# WAVE-OPTICAL PROPERTIES OF SYNCHROTRON RADIATION AND ELECTRON BEAM DIAGNOSTICS

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*Abstract*

This paper deals with wave properties of synchrotron radiation (SR) generated by a relativistic particle in an uniform magnetic field. Starting from exact solutions of the Maxwell equations, precise analytical formulae for the SR phases have been obtained. The ‘photography’ of one electron resulting from the focusing of its SR on the screen by the ideal lens, is computed. The lens aperture effect on the optical resolution of electron beam profile measurement by means of SR is analyzed. It is shown that the traditionally used geometrical approach is too crude and does not offer the best size of the lens aperture.

## 1 INTRODUCTION

Because of the wide-spread application of diverse optical devices in SR physical experiments, it is vital to study the wave-optical properties of SR. In particular, the knowledge of SR wave properties is of fundamental importance in the storage ring beam diagnostics based on the use of radiation interference [1-7]. The SR wave properties play a major role in conventional diagnostics of electron beams in storage rings [8-13], where a lens is used to focus the SR and form an image of the particle beam. The diffraction of SR on the lens aperture restricts the resolution of the beam profile measurement. In order to evaluate the optical resolution of electron beam diagnostics, the geometrical approach was used [14-16]. To estimate the depth of field errors, the SR from the moving electron was considered, within this approach, as the radiation from stationary long light source. In [9,17,18] the Gaussian beam was used as a model for SR beam. However both geometrical and Gaussian methods seem to be enough artificial. Furthermore, the detailed analysis demonstrates that phase distribution of SR from one electron is very close to the phase distribution of spherical wave was emitted by a point source, being at rest in the laboratory frame [2,7].

A consistent setting of the problem, in terms of wave optics, is as follows. Let a physical device consists of the optical parts, each has characteristics known beforehand. It is enough to know the amplitude and phase of the SR at every point of the device entrance window in order to take advantage of the Helmholtz-Kirchhoff integral theorem [19]. If so, one can calculate the radiation intensity distribution on the device output screen. The amplitudes of vertical and horizontal SR components are well known and expressed in terms of second-order modified Bessel functions. The analytical formulae for the SR phase distributions were derived in [20].

## 2 WAVE-OPTICAL PROPERTIES OF SR

Let as consider a physical experiment with a geometry shown in Fig. 1.

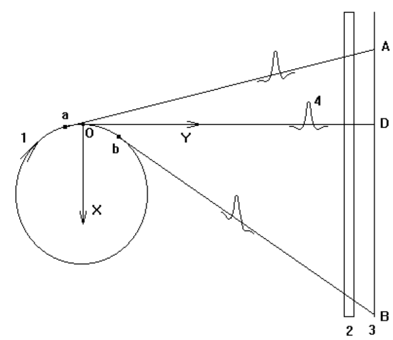


Fig.1: Typical layout of SR experiment. 1- electron orbit, 2 - light filter, 3 - device entrance window, 4 - SR pulse.

Let a relativistic electron with reduced energy $\gamma >> 1$ moves along its circular orbit $\vec{r}(t)$ in a horizontal plane *XOY* (the magnetic field is aligned with the vertical axis), see Fig. 1. Then the equations of particle motion are:

$$\vec{r}(t) = \{R - R\cos(\omega t), \quad R\sin(\omega t), \ 0\}. \quad (1)$$

Here R is the electron orbit radius and $\omega$ is the electron angular velocity. At time *t* the electron is at point $\vec{r}(t)$ of its orbit. The velocity vector is directed along a straight line intersecting the window plane (oriented normally to the *Y*-axis) at a point with abscissa *x(t).*

$$x(t) = R + D\,tg(\omega t) - \frac{R}{\cos(\omega t)}, \qquad (2)$$

where *D* is a distance from the origin of coordinate to the device entrance window. This relation can be reciprocated with respect to *t*: $x=x(t) \Rightarrow t=t_p(x)$. If $D>R$, the quantity $t_p(x)$ is determined from relation

$$tg(\omega t_p) = \frac{R\sqrt{(R-x)^2 + D^2 - R^2} - D(R-x)}{D^2 - R^2}. \quad (3)$$

Thus, $t_p(x)$ is the moment, when the particle located at $\vec{r}(t_p(x))$ points to *{x, D, 0}* of the device input window. The SR spectrum is determined by the Fourier transformation of the electric field:

$$\tilde{\vec{E}}(\vec{x}) = \int_{-\infty}^{\infty} \exp(i\frac{2\pi c}{\lambda}t)\vec{E}(t,\vec{x})dt \,, \qquad (4)$$

where $\lambda$ is the SR wavelength. As a result we have [20]:

$$\tilde{\vec{E}}(\vec{x}) = \exp(i\Phi(\vec{x}))\tilde{\vec{E}}_0 \,, \qquad (5)$$

$$\Phi(\vec{x}) = \frac{2\pi}{\lambda}\{ct_p(x) + |\vec{x} - \vec{r}(t_p(x))|\} \,, \qquad (6)$$

$$\tilde{E}_{0\sigma} = \frac{2\sqrt{3}\gamma|e|}{cD\sqrt{1+\gamma^2\theta^2}}\xi K_{2/3}(\xi) \qquad (7)$$

$$\tilde{E}_{0\pi} = -i\frac{2\sqrt{3}\gamma^2|e|\theta}{cD(1+\gamma^2\theta^2)}\xi K_{1/3}(\xi) \qquad (8)$$

where *e* is the electron charge, $\theta$ is the vertical angle of the observation point, $\lambda_c$ is the SR critical wavelength, $K_{1/3}$ and $K_{2/3}$ are the second-order modified Bessel functions, $\xi = \frac{\lambda_c}{2\lambda}(1+\gamma^2\theta^2)^{3/2}$.

The formulae (7) and (8) are well known expressions for the amplitudes of SR σ- and π-components. Their phases are constant, independent of the observation point. The phase dependence of SR from the observation point position is described only by the function $\Phi(\vec{x})$. The phase value is calculated as follows. The horizontal coordinate *x* of observation points $\vec{x}$ ={*x, D, z*} is used for finding the value of $t_p(x)$ by means of formula (3). The found value $t_p(x)$ indicates the instant when the electron points to {*x, D, z*}. Substituting this time value into (1), one finds the particle position. Then formula (6) gives the desired SR phase value. It is important to outline that the phase difference, rather than the phase absolute value $\Phi(\vec{x})$, has a physical sense.

Let us clarify the physical sense of the results derived above. Let there be an observer located in the storage ring median plane. Curve 4 in Fig.1 presents the time dependence of $E_\sigma(t,\vec{x})$ in the absence of light filter 2. The observer who is at the point *{0, D, 0}* of the entrance window will detect the peak of this function at moment $t = D/c$. This is obvious because, at time *t = 0*, the particle was at the origin of coordinates, and its velocity vector points to this observer. If there are other observers located at some different points *A* or *B* of this window, they will observe a similar SR pulse. Nevertheless, these observers will detect the signal peak at different times. Obviously, the time detected by each observer consists of two components. The first part $t_p(x)$ is the time, when the particle was at such point of its orbit from where the velocity vector pointed at the observer (points *a* or *b* in Fig.1). The other part is the time of radiation propagation from the trajectory point $\vec{r}(t_p(x))$ to the observation point. As a result, the recorded time of the maximum of function $E_\sigma(t,\vec{x})$ is equal to the quantity $2\pi c\Phi(\vec{x}) / \lambda$. Thus, we have obtained a physically sound result: the SR phase is proportional to the recorded arrival time of the maximum of function $E_\sigma(t,\vec{x})$.

## 3 'PHOTOGRAPHY' OF ONE ELECTRON

Let us consider the conventional experimental layout to measure the electron beam profile. In this case a lens is additionally mounted crosswise to the *Y* - axis. This lens is used to focus the SR and form the electron beam image on the screen 3, Fig.1 (the lens is not shown). It is possible, by using the SR amplitude, phase distributions and Helmholtz-Kirchhoff integral theorem, to compute the 'photography' of one electron. We will consider the ideal lens. It means that this lens adds an extra shift in the SR phases and does not change the radiation amplitudes. The magnitude of this phases shift is equal to

$\exp(\frac{-i\pi}{\lambda F}(x^2+z^2))$, where *F* is the lens focal length, *x* and *z* are the transverse lens coordinates. The computer code has been written for simulations of the relativistic electron optical image. Fig. 2 shows the computed image for the σ-polarized SR. The simulation was made under the following conditions. The electron beam energy is 2 GeV, electron orbit radius is 5.5 m (Daresbury SRS), SR wavelength is 500 nm, distance from the radiation point (origin of coordinates) to the lens is 10 m, distance from the lens to the screen is 10 m, the lens is infinite in transverse sizes, its focal length is 5 m.

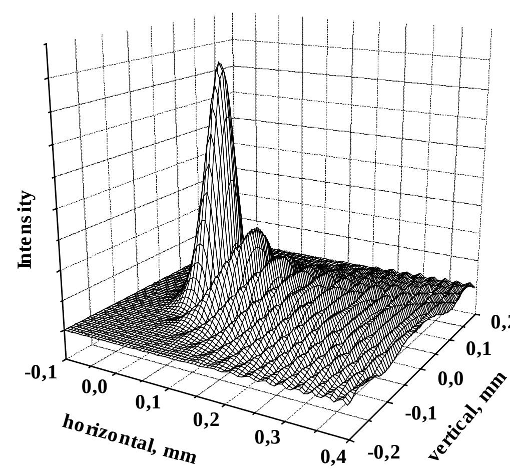

Fig.2: Computed photography of one electron, 'made' by the ideal lens focusses the σ-polarized SR on the screen.

## 4 IMAGING RESOLUTION

The most credible way to estimate theoretically the resolution of the electron beam profile measurements by SR is to compute the image of one electron under the real experiment geometry. It is obvious that the size of this image gives the required resolution, if the optical magnification is equal to 1. The horizontal case has been of our main interest here. Fig. 3 shows the normalized horizontal profiles of the images, computed for one electron under conditions presented above, but for the different lens apertures. The vertical slit with 200 mm height, but with different width, was assumed to be placed in front of the lens (vertical size of SR spot on the lens is 140 mm).

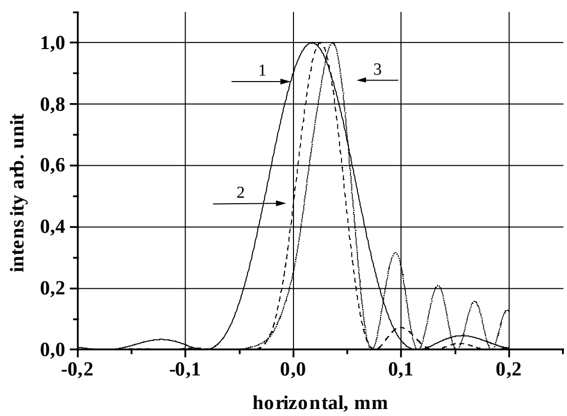


Fig. 3. Horizontal profiles of one-electron images for the different widths of vertical slit: 1 - 52 mm, 2 - 100 mm, 3- 400 mm.

One can see that the vertical slit with width about 100 mm provides the best horizontal resolution. At least such slit is much superior to the slit with 52 mm width. The additional superiority of 100 mm slit is that we will have much more SR flux as compared with 52 mm slit. But the slit with 52 mm width is a best size slit from the geometrical approach standpoint [11]. That is why the geometrical approach to the optical resolution problem does not give the best answer. One can also see from Fig. 2, that if we will increase the width of slit (curve 3, 400 mm width), the optical resolution will degrade due to the appearance of additional picks. That indicates on the existence of slit with optimal parameters which provides the best optical resolution.

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