DESIGN OF MINIMUM EMITTANCE LATTICES

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Abstract

We design minimum emittance n-bend achromat (nBA) lattices that can be used as a damping ring of the next linear collider or as a light source. Two dipole lengths are needed for a proper dispersion function matching. For the triple bend achromat (TBA) lattice with twelve superperiods, we obtain an emittance of 2.98 nm at 2 GeV that is four times less than 12.17 nm of the Pohang Light Source (PLS) in Korea. For nBA, two five bend achromat (FBA) lattices with six superperiods are designed. The concept can be extended to design any nBA with $\mathrm{n} \geq 4$ by varying the number of the common modules in a superperiod. The obtained emittances 5.32 nm and 5.17 nm at 2 GeV are about three times larger than the theoretical minimum of 1.82 nm. dynamic apertures of the proposed TBA and FBA lattices are also studied.

1 INTRODUCTION

Electron storage rings have been widely used as a light source for applications in basic physics, chemistry, biology, industry, etc. In many applications, it is desirable to have high brightness beam, which requires a small emittance. Due to the equilibrium between the radiation damping and quantum fluctuation, the horizontal natural emittance of electron beam in a storage ring is given by [1]

$$\epsilon_x = C_q \gamma^2 \frac{\langle \mathcal{H}/|\rho|^3 \rangle}{J_x \langle 1/\rho^2 \rangle}, \tag{1}$$

where $C_q = 3.84 \times 10^{-13}$ m, ρ is the bending radius of dipoles, γ the relativistic factor, J_x is the damping partition number, and

$$\mathcal{H} = \frac{1}{\beta_x}[D^2 + (\alpha_x D + \beta_x D')^2] \tag{2}$$

is the dispersion action. α_x and β_x are Courant-Snyder parameters, D is dispersion function, and $D' = dD/ds$. Acccelerator physicists have realized that there is an achievable minimum emittance. By minimizing $\langle \mathcal{H} \rangle / J_x$ in dipoles, a minimum emittance can be attained. A lot of efforts were taken to design minimum emittance lattices [2-7].

In the past, lattices used to obtain small emittance electron beams are double bend achromats (DBA), double bend triplet achromats, triple bend achromats (TBA), etc. More recently, a possibility of multiple bend achromats has also been considered. An advantage of a multiple bend achromat is its compactness and small emittance, and it is very suitable for a damping ring of the next linear collider or for a light source. Construction cost can be lowered by making a lattice compact. Thus a realistic design of a minimum emittance multiple bend achromat is important.

Recently, Lee finds that the necessary condition for a minimum emittance multiple bend achromat is that the outer dipole should be $3^{1/3}$ shorter than the center dipoles in order to attain the achromat matching condition [7]. The theoretical minimum emittance of the matched multiple bend achromat is

$$\epsilon = \frac{1}{4\sqrt{15}} \frac{C_q \gamma^2 \theta_1^3}{J_x}, \tag{3}$$

where θ_1 is the bending angle of the shorter dipoles (the outer dipole shown in Fig. 1) in each superperiod. Following basic design principles of Ref. [7], this paper attempts to design minimum emittance multiple bend achromat lattices. We design a triple bend achromat (TBA) lattice and then five bend achromat (FBA) lattices. In reality, some of the optical matching conditions stated in Ref. [7] have to be compromized to attain a compact lattice.

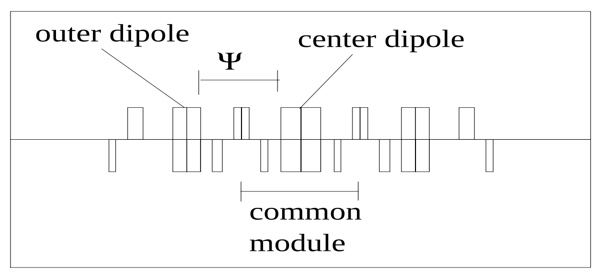

Figure 1: Layout of a triple bend lattice. Note that the center dipole is $3^{1/3}$ times longer than the outer dipole. Common module and Ψ the phase advance are marked. For the design of a nBA with $\mathrm{n} \geq 4$, additional constraints $\alpha_x = 0$, $\alpha_y = 0$, and $D'_x = 0$ are imposed at the both ends of common module. Symmetric common modules can be inserted into the lattice for a minimum emittance nBA.

This paper reports our efforts in the design of multiple bend achromat lattices. We organized our paper as follows. Section 2 discusses the basic design principle of TBA and FBA lattices. Section 3 discusses beam dynamics issues.

2 DESIGN OF MULTIPLE BEND ACHROMAT LATTICES

In this paper, we only consider separate function isomagnetic lattices. In the design of minimum emittance achromat lattices, the achromat matching condition requires dipoles with two lengths where the outer dipoles of an achromat module is a factor $3^{1/3}$ shorter than the inner dipoles. In order to increase dynamic aperture of the lattice, we also take into account half integer chromatic stopbands and third order resonances.

2.1 Triple bend achromat (TBA)

For TBA, Fig. 2 shows schematically the layout of our lattice. Optical match can be carried out easily by MAD and SYNCH. The proposed TBA lattice has twelve superperiods and a 6.8 m straight section per each superperiod for RF cavities, wigglers, etc. The nominal energy of the presented TBA lattice is set to 2 GeV for convenience in order to compare with the properties of the Pohang Light Source (PLS) in Korea. The circumference is 240.96 m and the emittance is 2.98 nm, while the theoretical minimum emittance is 1.41 nm. This is reflected from the fact that the required phase advance Ψ of $127.76°$ between the ends of adjacent dipoles mentioned in [7] is compromized to make a lattice compact. We choose betatron tunes (see Table I) to avoid half integer chromatic stopband as well as the third order resonances.

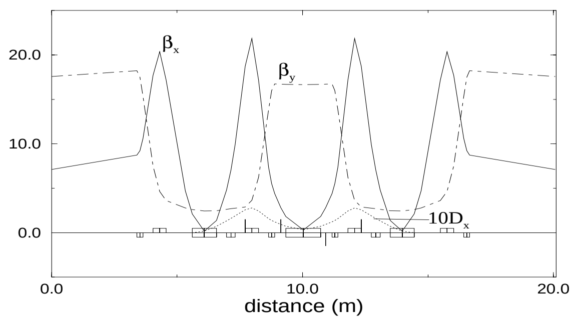

Figure 2: Plot of the triple bend achromat lattice. Plots of β_x, β_y, and $10 \times D_x$ are shown. The elements represented as single line are sextupoles, where the upward and downward lines correspond to focusing and defocusing sextupoles respectively.

2.2 Five bend achromat (FBA) as a multiple bend achromat

Based on the TBA lattice of previous section, a common module as shown in Fig. 1 with symmetric condition $\alpha_x = 0$, $\alpha_y = 0$ and $D'_x = 0$ at the both ends can be inserted into TBA module without affecting optical matching condition. By inserting multiple common modules, nBA can be attained. Figure 3 and 4 show β_x, β_y and $10 \times D_x$ of two different five bend achromat (FBA) lattices. This concept can be extended to design any multiple bend achromat by varying the number of common modules in a superperiod. For example, a ten bend achromat contains eight common modules in a superperiod. Minor adjustments in quadrupoles strength are necessary after the addition of the common modules to maintain achromat condition, otherwise a dispersion of magnitude $|D_x| \approx 0.001$ may arise in dispersion free section as a result of addition of common modules. Due to the additional constraints, the emittance of the designed FBA lattices is three times larger than the theoretical minimum, while the emittance of the designed TBA lattice is two times larger than the theoretical minimum.

The essential difference between these two FBA lattices are betatron tunes. $\nu_x = 15.163$ of FBA1 is pretty close to the half integer chromatic stop band $2\nu_x = 30$ while $\nu_x = 15.798$ of FBA2 is relatively far away. In case of FBA1, the operating point is far away from the third order resonances. But for FBA2, third order resonances such as $\nu_x + 2\nu_y = nP$ and $\nu_x - 2\nu_y = mP$ are located far away, while $3\nu_x = 8 \times 6$ systematic resonance is relatively close to the operating point. This may cause some problems for particles with large amplitude especially when sextupole detuning coefficients are large and it is necessary to correct this resonance.

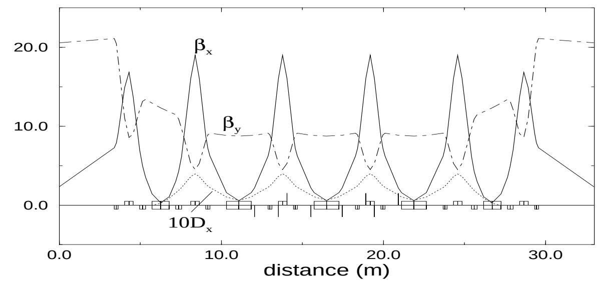

Figure 3: Plot of a five bend achromat FBA1 lattice. Plots of β_x, β_y, and $10 \times D_x$ are shown. The upward and downward lines are focusing and defocusing sextupoles.

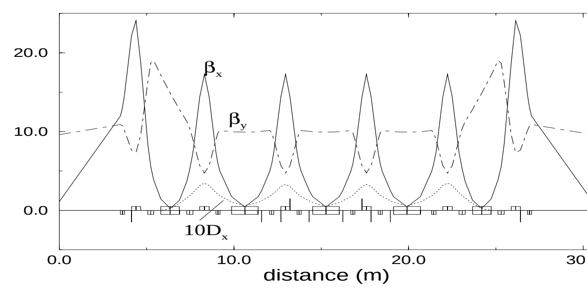

Figure 4: Plot of another five bend achromat FBA2 lattice. Plots of β_x, β_y, and $10 \times D_x$ are shown. The upward and downward lines are focusing and defocusing sextupoles.

For the FBA1 (FBA2) lattice with a 6.8 (7.0) m straight section in each superperiod, the resultant emittance is 5.32 (5.17) nm at 2 GeV nominal energy which is about three times larger than the theoretical minimum 1.82 nm. The circumference of ring is 197.798 (183.278) m which is even more compact than 240.96 m of the presented TBA lattice.

3 BEAM DYNAMICS ISSUES IN MINIMUM EMITTANCE ACHROMATS

We also take into account factors such as correction of first and second order chromaticities, dynamic aperture and sextupoles detuning coefficients. A comparison table (refer to Table I) is presented.

3.1 Chromaticity correction and third order resonances

In an effort to get lower emittance, stronger quadrupoles are used (they are about two times stronger than the quadrupoles of PLS). The strongest quadrupole used in the TBA (PLS) lattice has $\mathrm{K1} = -3.22$ ($\mathrm{K1} = 1.82$) m^{-2} and the corresponding $B' = 21.5$ (12.1) T/m, which is well within the limit. For the FBA1 (FBA2) lattice, the strongest quadrupole strength of $\mathrm{K1} = 2.34\,(2.54)\,\mathrm{m}^{-2}$ is lower than that of the TBA lattice. As a result of stronger focusing, the TBA (FBA) lattice has three (two) times larger horizontal natural chromaticity than PLS (refer to Table I) and these large chromaticities need to be properly corrected. The first and second order horizontal and vertical chromaticities are corrected. Chromaticity correction is performed by using the HARMON subroutine of MAD program.

In case of TBA, the variation in Q_x (Q_y) after correction is 0.0050 (0.020) over $-7.0 \times 10^{-3} \leq \Delta p/p \leq 7.0 \times 10^{-3}$ where ten times of the momentum spread of the equilibrium beam is $10\sigma_p = 6.97 \times 10^{-3}$. In case of FBA1, the variation in Q_x (Q_y) is 0.159 (0.0079) over the range $-7.0 \times 10^{-3} \leq \Delta p/p \leq 6.0 \times 10^{-3}$, where ten times of the momentum spread of the equilibrium beam is $10\sigma_p = 6.91 \times 10^{-3}$. Relatively large variation in Q_x is due to large betatron amplitude modulation stemming from the fact that $\nu_x = 15.163$ is pretty close to the half integer chromatic stopband $2\nu_x = 30$. In case of FBA2, the variation in Q_x (Q_y) is 0.018 (0.0058) over the range $-7.0 \times 10^{-3} \leq \Delta p/p \leq 7.0 \times 10^{-3}$ where ten times of the momentum spread of the equilibrium beam is $10\sigma_p = 6.97 \times 10^{-3}$.

3.2 Dynamic apeture and detuning coefficients

We defined dynamic aperture as the maximum betatron amplitude of particles that survive a damping time. Dynamic aperture in Table I is expressed in units of $\sigma_x = \sqrt{\epsilon_x \beta_x}$, the nominal beam size of natural emittance, i.e.,

$$\mathrm{dynamic\ \ aperture} = \frac{\sigma_{da}}{\sigma_x} = \sqrt{\frac{\epsilon_{da}}{\epsilon_x}}, \qquad (4)$$

where ϵ_x is the natural emittance of a lattice and ϵ_{da} is the emittance corresponding to the dynamic aperture. We find that the TBA lattice has a larger dynamic aperture than that of the FBA lattices. This may result from the less favorable betatron tunes of the FBA lattices which happen to be close either to half integer chromatic stopband or to a third order resonance. This can be avoided by choosing different n of nBA such that tunes may be placed far away from both half integer chromatic stopband and third order resonances. This would provide a larger dynamic aperture.

Table I shows the values of sextupole detuning coefficients (α_{xx}, α_{xy}, α_{yy}) for each lattices considered. Detuning coefficients for the TBA and FBA lattices are larger than those of PLS partly because of stronger sextupole strengths and interference of phase advance. Touschek lifetime is also studied. We calculated the necessary RF voltage and synchronous phase, when 12 hours of Touschek lifetime is required. The single bucket current is determined in such a way that harmonic number times the single bucket current is 400 mA. And the frequency of RF cavity is set to about 252 MHz. Summary of some important properties of the presented TBA, FBA1, and FBA2 lattices as well as PLS is presented in Table I.

In our design, we find also that the phase advance of each common module is $\Delta\nu_x \approx 0.43$, $\Delta\nu_y \approx 0.12$. By adding common modules, one should be able to find an optimal number of common modules in the minimum emittance lattice design. Employing the design strategy of our work, a high beam brightness can be achieved.

Table I
Comparison with an existing lattice

	PLS	TBA	FBA1
Emittance ϵ_x (nm)	12.17	2.965	5.324
Damping time (ms)	8.33	7.14	5.96
Dynamic aperture (σ_{xn})	94.49	53.97	41.30
$\alpha_{xx}/10^3$	-3.140	46.43	104.2
$\alpha_{xy}/10^3$	-6.240	223.1	11.49
$\alpha_{yy}/10^3$	2.028	442.9	17.85
Circumference (m)	280.56	240.96	197.798
Nominal energy	2 GeV	2 GeV	2 GeV
Superperiod	12	12	6
Number of dipoles	36	36	30
ν_x	14.280	18.696	15.163
ν_y	8.180	6.823	3.218
Horizontal chromaticity	-23.44	-70.16	-42.83
Vertical chromaticity	-17.70	-26.45	-11.19
Touschek Lifetime (min)	721	724	743
RF voltage (MV)	0.852	1.560	1.260
Synchronous phase (deg)	164.7	171.7	169.9
Harmonic number h	236	203	166

PLS: Pohang Light Source in Korea

4 REFERENCES

[1] M. Sands, in *Physics with Intersecting Storage Rings*, edited by B. Touschek (Academic, New York, 1971), pp. 257- 411.

[2] M. Sommer, Internal Report DCI/NI/20/81 (1981) (unpublished); D. Potaux, Internal Report DCI/NI/30/81 (1981) (unpublished); Y. Kamiya and M. Kihara, KEK Lab. Report (unpublished); H. Wiedemann, ESRP Report ESRP-IRM-71/84 (1984) (unpublished).

[3] L.C. Teng, Argonne Lab. Report LS-17 (1985) (unpublished) and L.C. Teng, TM-1269, Fermilab Internal Report, June 1984 (unpublished).

[4] S. Tazzari, in *CERN School Proceedings CERN 85-19*, edited by S. Turner (CERN, Geneva, 1985), pp. 566-585.

[5] S.Y. Lee and L.C. Teng, 1991 IEEE Conf. Proceeding, May 6-9, 1991, San Francisco, California, pp. 2679-2681.

[6] D. Trbojevic and E. Courant, *EPAC Conf. Proc.* (London, UK, 1994), pp. 1000-1002.

[7] S.Y. Lee, Phys. Rev. E **54**, 1940 (1996).