Dynamic aperture in the luminosity upgraded HERA-e lattice.

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Abstract

The electron emittance will be reduced in the luminosity upgrade by increasing the horizontal focusing in the arcs. Consequently the sextupole strengths will nearly double to compensate the larger chromaticity. The dynamic aperture of the new lattice in the presence of orbit errors has been calculated with and without the effects of radiation damping and quantum fluctuations. Under the same conditions, the dynamic aperture is nearly halved compared to the value for the current operational lattice. This agrees with an approximate scaling law for the dependence of the dynamic aperture on the sextupole strength. While the detuning with amplitude terms are large, especially the cross-detuning term, we find that the dynamic aperture does not depend sensitively on them. However the dynamic aperture does change significantly as the strengths of the third integer coupling resonances are varied. We also find that synchro-betatron resonances affect the off-momentum aperture significantly.

1 INTRODUCTION

The design of the HERA electron lattice for the upgrade and the nonlinear chromaticity correction has been described in a companion paper [1]. The phase advances in the FODO cells have been chosen as (90,60)° in the horizontal and vertical planes respectively. The nonlinear chromaticity correction is done with pairs of non-interleaved sextupoles grouped around the north and south insertions while the linear chromaticity is corrected with a single family of sextupoles in each plane placed in the rest of the arcs. The strengths of the linear chromaticity correcting sextupoles are (1.68,-1.16) m^{-3} while the strengths of the local non-interleaved sextupoles are smaller. There are two versions of this chromaticity correcting scheme and for the tracking calculations reported in this paper, only the second version was used. In the present HERA-e ring now operational with 60° phase advance in both planes, sextupoles are grouped into 3 interleaved families in each plane. The families are the same in the north and the south quadrants but different in the east and west quadrants. The absolute minimum and maximum sextupole strengths are (0.41, 0.88) m^{-3} respectively. The stronger focusing in the arcs and in the interaction regions in the upgraded optics have therefore almost doubled the sextupole strengths. From the fact that the sextupole strengths can be scaled out of the equations of motion of on-momentum particles, it can be shown [2] that, everything else being the same, the dynamic aperture scales with the sextupole strengths. Since the upgrade lattice differs from the present lattice not just in the arcs but also in the insertions, this scaling relation can only be approximate. In the present lattice the dynamic aperture for on-momentum particles is at an amplitude of about 7.4mm at the east IP. The scaling law would predict that in the upgrade lattice, the dynamic aperture will be at half this amplitude or an aperture of $15\sigma_x$ measured in units of the horizontal rms beam size.

2 SYMPLECTIC TRACKING

We use two tracking programs to find the dynamic aperture. SIXTRACK [3] for fast, symplectic tracking to find the optimal tunes, and MAD[4] for slower tracking with the inclusion of radiation damping and quantum excitation at the optimal tunes.

In SIXTRACK pairs of particles are tracked with the particles in a pair having a very small difference in their initial amplitudes. The Lyapunov exponent or rather the normalized distance between these two neighbouring particles is used to identify the dynamic aperture as the region where there is a “sensitive dependence on initial conditions” and chaotic motion begins. Use of the Lyapunov exponent to determine the dynamic aperture usually results in a conservative estimate compared to other methods [5]. Typically 32 particles are tracked in pairs, with members of a pair differing in horizontal amplitude by tens of nanometers. Particles are tracked for 1024 turns which is larger than the damping time of about 613 turns. On longer time scales we expect that the effect of the nonlinearities will be reduced by the synchrotron radiation damping. Hence most particle losses due to the instability of single particles should occur within a few damping times. Synchrotron oscillations are modelled in SIXTRACK by the inclusion of a single RF cavity. This is adequate when damping effects are not included. In the tracking runs reported in this section, no errors - neither of alignment, field nor multipolar - were included in the lattice. The only nonlinearities were those of the chromaticity correcting sextupoles.

The tracking calculations were done assuming an emittance coupling ratio of 10%. In the initial calculations, tunes were varied in the (Q_x, Q_y) plane in order to find the tunes at which the dynamic aperture is the largest. The dynamic aperture was calculated without synchrotron oscillations at more than 200 tunes. The settings of the local non-interleaved sextupoles were also varied in order to minimize the third order resonance driving terms while at the same time keeping the chromatic variations to reasonable values. The results of the tune scan showed that at the most favourable tunes, the dynamic aperture of the on-momentum particles is about $18\sigma_x$ in this lattice. For the majority of possible tunes, the dynamic aperture was closer to $15\sigma_x$, as predicted by the scaling relation men-

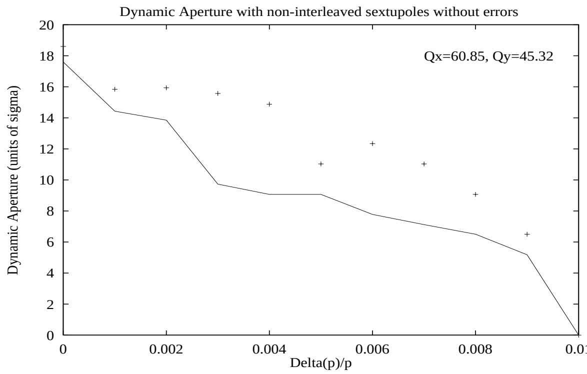

Figure 1: *Dynamic aperture of the REV2 lattice with the non-interleaved sextupole scheme, version 2, as a function of the synchrotron oscillation amplitude. The solid line shows the chaotic boundary while the crosses show the smallest amplitude at which particle loss occurs within a 1000 turns. This is the dynamic aperture for the perfect machine with only the sextupole nonlinearities and without errors and radiation damping.*

tioned above.

The dynamic aperture was then calculated with synchrotron oscillations at one of these favourable tunes ($Q_x = 60.85, Q_y = 45.32$). Figure 1 shows the dynamic aperture as a function of the synchrotron oscillation amplitude. Both the chaotic boundary (indicated by the solid line) and the smallest amplitude at which particles are lost within 1000 turns (indicated by the crosses) are shown. The dynamic aperture indicated by the chaotic boundary drops at $\delta = 0.003$ to about 10σ and then decreases to 6σ at $\delta = 0.008$. At larger synchrotron amplitudes, the dynamic aperture is effectively zero. These results show that the lattice needs further optimization in order to improve the dynamic aperture. The relatively large distance (in the range of 2-5 σ) between the chaotic boundary and the unstable boundary indicates that the resonances are not very strong and that the dynamic aperture might be improved by the beneficial aspects of radiation damping. Typically damping prevents resonances from transporting particles to large amplitudes unless the resonances are strong. However at high enough energies radiation damping can also cause unstable behaviour and we consider these in the next section.

3 TRACKING WITH DAMPING AND QUANTUM EXCITATION

Two dimensionless parameters can be used to characterize the strength of the radiation damping and quantum excitation: τ_x/T_0 the horizontal damping time in numbers of turns and Q_γ the ratio of the variance of the total photon energy emitted per turn to the square of the beam energy [6]. Smaller τ_x/T_0 implies that the effects of damping are more significant. Q_γ is given by

$$Q_\gamma = \frac{\langle N_\gamma \rangle \langle u^2 \rangle}{E^2} = \frac{55\pi}{36} \frac{\alpha_f (\hbar c)^2}{(m_0 c^2)^2} \frac{\gamma^5}{|\rho_0|^2}, \qquad (1)$$

where the average number of photons emitted per turn is $\langle N_\gamma \rangle = 5\pi \alpha_f \gamma / \sqrt{3}$ and $\alpha_f = 1/137.036$ is the fine structure constant. The variance of the photon energy distribution is

$$\langle u^2 \rangle = \frac{11}{27} u_c^2 = \frac{11}{27} \left[\frac{3}{2} \frac{\hbar c \gamma^3}{|\rho_0|} \right]^2 . \qquad (2)$$

	HERA-e Upgrade
Beam Energy E [GeV]	27.5
Radiation Loss per turn U_γ [MeV]	91
Relative Energy Loss U_γ/E [%]	0.33
Damping time τ_x/T_0 [turns]	613
Quantum Excitation Q_γ [times 10^9]	6.4

Table 1: *Damping and quantum excitation parameters for HERA-e in the upgrade lattice.*

Table 1 shows these damping and quantum parameters for the upgrade HERA-e lattice.

Jowett [6] has pointed out that radiation may in fact reduce the dynamic aperture at high energies. There are in essence two effects: a) The correct closed orbit and hence the synchronous phase must be found by including the energy loss. Ignoring the radiation might lead to an optimistic estimate of the dynamic aperture by ignoring e.g. the effects of synchro-betatron resonances excited by this additional energy variation. At the energy and parameters of HERA-e, these effects should not be significant and tracking results confirm this. b) An additional potentially unstable effect of radiation is the coupling of the transverse to the longitudinal dynamics. Particles with large betatron amplitudes in quadrupoles lose additional energy generating quadrupole synchrotron radiation. This radiation is significant in the high beta quadrupoles in the interaction regions. Consequently the stable phase angle $\phi_s \simeq \sin^{-1}[U_{loss}/eV_{RF}]$ will change due to the extra loss of energy. The synchrotron oscillations will be about a shifted stable fixed point and it may happen that even particles with small energy deviation but large transverse amplitudes will make excursions outside the RF bucket and be lost. In the upgrade lattice, calculation shows that the shift in the synchronous phase due to this quadrupole radiation is about 0.58mrad which is small compared to the synchronous phase angle of 0.82rad. Tracking studies confirm that the dynamic aperture is larger in the presence of damping so the effect of the betatron amplitude on the synchrotron motion is not significant in HERA.

4 DYNAMIC APERTURE WITH ERRORS

The different errors which inevitably exist in the magnets - both in their fields and alignments - enhance the unstable motion of particles and reduce the dynamic aperture.

Orbit errors change the trajectory of particles through the nonlinear elements which also causes a detuning. Typically orbit errors alone can reduce the dynamic aperture by a significant fraction. The dynamic aperture in the presence of the orbit errors due to quadrupole misalignments and radiation damping has been studied in the REV2 lattice using the tracking model in MAD. In these calculations, the dynamic aperture is interpreted as the largest stable amplitude. Multipole errors of the superconducting combined function magnets placed around the IPs in both the north and south IRs have also been included. However because of the relatively small beta functions in these magnets and the high field quality specified in the design of these magnets, the effects of these multipole errors on the dynamic aperture is negligible.

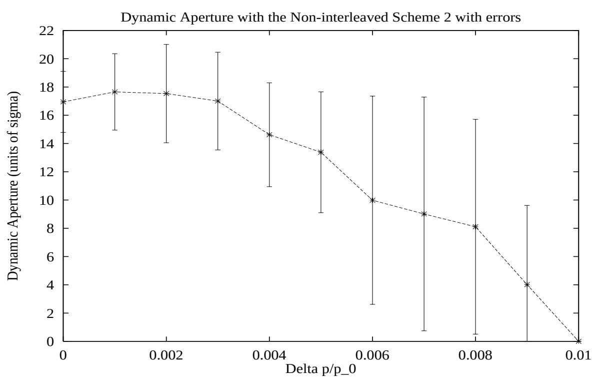

Figure 2: *Dynamic aperture of the upgrade lattice with the non-interleaved sextupole scheme, orbit errors due to random quadrupole misalignments and radiation damping as a function of the synchrotron oscillation amplitude. The line represents the average dynamic aperture over five seeds for the orbit errors while the error bars show the variation.*

Figure 2 shows the results of the tracking calculation in this more realistic model. Orbit errors were generated by random misalignments of all the quadrupoles in the ring. Closed orbit correction using the MICADO algorithm in MAD was done to reduce the rms orbit errors to approximately 1mm in each plane. This figure shows that even with orbit errors, radiation damping does improve the dynamic aperture significantly. At small synchrotron oscillation amplitudes $\leq 0.004 \approx 5\sigma_E$ the average dynamic aperture lies between $14\sigma_x$ and $18\sigma_x$. At larger amplitudes, the dynamic aperture drops rapidly and effectively vanishes at $\Delta p/p_0 \geq 0.008$. Tracking calculations with constant momentum offsets but no synchrotron oscillations show that the drop in dynamic aperture at $\Delta p/p_0 > 0.005$ in Figure 2 is largely attributable to synchro-betatron resonances. The dynamic aperture obtained in this lattice is marginally acceptable but it is clear that measures to improve the dynamic aperture are very desirable.

One way to improve the dynamic aperture is to compen-

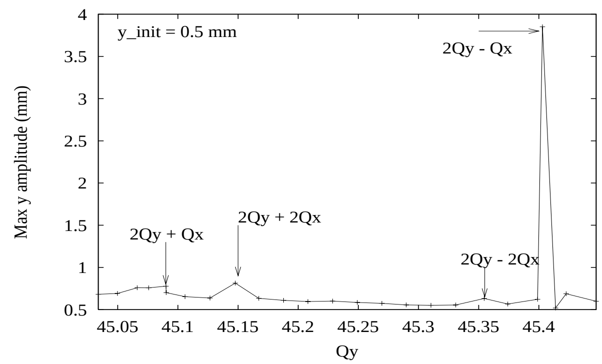

Figure 3: *The maximum vertical amplitude for a particle started with a small initial amplitude as a function of Q_y.*

sate the resonances driven by the chromaticity correcting sextupoles. First the important resonances have to be identified. Particles were launched with a small initial vertical amplitude and a range of horizontal amplitudes and tracked with different vertical tunes Q_y keeping the horizontal tune constant. Figure 3 shows that the dominant resonances driving particles to large amplitudes are the 3rd order resonances, primarily $2Q_y - Q_x = p$ and also $2Q_y + Q_x = p$. Correcting both these resonances with two neighbouring azimuthal harmonic numbers p for each will require in all 8 sextupoles. A study to increase the dynamic aperture with such a resonance compensation scheme using sextupoles placed in the zero dispersion west straight is now underway.

Another possibility is to increase the RF frequency by a few hundred Hz to reduce the horizontal emittance. The phase advance would then have be increased to an intermediate value e.g. 75 degrees. This would result in a smaller increase in the chromaticity, a smaller increase in the strength of the chromaticity correcting sextupoles and hence a larger dynamic aperture. Finally, the effect of synchro-betatron resonances at large $\Delta p/p_0$ could be minimized by optimizing the lattice to reduce the dispersion in the cavities even further.

5 REFERENCES

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