leads to an excitation of the high and low mode simultaneously. The parametric resonance model gets more complicated if the rms emittances are changing. Reasons for emittance change can be an envelope instability, particle redistribution under high space charge forces and temperature exchange.

Fig. 7 and 8 show the 99.9% total to rms emittance ratio in x-direction. In Fig. 7 the matched case is compared to a 20% quadrupolar mode excitation. A substantial increase of the 99.9% emittance is visible due the 1/2 parametric resonance excitation. The resonance condition is fulfilled for 65^o radial single particle tune. Particles with such a tune start close to the core. In Fig. 8 the same emittance ratio as in Fig. 7 is shown but here comparing the matched case with a by 20% radially and 30% longitudinally excited high mode. As predicted no resonance effect can be seen because radially a single particle tune of 84^o is needed to excite the 1/2 parametric resonance. There are no particles with such a tune in the distribution.

5 MONTE CARLO SIMULATION OF THE ESS LINAC

All the results above are for a bunched beam transfer line, where particle are not accelerated. The conclusions are also valid for the design of a high current linac. As an example Monte Carlo results are shown for the 214 mA ESS coupled cavity linac which accelerates the beam from 70 MeV up to 1.334 GeV. The injection parameters at 70 MeV are about the same as for the discussed transferline. The input distribution is 6d waterbag. The ratio between full and

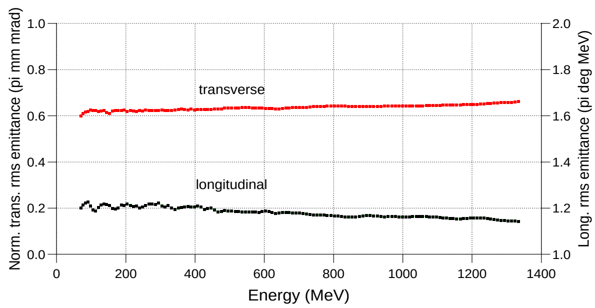

Figure 9: Rms emittances along the ESS linac for the matched case

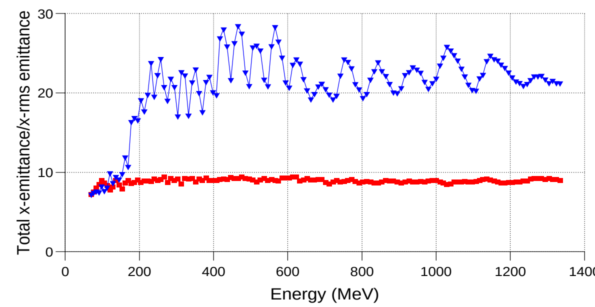

Figure 11: 99.9% total to rms emittance ratio in x-direction for a matched (bottom) and a mismatched case (top)

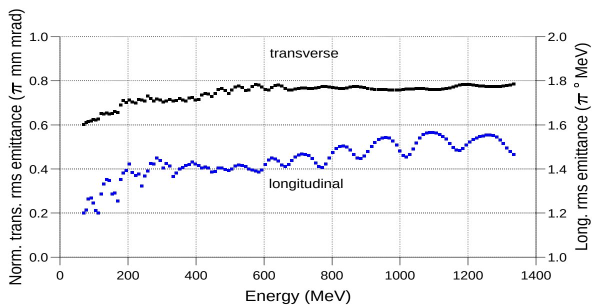

Figure 10: Rms emittances along the ESS linac for the mismatched case

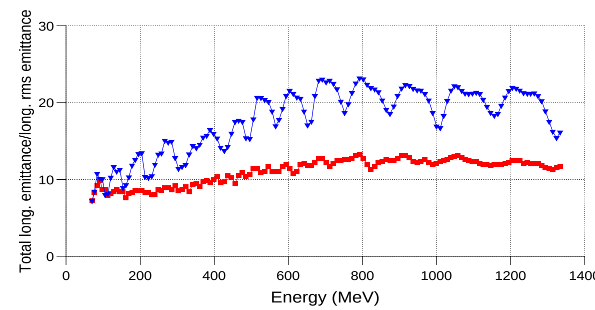

Figure 12: 99.9% total to rms emittance ratio in z-direction for a matched (bottom) and a mismatched case (top)

zero current tunes is greater than 0.7 both transversely and longitudinally, all along the linac. The Monte Carlo simulations are done with 50 000 particles.

Fig. 9 shows the normalized rms emittances along the linac. Fig. 10 shows the rms emittances for a mismatched case where all three modes are excited simultaneously by injecting the beam with +20% horizontal, -20% vertical and -20% longitudinal mismatch. In the radial plane the 1/2 parametric resonance is excited by the quadrupolar mode. Longitudinally one has a excitation of the 1/2 parametric resonance by the low mode. As a consequence all three rms emittances are increased by more than 30%. The 99.9% to rms emittance ratio are significantly increased transversely and longitudinally compared to the matched case, see Fig. 11 and Fig. 12.

During the startup of period of high intensity linacs more than 20% initial mismatch are hard to avoid especially in the longitudinal plane due to adjustment of the bunch length. Pariticle loss should be limited for the startup period. In space charge dominated linac designs where the tune depression is below 0.4 chaotic single particle motion even for 20% initial mismatch have been observed [5,8]. For spallation source linacs with its restrictions on loss free ring injection a design is required which is insensitive to transverse and longitudinal mismatch.

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