MODEL-INDEPENDENT ANALYSIS WITH BPM CORRELATION MATRICES *

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Abstract

We discuss techniques for Model-Independent Analysis (MIA) of a beamline using correlation matrices of physical variables and Singular Value Decomposition (SVD) of a beamline BPM matrix. The beamline matrix is formed from BPM readings for a large number of pulses. The method has been applied to the Linear Accelerator of the SLAC Linear Collider (SLC).

1 INTRODUCTION

The BPM readings of a beamline are highly correlated. To exploit this correlation, one can assemble them into a matrix. Without the need of referring to a beamline model, one can perform operations on this matrix including an SVD to obtain eigenvalues and two sets of eigenvectors. Most of the eigenvalues are due to BPM noise and are small. The number of eigenvalues that are above the noise floor determines the number of changing physical variables which measurably affect the beam centroid motion. The spatial eigenvectors and their corresponding temporal eigenvectors form two complete orthogonal bases respectively for the spatial and the temporal linear space spanned by the underlying physical changes. Techniques for identifying the physical variables will be described and results from analyzing the Linear Accelerator of the SLAC Linear Collider (SLC) will be presented.

2 EXPECTATIONS FOR THE BEAMLINE MATRIX

The data acquired from BPM readings can be stored in a matrix B of P rows by M columns, where M is the total number of BPM readings on each pulse and P is the total number of pulses. In other words, the p^{th} row vector $\vec{b}_p \equiv (b_p^1, b_p^2, ...b_p^M)$ represents the complete set of the M readings on the p^{th} pulse. These M readings are correlated since there are a finite number of degrees of freedom in the motion of the beam. Assuming there are D physical variables that are changing and can affect the beam centroid motion, one can write

$$\vec{b}_p = \vec{b}_0 + \sum_{s=1}^{D} \Delta v_p^s \frac{\partial \vec{b}}{\partial v^s} + \sum_{r=1}^{D}\sum_{s=r}^{D} \Delta v_p^r \Delta v_p^s \frac{\partial^2 \vec{b}}{\partial v^r \partial v^s} + ... + \vec{n}_p,$$

where $\vec{b}_0$ is a constant arising from centroid or BPM offsets, Δv_p^s is the s^{th} variable value (the difference from a nominal value) on the p^{th} pulse, and $\vec{n}_p$ is the random noise in each BPM. In addition to the frist derivative terms, the second derivative terms may play an essential role in beam centroid motion, such as the change in the betatron motion as a result of changes in energy. The constant term $\vec{b}_0$ is of no interest. Thus we usually consider

$$\begin{aligned}\vec{b}_p - <\vec{b}> &= \sum_{s=1}^{D} (\Delta v_p^s - <\Delta v^s>) \frac{\partial \vec{b}}{\partial v^s} \\ &+ \sum_{r=1}^{D}\sum_{s=r}^{D} (\Delta v_p^r \Delta v_p^s - <\Delta v^r \Delta v^s>) \frac{\partial^2 \vec{b}}{\partial v^r \partial v^s} \\ &+ ... + \vec{n}_p, \qquad (1)\end{aligned}$$

where $<>$ denotes an average over pulses.

We normalize the physical variable changes, their products and the corresponding derivatives as follows: letting

$$\Delta v_{rms}^s \equiv \sqrt{\sum_{p=1}^{P} (\Delta v_p^s - <\Delta v^s>)^2 / P},$$

$$\Delta v_{rms}^{r,s} \equiv \sqrt{\sum_{p=1}^{P} (\Delta v_p^r \Delta v_p^s - <\Delta v^r \Delta v^s>)^2 / P},$$

and the rms of the derivatives over the BPMs be denoted as

$$\beta_s^{rms} \equiv \sqrt{\sum_{m=1}^{M} \left(\frac{\partial \vec{b}}{\partial v^s}\right)^2 / M},$$

$$\beta_{r,s}^{rms} \equiv \sqrt{\sum_{m=1}^{M} \left(\frac{\partial^2 \vec{b}}{\partial v^r \partial v^s}\right)^2 / M},$$

we define dimensionless temporal unit vectors with elements given by

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$$q_p^s \equiv \frac{\Delta v_p^s - < \Delta v^s >}{\sqrt{P}\Delta v_{rms}^s}$$

or

$$q_p^{r,s} \equiv \frac{\Delta v_p^r \Delta v_p^s - < \Delta v^r \Delta v^s >}{\sqrt{P}\Delta v_{rms}^{r,s}},$$

and similarly, we define dimensionless spatial unit vectors

$$\vec{f}_s \equiv \frac{\partial \vec{b}}{\partial v^s} / (\sqrt{M}\beta_s^{rms}),$$

and

$$\vec{f}_{r,s} \equiv \frac{\partial^2 \vec{b}}{\partial v^r \partial v^s} / (\sqrt{M}\beta_{r,s}^{rms}).$$

Eq. 1 can then be re-written as

$$\hat{\vec{b}}_p \equiv \frac{\vec{b}_p - < \vec{b} >}{\sqrt{PM}} = \sum_{s=1}^{D(D+3)/2} q_p^s \sigma_s \vec{f}_s + \frac{\vec{n}_p}{\sqrt{PM}}. \quad (2)$$

where we have replaced double indices (r,s) with a new single index and set $\sigma_s \equiv \Delta v_{rms}^s \beta_s^{rms}$.

In the matrix form, Eq.(2) becomes

$$B/\sqrt{PM} \equiv [\hat{\vec{b}}_p] \equiv \hat{B} = Q\Sigma F^T + N/\sqrt{PM}, \quad (3)$$

where $Q \equiv [\vec{q}^s]$ is a matrix of P rows by $D(D+3)/2$ columns, Σ is a diagonal matrix of the rms values σ_s's. $F \equiv [\vec{f}_s]$ is a matrix of M rows by $D(D+3)/2$ columns, and $N \equiv [n_p^m]$ is a matrix of P rows by M columns containing the random BPM noise.

3 MINIMUM-CORRELATED SUBSETS

If a subset Q_s of the temporal patterns are known and are conjectured to be uncorrelated to temporal patterns outside the subset, then one can obtain the corresponding subset F_s of the spatial patterns. Defining the temporal correlation matrix of the subset as $C_s \equiv Q_s^T Q_s$, we obtain

$$\Sigma_s F_s^T = C_s^{-1} Q_s^T \hat{B} + O(\sigma_{noise}/\sqrt{PM}). \quad (4)$$

Note that in Eq. 4, the error due to noise is inversely proportional to the square root of the number of pulses and the number of BPMs, indicating that collecting data over more pulses with more BPM readings potentially enhance the analysis resolution.

4 SINGULAR VALUE DECOMPOSITION (SVD)

On the other hand, some important variables may be unknown or not measured at the time of BPM data acquisition. In this case, one can perform an SVD of the matrix $\hat{B}$ after removing the known patterns. The SVD equation is given by

$$\hat{B} = U\Lambda V^T, \quad (5)$$

where both U and V are unitary matrices representing orthogonal temporal patterns and spatial patterns respectively, and Λ is a diagonal matrix containing the corresponding eigenvalues. The number of eigenvalues above the noise floor determines the number of significant physical variables that are changing and affecting the beam centroid motion. The corresponding spatial eigenvectors in V and the corresponding temporal vectors in U form two complete orthogonal bases respectively for the spatial and the temporal linear space spanned by the underlying physical changes. Eq. 5 resembles Eq. 3. Although they share the same linear space, each of the eigen-modes in Eq. 5 do not correspond one-to-one to the physical patterns in Eq. 3. Indeed, the ultimate goal of MIA is to identify as many of the physical modes in Eq. 3 as possible with the help of the SVD analysis.

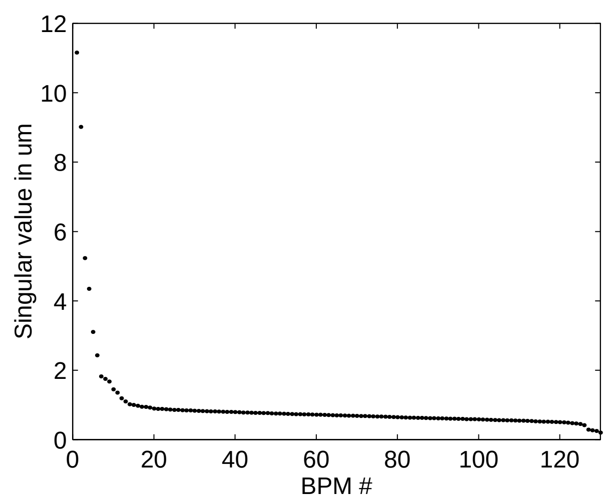

Figure 1: Eigenvalue plot

5 EIGENMODES AND NOISE

As an example, some typical results from SVD analysis of a set of SLAC linac horizontal motion data of 5000 pulses and 130 BPMs are shown in Figures 1-2. Figure 1 shows the eigenvalues obtained while Figure 2 shows the 6 eigenvectors corresponding to the largest eigenvalues. Except for small curved tails on the high and low ends, the noise floor is typically linear and its slop decreasing as $1/\sqrt{P}$. Without knowing temporal patterns, the magnitude of the floor obtained from SVD is inversely proportional to $\sqrt{M}$. Besides those 6 significant eigenvalues, there are additionally about 3 eigenvalues that can be categorized as above the noise floor and therefore one can conclude that there are about 9 physical variables that are changing and affecting the beam centroid motion. The most significant of these physical variables are the beam injection position and injection phase which correspond to two degrees of freedom in betatron motion. As shown by the top two plots in Figure 2, these two spatial eigenvectors basically represent the two betatron modes with a little mixture of other physical modes from, for example, jittering of beam bunch length, beam energy, beam intensity, vertical injection position and phase, incoming longitudinal phase, etc. due to correlation. Note that bad BPMs can be identified very easily. Each of the 5^{th} and the 6^{th} plots of Figure 2 clearly shows an eigenvector with a single outstanding component that corresponds to an abnormal BPM. Before further MIA analysis, one should identify the bad BPMs and remove their corresponding columns of data. One could also cut the noise by simply re-assigning the noise floor eigenvalues to 0 in the diagonal matrix Λ and then using the right-hand side of Eq. 5 to get a cleaner beamline BPM matrix. The advantage of doing so is that one can perform an SVD of a subset of the beamline BPM matrix (same number of rows but fewer columns) after cutting the noise to get a more accurate subset of eigenvalues and eigenvectors for easier identification of physical variables.

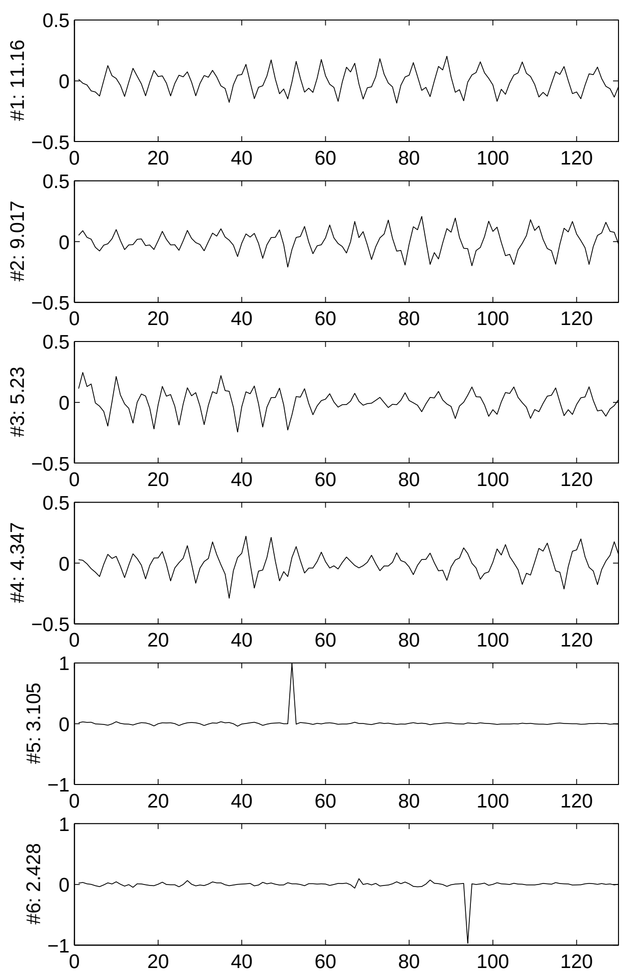

Figure 2: Eigenvector plot

6 THE DEGREE-OF-FREEDOM PLOT

Figure 3 shows what we call the degree-of-freedom plot. This plot is obtained by performing SVDs of the beamline BPM matrix subsets of increasing number of BPMs. The eigenvalues for different subsets are connected into curves. Such systematic SVD analysis can reveal not only the total number of the degrees of freedom but also the locations where subsequent new degrees of freedom appear. The blank strips indicate removed nosiy BPM locations. Different from Figure 1, the eigenvalues plotted here are not normalized by the number of BPMs. This enhances the curve visualability such that the coherent signal curves grow with the number of BPMs and the slopes of the curves indicate the local strength of signals. The two largest eigenvalue curves are the betatron modes. There are also other eigenvalue curves representing additional measurable physical variables that are yet to be identified.

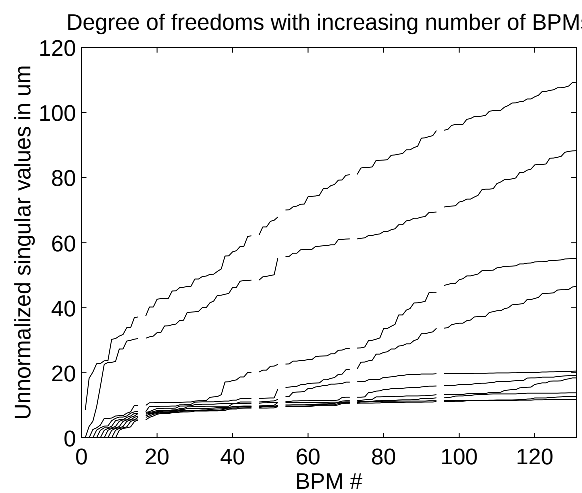

Figure 3: Degree-of-Freedom plot

7 SUMMARY

Preliminary studies for Model-Independent Analysis (MIA) of a beamline using a Singular Value Decomposition (SVD) was presented. MIA has many advantages in comparison with other measurement techniques. To name a few: the resolution of BPMs can be measured directly and improved by using more beam pulses and BPMs; systematic BPM errors can be immediately identified and removed; the BPM noise can be mostly cut by performing the SVD; the primary effects, such as betatron motion, can be identified and separated from the secondary effects easily. More detailed discussions will be given in a forthcoming article.

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