ELECTRO-MAGNETIC BUNCH LENGTH MEASUREMENT IN LEP

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Abstract

Bunch lengths between 3 and 12 *mm* have been measured routinely in LEP in 1997 with a small (7*mm* diameter) button electrode. The measurement method is based on the spectral analysis of the electrode signal and relies on the fact that the transfer function of the complete set-up, including the signal cable, can be computed rather exactly thus eliminating the need for external calibration. The information of beam intensity is recovered as a by-product. It provides an interesting internal validation of the measurement by comparison with the normal intensity measurement. The system has been used to detect subtle but real bunch length changes with bunch intensity which can be attributed to the inductive impedance in LEP. A value for the imaginary (inductive) longitudinal impedance is derived from the observations. An indication for the resistive part of the impedance is given as well.

1 INTRODUCTION

Bunches in LEP are short (a few *mm*) and Gaussian. The Gaussian time profile also produces a Gaussian frequency spectrum. The transfer function of an electro-magnetic observation system will distort the bare spectrum of the bunch. If the transfer function of the elements between the beam and final observation station are known with sufficient accuracy, then the bare spectrum can be recovered from the measured one and the determination of the bunch length is trivial. The LEP bunch length measurement consists of a button electrode, a transmission cable, a spectrum analyser and an acquisition system. The transfer function and performance limitations will be discussed in following sections.

2 TRANSFER FUNCTION OF BUTTON ELECTRODE

The beam monitor in LEP is an electrostatic button electrode. The properties can be derived from the geometry sketched in Fig. 1. The half aperture A=45 *mm*. The capacitance of the electrode is C. The transfer function can be computed as follows.

The charge collected on the electrode is :

$$q = \int_{t}^{t+\Delta t} i dt \cdot \tag{1}$$

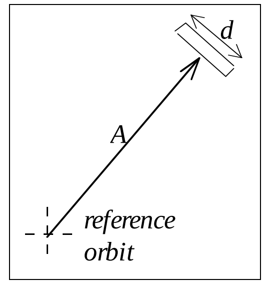

Figure 1 : Geometry of button electrode

It should be mentioned that the relation between the beam current i_b and the fraction i of it that couples with the electrode follows from the geometry: $i/i_b = d/2\pi A$. The electrode is a current source and delivers:

$$i_e = \frac{dq}{dt} = i(t+\Delta t) - i(t) \cdot \tag{2}$$

The currents can also be expressed as :

$$i(\omega, t) = I(\omega) e^{j\omega t} \tag{3}$$

$$i_e(\omega, t) = I(\omega)\left(e^{j\omega(t+\Delta t)} - e^{j\omega t}\right) = 2jI \sin\frac{\omega\Delta t}{2} e^{j\frac{\omega\Delta t}{2}} \tag{4}$$

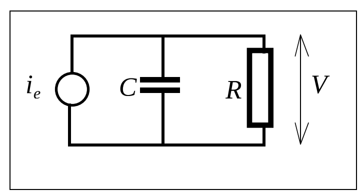

Figure 2 : Equivalent diagram of button electrode.

The current i_e flows through a RC parallel circuit as shown in Fig. 2 and produces a voltage V. The transfer impedance is defined as:

$$Z = \frac{V}{i_b} = \frac{i_e}{i_b}\frac{R}{1+j\omega\tau} \text{ or } Z = e^{j\frac{\omega\Delta t}{2}} \frac{d}{2\pi A} \frac{2j\tau \sin\frac{\omega\Delta t}{2}}{1+j\omega\tau}, \tag{5}$$

where R = 50 Ω, the load impedance, C the capacitance, $\tau = RC$ the low frequency time constant, $\Delta t = \pi d/4c$ is the transit time for the (round) electrode and c the speed of light.

For low frequencies ($\omega\Delta t/2 << 1$) this reduces to the standard low frequency formula

$$Z = \frac{d}{2\pi A}\frac{\Delta t}{C}\frac{j\omega\tau}{1+j\omega\tau} = Z_\infty \frac{j\omega\tau}{1+j\omega\tau} \cdot \tag{6}$$

3 THE SIGNAL TRANSMISSION CABLE

The attenuation of a cable is the sum of two components. The first one is caused by the skin effect which follows a square root law with frequency and the second one is caused by the losses in the dielectric which increases linearly with frequency. The general expression for the attenuation constant can be written as :

$$\alpha = \alpha_{skin} + \alpha_{dielectric} \cdot \tag{7}$$

$$\alpha_{skin} = -\frac{g}{v/c}\sqrt{\frac{\mu\rho}{2}}\left(\frac{1}{a}+\frac{1}{b}\right)\frac{1}{240\pi\ln\frac{b}{a}}\sqrt{\omega}, \tag{8}$$

where a and b are the radii of the inner and outer conductor, v is the signal velocity in the cable, ρ the resistivity of the conductor and g a geometrical factor. The factor g=1 for smooth coaxial cables but becomes 1.1 for the air filled cable that is used in LEP since the outer conductor is undulated.

$$\alpha_{dielectric} = -\frac{\omega}{2c}tg\delta, \tag{9}$$

where $tg\delta$ is the loss factor of the dielectric. Taken into account that only 1/6 of the cable length is occupied by solid insulation (the rest is air) we find $tg\delta$ =0.00014. It was found that these values remain trustworthy up to 9..10 *GHz*. The equivalent length of the cable , including the final 1.6 *m* flexible connection to the spectrum analyser, is l= 95 *m*. All elements in the measurement chain are now defined and the transfer impedance can be computed and is shown in Fig. 3:

$$Z_t = \frac{d}{2\pi A}\frac{\Delta t}{C}\frac{j\omega\tau}{1+j\omega\tau}e^{(\alpha_{skin}+\alpha_{dielectric})l}. \tag{10}$$

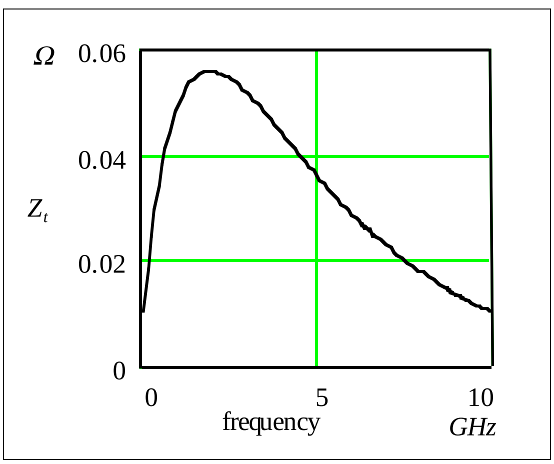

Figure 3 : Transfer impedance of button electrode and transmission cable.

4 MEASUREMENT

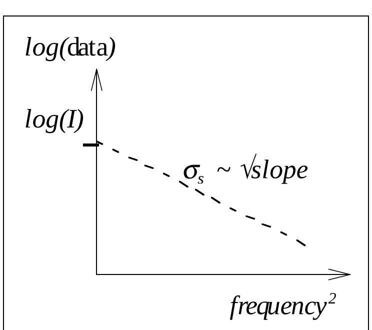

Figure 4 : Gaussian plot in *log/frequency²* coordinates.

The measurement is done with a HP8563A spectrum analyser. The output signal is given by :

$$V(\omega) = Z_t I\sqrt{2}\Delta f T e^{-\frac{1}{2}\left(\frac{\omega\sigma_s}{c}\right)^2}, \tag{11}$$

where Δf= 2 *MHz* is the bandwidth used, σ_s the bunch length, I the total beam current and T the revolution period. Since the data are available in *dB* it is straightforward to subtract logarithmically the transfer impedance and the operational constants. Fitting a Gaussian (which in fact is a linear fit when the data are displayed as function of frequency^2) then yields I and σ_s (Fig.4). The value of I is formally only correct when the beam is centered in the pick-up body and is indeed at a distance A from the electrode.

5 LIMITATIONS

The first limit is related to the signal to noise ratio. The noise level of the analyser used in LEP is -70 *dBV (320 μV)* averaged over the operational frequency range in a bandwidth of 2 *MHz*. That corresponds with a noise figure of 48 *dB* of which 10 *dB* are due to an input attenuator that is switched in with the option of remote control. The most critical part of the spectrum is near 10 *GHz*. For a 10 *mm* long bunch at least 200 *μA* total beam current is required to make a proper measurement. Shorter bunches can be measured with less intensity.

The expression for the transfer impedance is known to be wrong for a certain number of frequency bands. First there is a structure resonance at 5.5 *GHz* [1]. Then three other bands have been identified at 7, 8.5 and 9.2 *GHz*. They correspond to longitudinal and transverse reflections in the air-filled cable caused by the fact that the insulator is not continuous. These frequency bands have been excluded from the analysis.

The finite sample length causes fluctuations in the spectrum which are attenuated by averaging. It turns out that the resolution obeys the following simple scaling law: $(d\sigma_s.\ \sigma_s)$~1.4 mm^2.

6 RESULTS OBTAINED WITH THE SYSTEM

It was noticed during the LEP physics runs that the bunch length decreased with decreasing bunch current. The effect is small but significant. A number of data were collected mainly at 91.5 *GeV/c* and are shown in Fig. 5. The measured synchrotron tune was q_s = 0.117 except in 3 cases where it was 0.112, 0.112 and 0.114. A small correction on the measured bunch length was applied for these cases in order to keep the parameter space consistent. The *rf* frequency shift was 120 *Hz*, hence J_x=1.58.

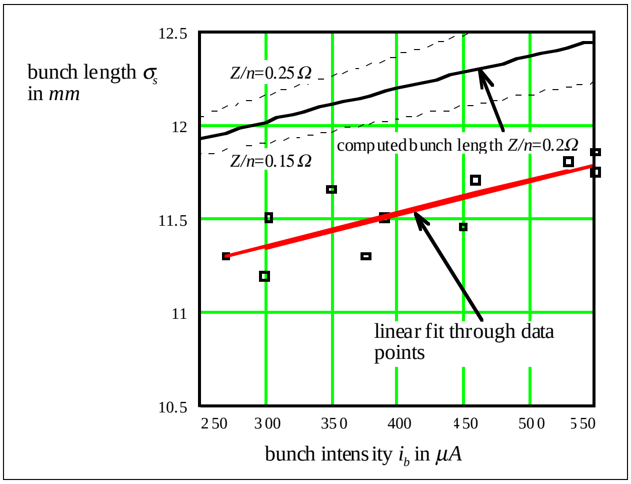

Figure 5: Bunch length as a function of single bunch intensity at 91.5 *GeV/c* compared with expected values when *Imag(Z/n)*=0.2 *Ω*.

The computed bunch length for zero intensity is 11.45 *mm*. The *computed bunch length* dependence on bunch intensity was found as follows. A longitudinal inductive impedance produces a longitudinal defocusing force. That force lengthens the bunch which will decrease in its turn the defocusing force,...The equilibrium bunch length can be found with an iterative process. The unknown in that computation is the effective inductive impedance seen by the basic longitudinal dipole mode of the bunch. Its value was chosen with trial and error such that the *slope* of the computed and measured impedance were about the same. This procedure yielded *Imag*(*Z/n*)= 0.2 *Ω*. The computed curves for 0.15*Ω* and 0.25*Ω* are shown as well for comparison. The systematic difference between the measured and computed bunch length is ~0.7 *mm* or ~6%. A possible explanation may be the fact that the loss-factor of the dielectric of the signal cable is ~10% lower than the value that was actually taken. Published data, only valid for frequencies below ~5 *GHz*, yield ~1.6 10 $^{-4}$ for this quantity. Reducing this to 1.4 10 $^{-4}$ (see section 3) is sufficient to eliminate this (small) discrepancy in some circumstances bearing in mind that the frequency range of the measurement extends from 4 to 10 *Ghz*. A slight dependance of *tg*δ on frequency may not be excluded.

Similar observations with the same conclusions were done at 65 *GeV/c*. A more interesting case is shown in Fig. 6 which refers to data taken at 22 *GeV/c* at very low bunch currents. The bunch length is computed for *Imag*(Z/n)=0.2 *Ω*. The synchrotron frequency was q_s=0.12 and J_x=1.

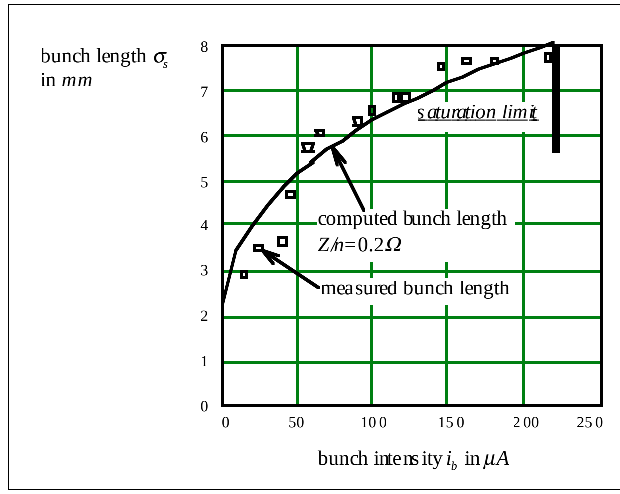

Figure 6 : Computed and measured bunch length at 22 GeV/c.

The measurement confirms that the inductive part of *Z/n* = 0.2 *Ω*. This value agrees well with transverse detuning measurements. Longitudinal turbulence is provoked by the resistive part of *Z/n*. The threshold can be written as :

$$\Re(Z/n)\langle i\rangle = \frac{V}{h}\left(h\Omega\sigma_s\right)^2, \qquad (12)$$

where *V* is this time the accelerating voltage and *h* the harmonic number. That condition can also be expressed as follows:

$$\Re(Z/n) = \frac{hV}{\sqrt{2\pi e}}\left(\frac{\sigma_s}{R}\right)^3\frac{1}{i_b}. \qquad (13)$$

Assuming that this threshold was reached at the point where the accumulation in the bunch saturated (i_b=220 μA) we find *Real(Z/n)*=0.07*Ω*.

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REFERENCE

[1] W.C. Barry, "Broad-band Characteristics of circular Button Pickups", LBL-32939, 1992