# A HILBERT TRANSFORM SPECTROMETER USING A HIGH-$T_C$ JOSEPHSON JUNCTION FOR BUNCH LENGTH MEASUREMENTS AT THE TESLA TEST FACILITY LINAC


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*Abstract*

The longitudinal charge distribution of an electron/positron bunch can be determined from the coherent transition radiation emitted as the bunch crosses a thin metal foil. A Josephson junction made from a thin film of $YBa_2Cu_3O_{7-x}$ on a bicrystal substrate is used as a detector for transition radiation in the millimeter and submillimeter range. The spectral intensity of the radiation and the longitudinal form factor of the bunch are derived by applying a Hilbert transformation to the radiation-induced modification of the current-voltage characteristic of the Josephson junction. The physical principles of a Josephson junction as a detector for submillimeter radiation are outlined and a first bunch length measurement is presented.


## 1 INTRODUCTION

Future electron-positron linear colliders as well as electron drive linacs for Free Electron Lasers (FEL) in the X-ray regime require the production and acceleration of bunches whose length is in the 50-100 $\mu$m range [1]. To determine the bunch length frequency-resolved techniques are adequate such as far-infrared grating spectroscopy [2] or Fourier-transform spectroscopy [3, 4, 5]. If the wavelength exceeds the bunch length, all electrons in the bunch radiate coherently and the longitudinal charge distribution in the bunch can be obtained by Fourier transformation of the measured frequency spectrum. The spectrometers used for such measurements are usually equipped with mechanically movable elements like mirrors or gratings, hence the recording of the entire frequency spectrum may last several minutes and an average over many successive bunches has to be taken.

Hilbert-transform spectroscopy based on the ac Josephson effect offers the possibility of high-speed spectroscopy in the millimeter- and submillimeter-wavelength range [6]. This technique might even permit single-bunch measurements. The principle is to investigate the modification of the current-voltage characteristic of a Josephson junction due to incident radiation. Applying a Hilbert transformation the frequency spectrum of the radiation can be derived and, after Fourier transformation, the charge distribution in the bunch can be calculated. A Hilbert transform spectrometer has been tested at the TESLA Test Facility (TTF) linac. In a first stage the device is able to measure the average bunch length during a macropulse.

## 2 PRINCIPLE OF THE BUNCH LENGTH MEASUREMENT

Coherent transition radiation is produced when relativistic charged particles pass the interface between two materials of different dielectric properties. By arranging the radiatior, here a thin aluminum foil, at an angle of 45$^o$ with respect to the beam direction, the radiation is emitted at a right angle and can easily be extracted from the vacuum chamber through a quartz window. The spectral intensity emitted by a bunch of $N$ particles can be expressed as

$$I_{tot}(\lambda) = I_1(\lambda)\{N + N(N-1)|f(\lambda)|^2\} \qquad (1)$$

where $I_1(\lambda)$ is the intensity radiated by a single electron at a given wavelength $\lambda$ and $f(\lambda)$ is the bunch form factor [7, 8, 9], defined as the three-dimensional Fourier transform of the normalized charge distribution $\rho(\mathbf{r})$. If the radiation is observed along the direction of the beam and the transverse charge distribution is neglected the form factor $f(\lambda)$ becomes

$$f(\lambda) = \int \rho(z) \exp\left(\frac{2\pi i z}{\lambda}\right) dz \, . \qquad (2)$$

For wavelengths in the order of the bunch length and longer, the form factor approaches unity. The emitted radiation is then coherent and permits a direct measurement of $|f(\lambda)|^2$.

## 3 PRINCIPLE OF HILBERT TRANSFORM SPECTROSCOPY

A Josephson junction serves as a Hilbert transform spectrometer. The non-linear electric properties of such a device are determined by Cooper-pair tunneling which leads to the I-U characteristic shown as the dashed curve in Fig. 1. A dc current $I_0$ can be passed through the junction without observing a voltage drop as long as the current

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stays below a critical value $I_c$ (dc Josephson effect). For currents above $I_c$ a voltage drop across the junction is observed accompanied with an alternating current whose frequency is given by the relation $\omega = 2eU/\hbar$ (ac Josephson effect, $f_{Jos} = \omega/2\pi = 483.6$ GHz for $U = 1$ mV). When

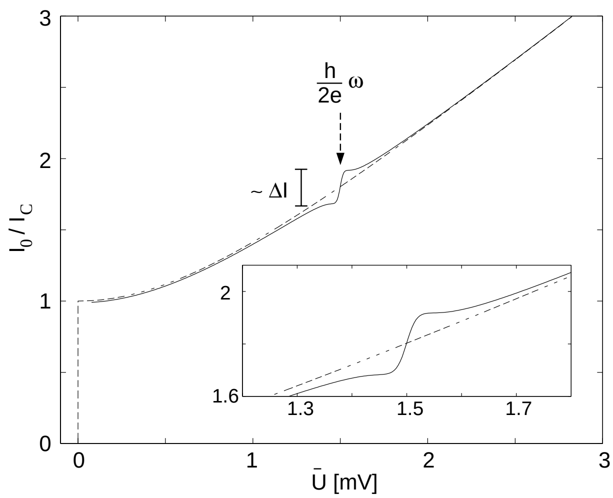


Figure 1: Dashed curve: voltage across the junction as a function of the dc bias current. Solid curve: modification of dc characteristic curve due to monochromatic incident radiation.

the Josephson junction is exposed to monochromatic radiation of (angular) frequency $\omega$ the current-voltage characteristic acquires a current step $\Delta I$ at the voltage $\overline{U} = (\hbar\omega/2e)$ ($\overline{U}$ is obtained by averaging over the Josephson oscillation). Within the framework of the Resistively Shunted Junction (RSJ) model [11], and in small-signal approximation, the magnitude of this step is proportional to the power of the incident radiation. Hence the junction acts as a quadratic detector and can be used to measure the spectral intensity of a continuous radiation spectrum. For this purpose we define the function

$$g(\overline{U}) = \frac{8}{\pi}\frac{\hbar}{2e} \cdot \frac{\Delta I(\overline{U}) I(\overline{U}) \overline{U}}{R^2 I_c^2} \qquad (3)$$

where $R$ is the ohmic resistance of the junction. The spectral intensity is derived from $g$ by an inverse Hilbert transform [6]

$$S(\omega) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{g(\omega_0)\, d\omega_0}{\omega - \omega_0} \quad \text{where} \quad \omega_0 = \frac{2e}{\hbar}\overline{U} . \qquad (4)$$

Here $\mathcal{P}$ denotes the principal value of the integral.

To determine the function $g$ the voltage-current characteristic of the Josephson junction is scanned with and without incident radiation, increasing the bias current $I_0$ in small steps. At each step the voltage $\overline{U}$ and its modification $\Delta U$ due to the radiation are measured. $\Delta I$ is computed using the differential resistance $R_d = d\overline{U}/dI$ derived from the unperturbed $I$-$U$ curve.

# 4 MEASUREMENT OF THE COHERENT SPECTRUM

High-$T_C$ Josephson junctions were fabricated by epitaxial growth of $YBa_2Cu_3O_{7-x}$ on $NdGaO_3$ bicrystal substrates. A schematic view of the detector which incorporates the antennas for millimeter and submillimeter wave detection is shown in Figure 2. The grain boundary leads to a thin resistive barrier between the two superconductors, which then work as a Josephson junction. Electrical circuits to bias the junction with a dc current and to measure the potential difference across the junction are connected to the antennas. The junction features a large dynamic range of

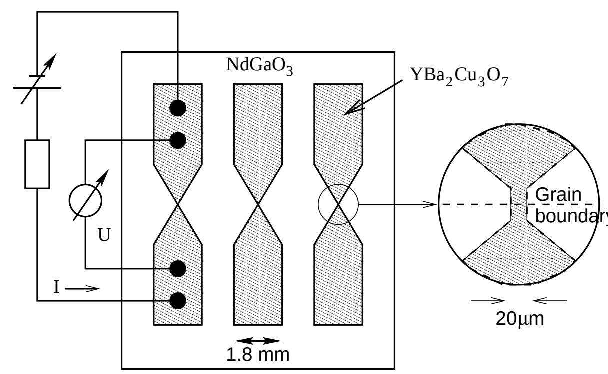


Figure 2: A schematic view of the Josephson junction used as a detector for millimeter and submillimeter wave radiation.

about $10^5$ and a high sensitivity of $\approx 10^{-14}$ W/Hz$^{1/2}$ Noise Equivalent Power (NEP) to millimeter- and submillimeter radiation [13]. The resolution is around 1 GHz in the temperature range from 4 to 78 K [14].

The TTF linac was operated with a thermionic gun producing bunches with $2.3 \cdot 10^8$ electrons at a repetition rate of 216 MHz. The macropulse length was 30 $\mu$s at a repetition rate of 2 Hz. Using the compression of a sub-harmonic buncher and a superconducting cavity an rms bunch length of $\sigma_t = 1.2$ ps was achieved [5, 12].

Fig. 3 shows the measured voltage response of the junction to incident transition radiation and the function $g(\overline{U})$ (as defined by Equation (3)). At small voltages $\overline{U}$ the internal noise of the detector becomes large hence $g$ is obtained in this region by a smooth extrapolation. The intensity spectrum is calculated using an algorithm of discrete Hilbert transform. Fig. 4 shows the evaluated coherent radiation spectrum. The spectrum is plotted in the frequency range between 60 and 260 GHz and has a maximum at about 100 GHz. Points marked by crosses have to be treated with care. The decrease towards smaller frequencies is due to the cut-off frequency (60 GHz) of the WR-10-type waveguide in front of the detector. The main systematic uncertainty of the present, preliminary, experiment originates from the wavelength-dependent acceptance

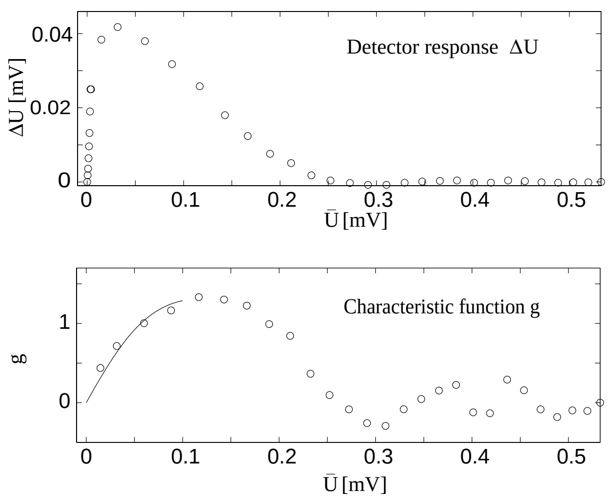


Figure 3: Upper graph: The detector response $\Delta U$ as a function of $\overline{U}$. Lower graph: The characteristic function $g$, as defined by Equation (3), plotted versus $\overline{U}$. The solid line shows an extrapolation of $g$ for small $\overline{U}$.

of the transmission line guiding the radiation to the detector. The point-to-point errors are dominated by the read-out errors of the voltage response $\Delta U$ of the junction. These values were determined from a digital oscilloscope after averaging over 15 seconds. The errors in the quantities $U$, $I$ ($< 1\ \mu$V resp. $1\ \mu$A) and $\Delta I$ are carried through the Hilbert transform and result in the error bars shown in Fig. 4. The present data are not accurate enough to determine the detailed shape of the longitudinal charge distribution. Therefore, a Gaussian has been assumed to determine the rms bunch length. The Gaussian fit applied to the data, shown as a solid line in Fig. 4, yields

$$\sigma_f = (98 \pm 16)\ \text{GHz} \quad \text{or} \quad \sigma_t = (1.2 \pm 0.2)\ \text{ps}\ . \qquad (5)$$

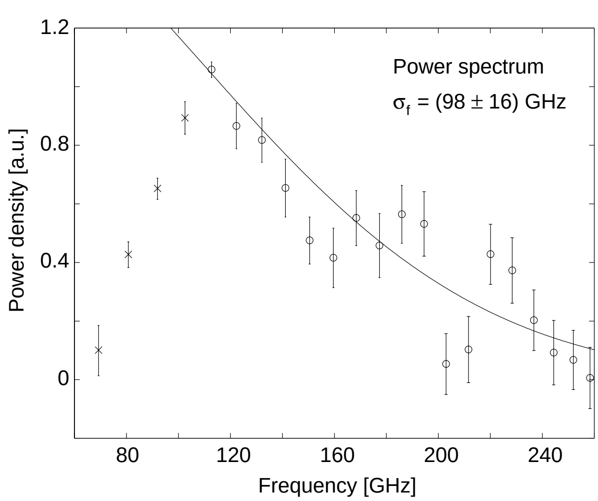


Figure 4: The coherent radiation spectrum as obtained from a discrete Hilbert transform of the characteristic function $g$.

# 5 CONCLUSION AND OUTLOOK

A Josephson junction has been successfully used as a frequency-selective detector for millimeter and submillimeter wave radiation and for a first bunch length measurement at the TESLA Test Facility Linac. It is planned to improve the detector by mounting it into a cryostate with direct optical coupling of the radiation onto the detector. The bandwidth of the read-out electronics will be enhanced to permit measurements of selected bunches within a macropulse.

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