A GUIDING CENTRE APPROXIMATION APPROACH FOR THE SIMULATION OF ELECTRON TRAJECTORIES IN ECR AND MICROWAVE ION SOURCES

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Introduction

- Objective: Investigation of the validity and stability of the Guiding Centre (GC) algorithm to speed up particle propagation in magnetised plasma simulation.

- 2 ion sources to consider:
  - ECRIS (Phoenix V2) confinement up to ~1ms.
  - Microwave discharge (MDIS) ion source (SILHI@GANIL) confinement ~1μs.

- Methodology:
  - Implementation of two, otherwise identical, algorithms in c to simulate electron trajectories by Boris and the GC approximation.
  - Consistency check and computation time measurement of trajectories for equivalent initial conditions with different time-steps (dt).
  - Identify a range of validity for dt.

Implementation

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General initialization

- Number of electrons, maximum propagation time and time-step are given
- Read provided magnetic field map files, merge them into a 3D grid.
- Set electrons' position ($\mathbf{r}$) (uniform disc) and velocity ($\mathbf{u} = \gamma \mathbf{v}$) (Maxwell-Boltzmann)

GC initialization

- Map ∇B (speeds up the computation per step without loss of accuracy)
- For each electron:
  - $u_\parallel = \boldsymbol{u} \cdot \hat{b}$
  - $\mathbf{R} = \mathbf{r} + \boldsymbol{\rho}$
  - Compute magnetic moment μ

Boris Algorithm

dt

GC Approximation

dt

- $\frac{d\mathbf{R}}{dt}$ and $\frac{du_\parallel}{dt}$ integrated concurrently by RK4

Output

- Track file (r, u, μ (GC), n_e, ε, t, t_{cpu})

Boris algorithm

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- The standard for magnetised plasma simulations.
- Second order, explicit method. Energy conserving while only a magnetic field is present.
- For B= 1T, $T_B=1/\omega_B$~35ps → dt~3ps.

$$\underbrace{\frac{\mathbf{u}^{n+1}-\mathbf{u}^{n}}{\Delta t} = \frac{q}{m}\left(\mathbf{E}(\mathbf{x}^{n+\frac{1}{2}}) + \left(\frac{\mathbf{u}^{n+1}+\mathbf{u}^{n}}{2\gamma^{n+\frac{1}{2}}}\right)\times \mathbf{B}(\mathbf{x}^{n+\frac{1}{2}})\right)}_{\text{Discretised Lorentz force with: } \bar{\mathbf{v}}=(\mathbf{u}^{n+1}+\mathbf{u}^{n})/2\gamma^{n+\frac{1}{2}}} \Rightarrow$$

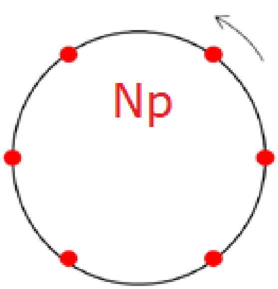
$$\mathbf{u}^{-} = \mathbf{u}^{n} + \frac{q\Delta t}{2m}\mathbf{E}(\mathbf{x}^{n+\frac{1}{2}})$$

$$\mathbf{u}^{+} = \mathbf{u}^{-} + (\mathbf{u}^{-} + (\mathbf{u}^{-}\times \mathbf{t}))\times \mathbf{s}$$

$$\mathbf{u}^{n+1} = \mathbf{u}^{+} + \frac{q\Delta t}{2m}\mathbf{E}(\mathbf{x}^{n+\frac{1}{2}})$$

$$\text{with: } \underbrace{\mathbf{u}^{n} = \mathbf{u}(t^{n}-\Delta t/2) = \gamma^{n}\mathbf{v}(t^{n}-\Delta t/2)}_{\text{Proper velocity}} \quad \text{and} \quad \underbrace{\Delta t = \frac{2\pi m_e[eV]}{c^2 B_{min}\,\mathrm{Np}}}_{\text{time-step}}$$

T_{Bmin}/Np

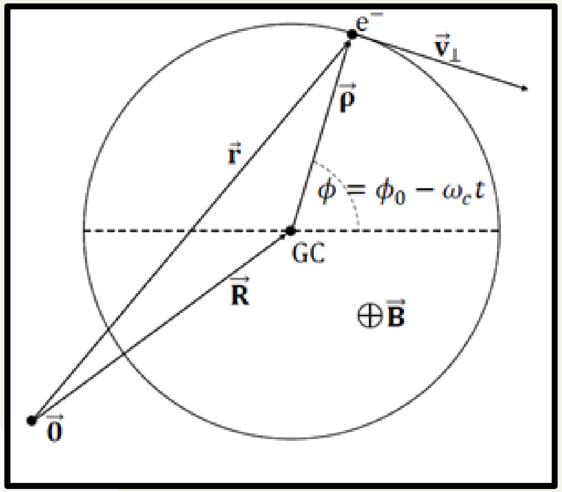


Np is the number of points to calculate per orbit at B_{min}

Guiding Centre (GC) approximation

- Condition:
  - The magnetic dipole moment (μ) is an adiabatic invariant if the motion along the plasma chamber is slow compared to the cyclotron motion .
  - Equivalently, the Larmor radius should be much smaller than the length scale of ∇B.

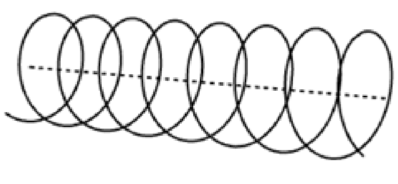
$$\rho_L \ll \frac{B}{|\nabla B|}$$



$$\frac{d\mathbf{R}}{dt} = v_{\|}\hat{\mathbf{b}} - \overbrace{\frac{\hat{\mathbf{b}}\times\mathbf{E}}{B}}^{\text{crossed fields}} + \frac{\hat{\mathbf{b}}}{B\left(1-\frac{E_\perp^2}{B^2}\right)} \times \left\{ \frac{m\gamma}{q} \overbrace{\left(v_{\|}^2(\hat{\mathbf{b}}\cdot\nabla)\hat{\mathbf{b}} + v_{\|}(\mathbf{u}\cdot\nabla)\hat{\mathbf{b}} + v_{\|}(\hat{\mathbf{b}}\cdot\nabla)\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right)}^{\text{curvature and polarisation drift}} + \right.$$

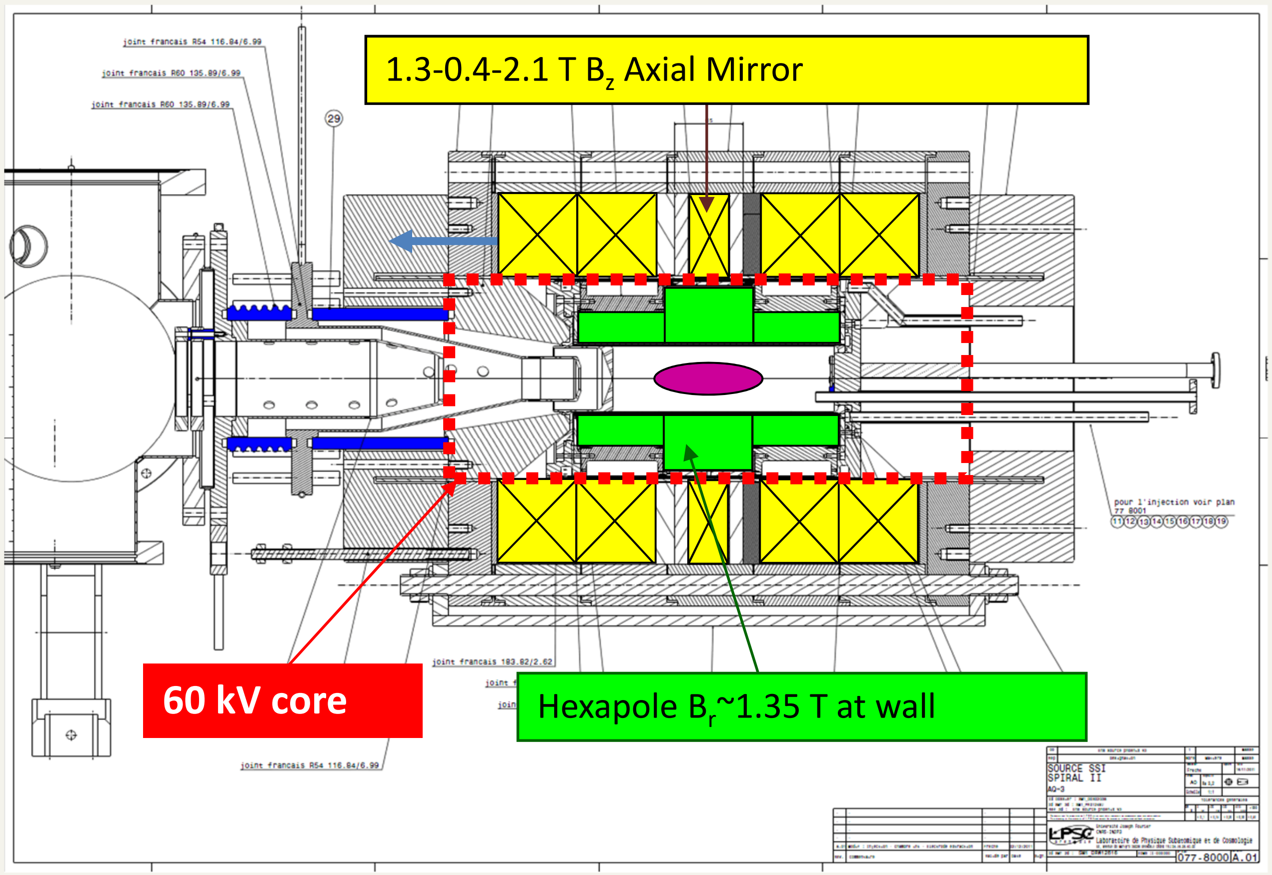
$$\left. \underbrace{\frac{\mu}{\gamma q}\nabla\left[B\left(1-\frac{E_\perp^2}{B^2}\right)^{1/2}\right]}_{\text{grad B drift}} + \underbrace{\frac{v_{\|}E_{\|}}{c^2}\mathbf{u}}_{\text{relativistic}} \right\} \quad \text{with} \quad \mathbf{u} = \mathbf{E}\times\frac{\hat{\mathbf{b}}}{B} \quad \text{and} \quad \underbrace{\frac{d(m\gamma^{*2}v_\perp^{*2}/(2B^*))}{dt} = \frac{d\mu^*}{dt} = 0}_{\mu \text{ invariance } (*\text{=frame of reference at } \mathbf{u})}$$

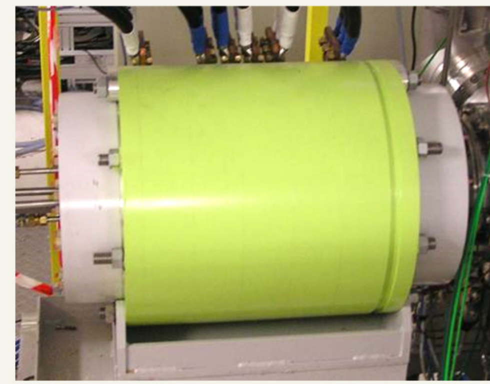
$$\frac{d(\gamma v_{\|})}{dt} = \underbrace{\gamma\mathbf{u}\cdot(v_{\|}(\hat{\mathbf{b}}\cdot\nabla)\hat{\mathbf{b}} + (\mathbf{u}\cdot\nabla)\hat{\mathbf{b}})}_{\text{direction change: } \mathbf{u}, v_{\|} \text{ exchange}} + \underbrace{\frac{q}{m}E_{\|}}_{\text{E acc.}} - \underbrace{\frac{\mu}{\gamma m}\hat{\mathbf{b}}\cdot\nabla\left[B\left(1-\frac{E_\perp^2}{B^2}\right)^{1/2}\right]}_{\text{grad. B mirror force}}$$

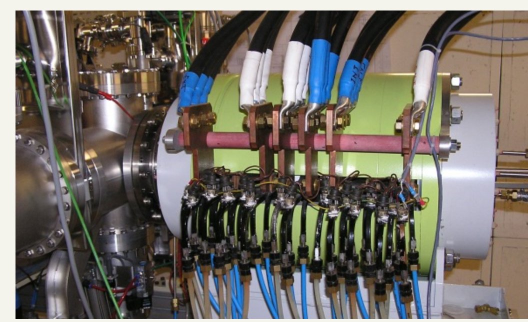


Phoenix V2 ECRIS

- Compact ECRIS commissioned for the SPIRAL 2 accelerator.
- 0.6L plasma chamber volume (L204mm , Ø63mm)
- Operation frequency of 18GHz







Phoenix V2 min-B field

Map files provided to simulation

- 2D for axisymmetric solenoidal fields
- 3D for hexapolar fields

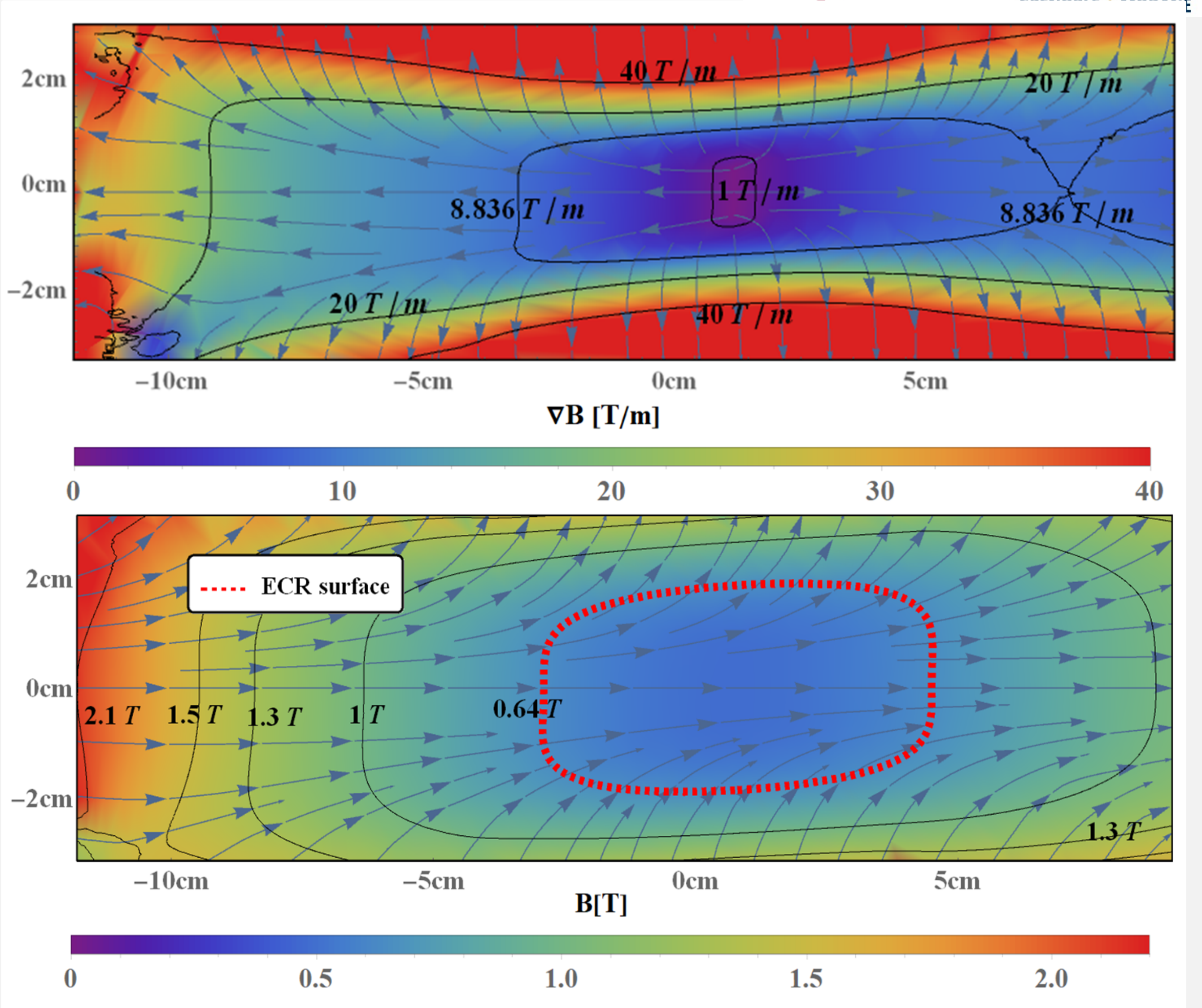


Figure: Phoenix V2 magnetic field and it’s gradient, longitudinal cross section map

ECR orbit Far From Axi

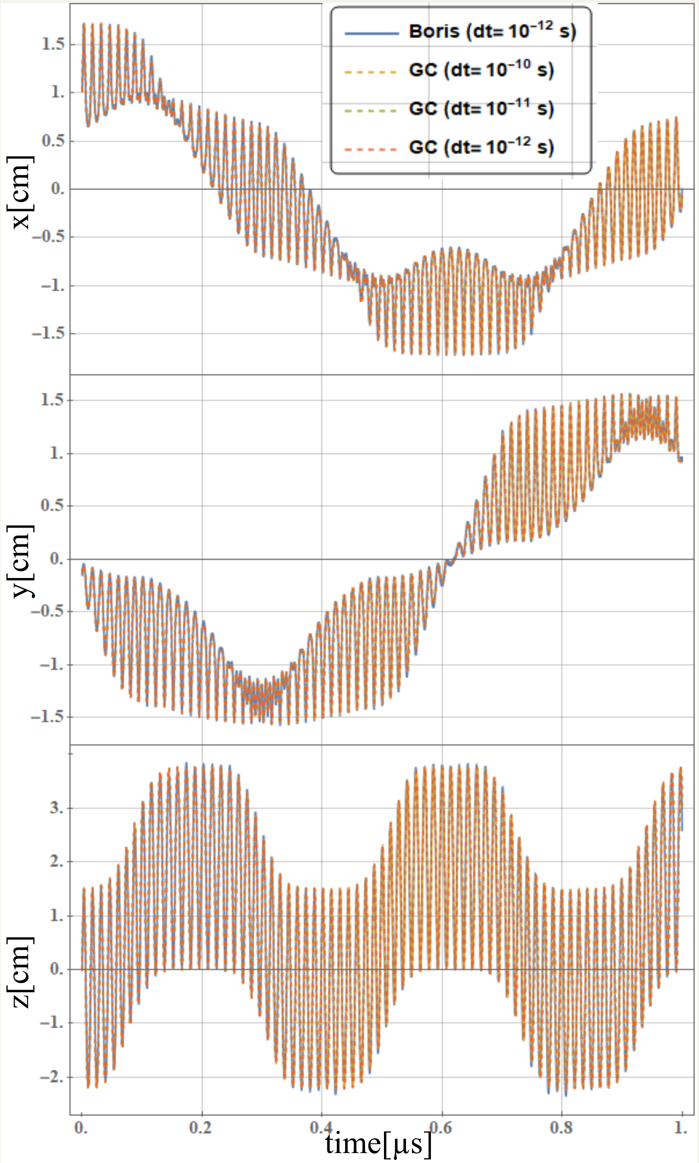


Figure: Confined electron orbit components

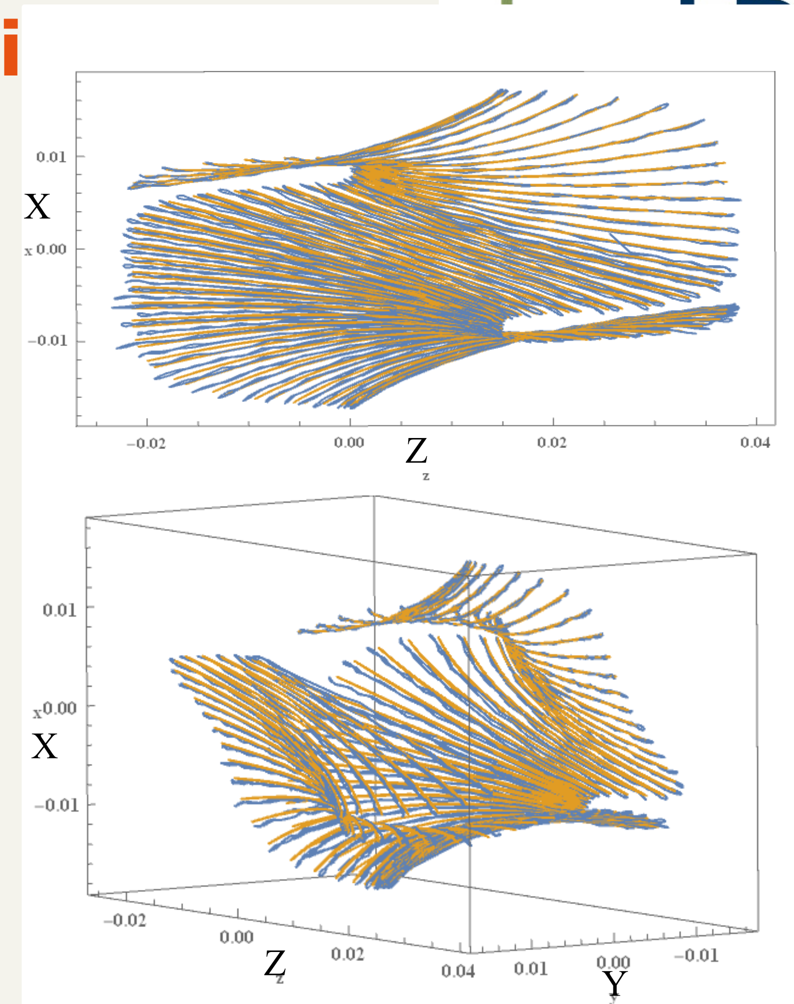


Figure: Confined electron orbit 3D plot

- Good agreement between Boris and GC with ~1μs of propagation.

ECR orbit analysis (FFA)

- GC algorithm ~20 times more computationally expensive per step.
  - ~100 times larger timestep rends it faster.
- The GC trajectories are stable up to a 100ps time-step, broken at a 1000ps.
- T_{boris} = Boris computation time

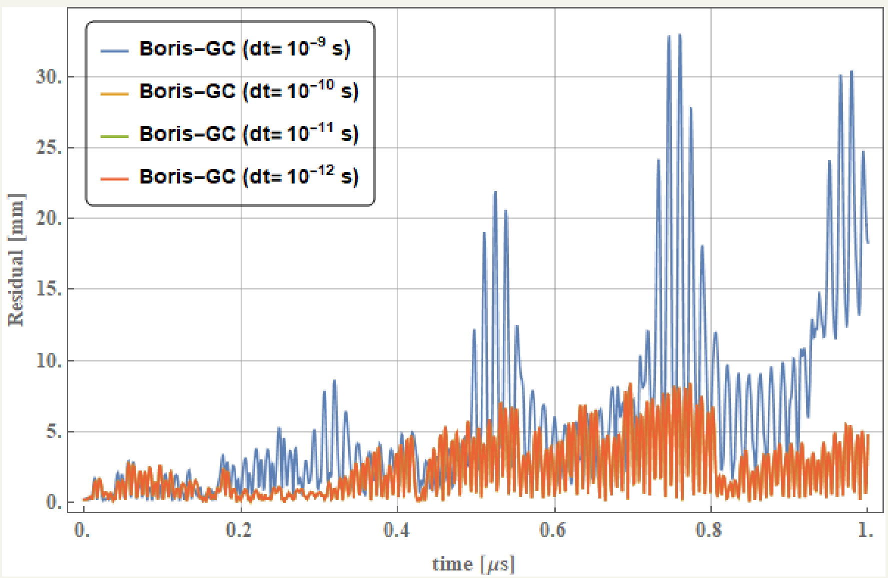
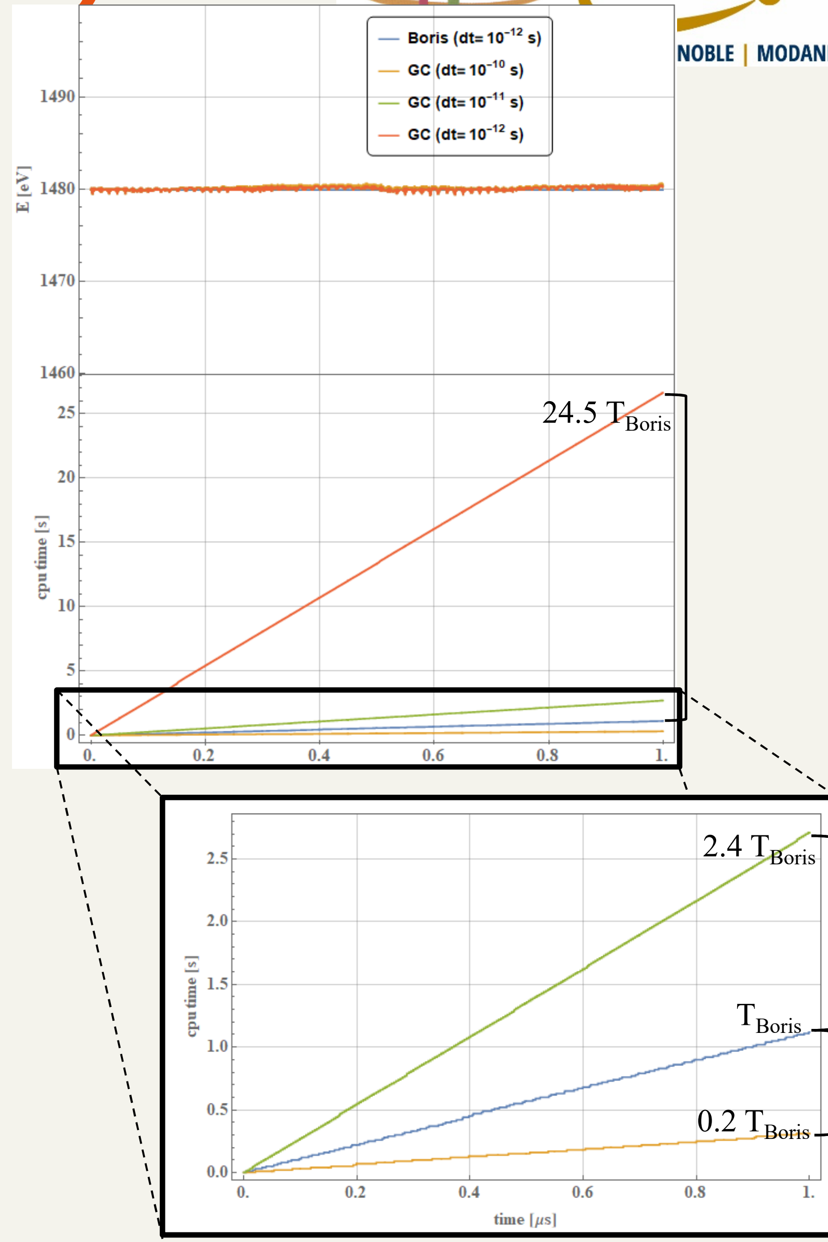


Figure: Boris-GC orbit residual at corresponding propagation times.



ECR orbit Close To Axis (CTA)

- Disphasement observed between the GC and Boris orbits
- Spatial envelopes are consistent.

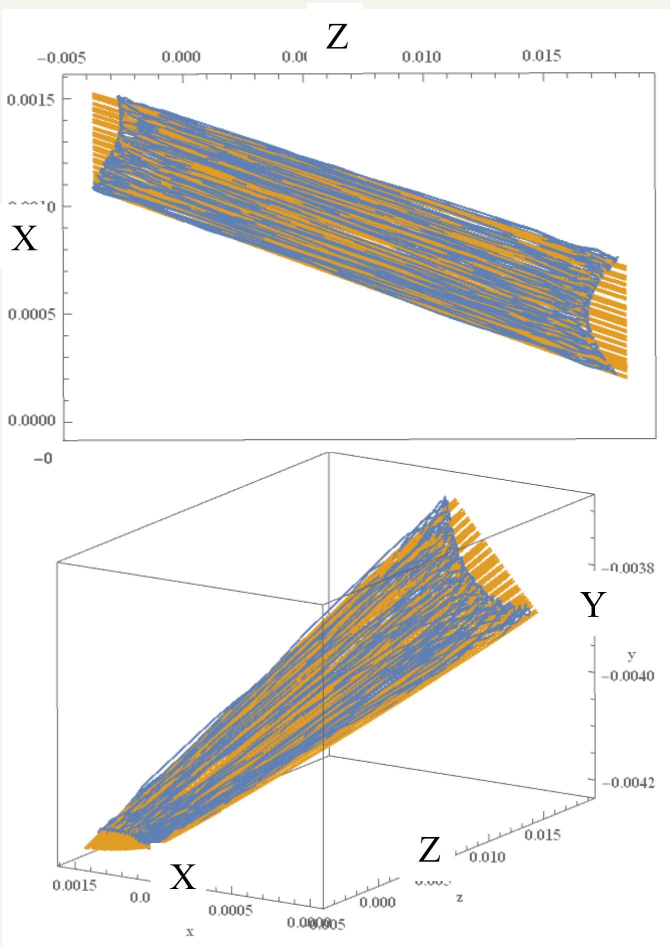


Figure: Confined electron orbit 3D plot

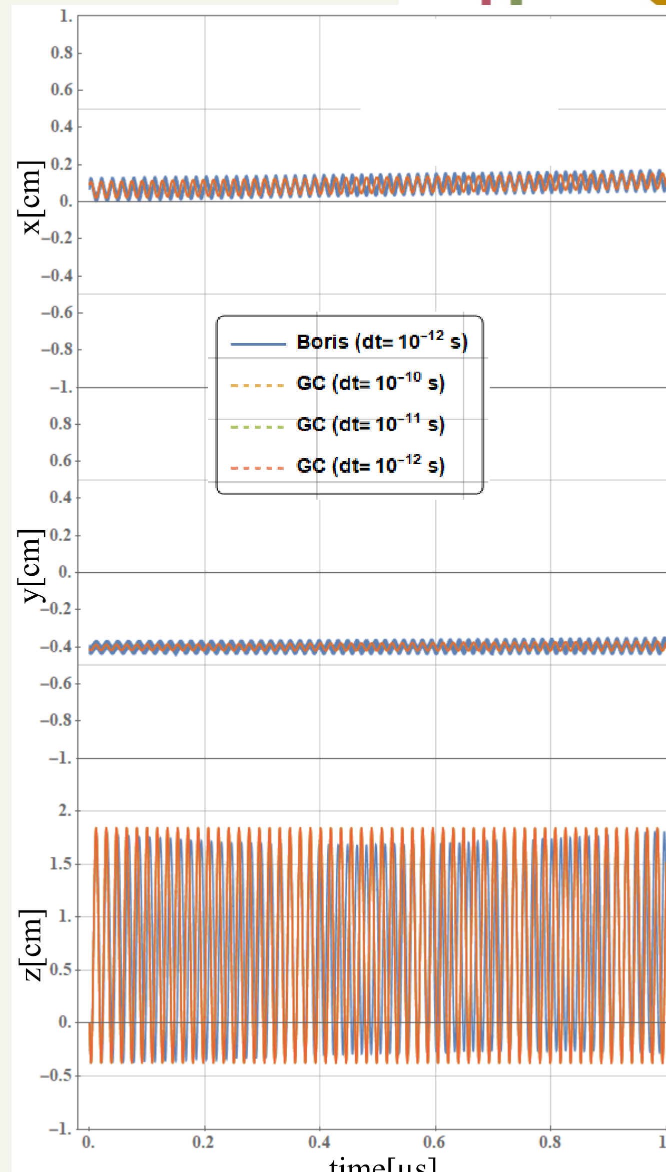
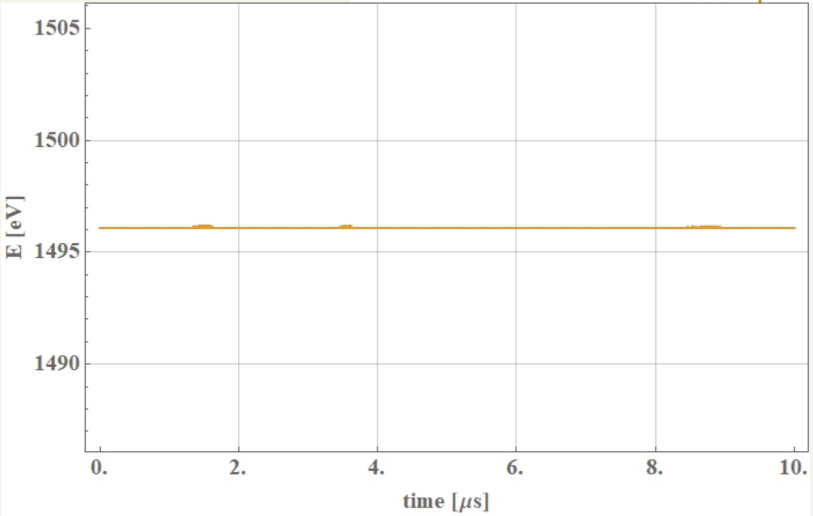


Figure: Confined electron orbit components

ECR orbit analysis (CTA)

- The observed disphasement is evident
- Good agreement between Boris and GC
- This orbit is one of the hardest for GC, (high axial oscillation frequency)
- Stable at this time-scale



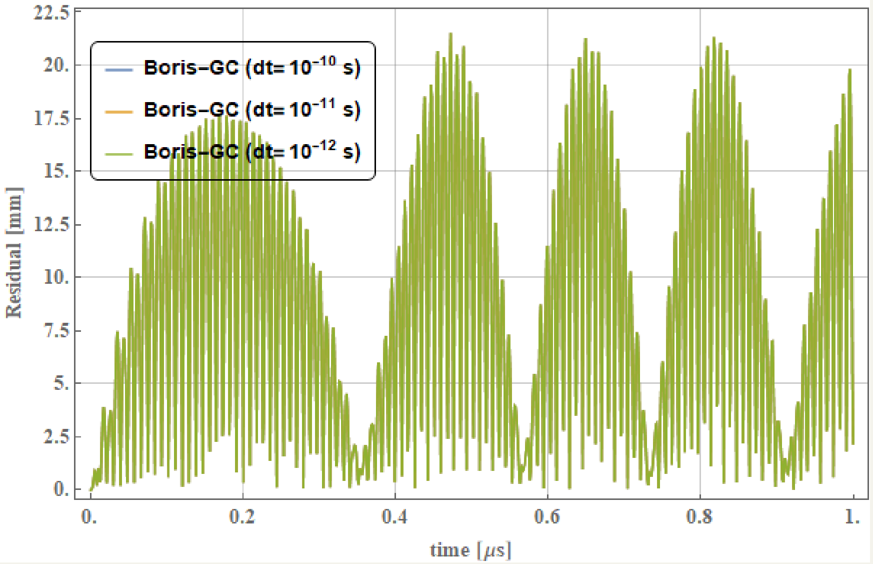


Figure: Boris-GC orbit residual at corresponding propagation times.

Energy conserved with both methods

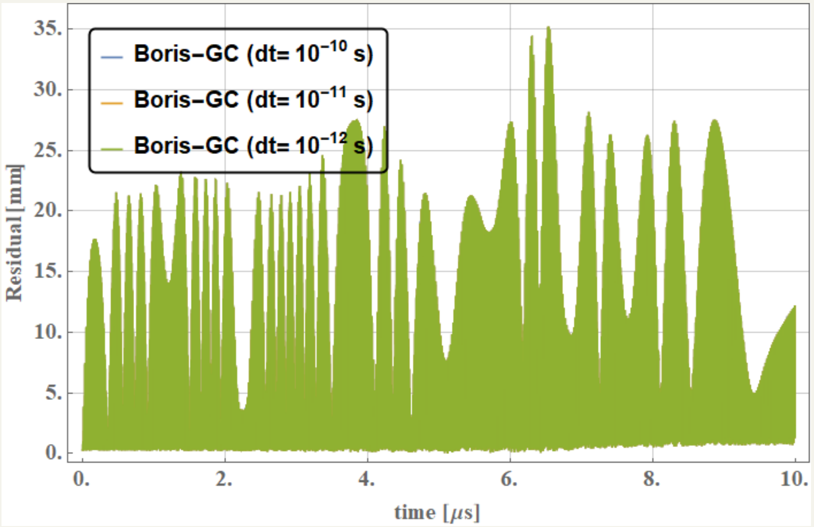
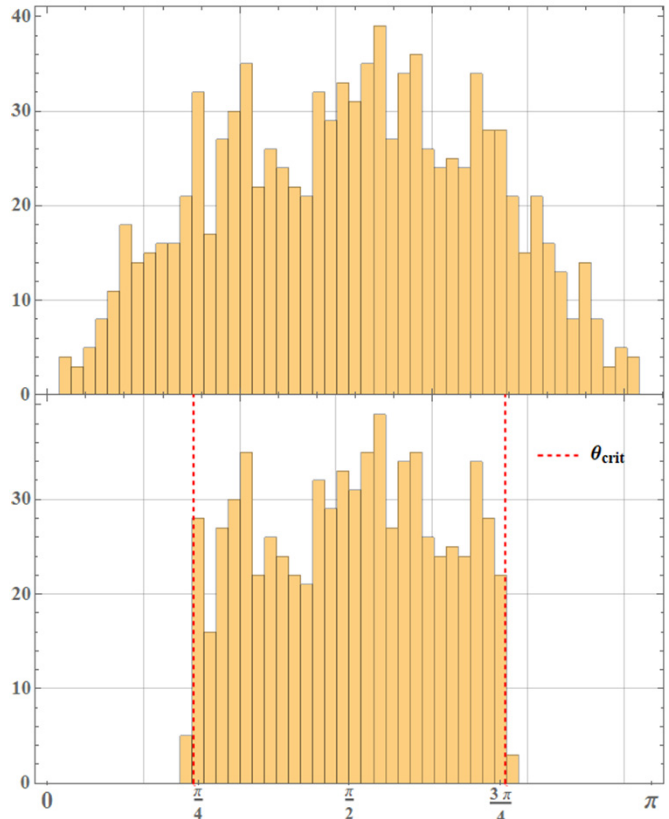


Figure: Boris-GC orbit residual at corresponding propagation times.

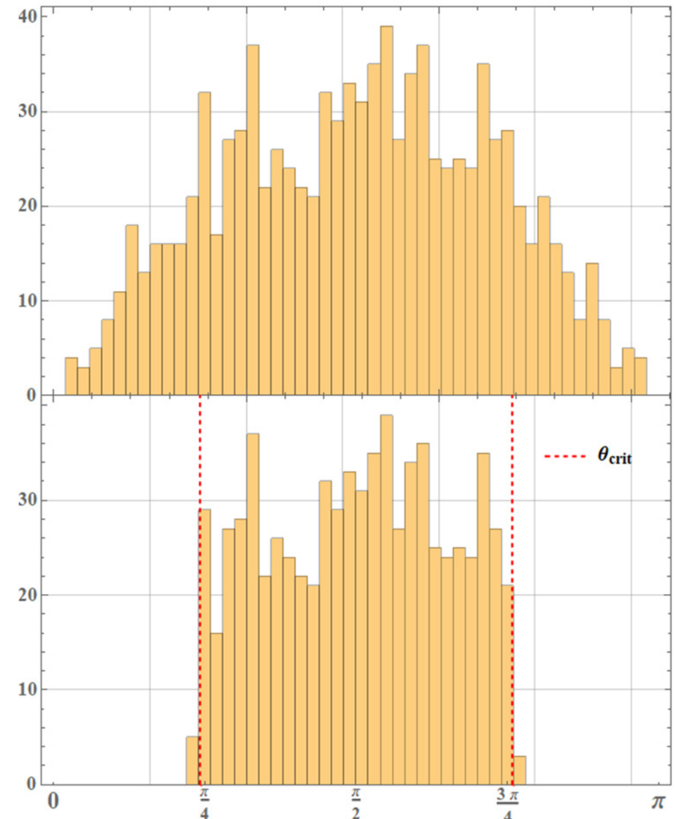
ECR confinement

- Good agreement for the confined electron distribution with both propagation methods
  - Sample size 1000, largest valid dt (10^{-10}s for GC and 10^{-12}s for Boris)

Boris propagated e⁻



GC propagated e⁻



$$\tan(\theta_{pitch}) = \frac{v_\perp}{v_\parallel}$$

$$\frac{1}{\mathcal{R}} = \frac{B_0}{B_{max}}$$

$$\sin(\theta_{crit}) = \frac{1}{\sqrt{\mathcal{R}}}$$

SILHI@GANIL ion source

- 2.45GHz operation frequency
- Microwave discharge ion source (MW)
- ~0.1T maximum solenoidal magnetic field. (2D map given)
- No magnetic confinement, one magnetic mirror towards extraction

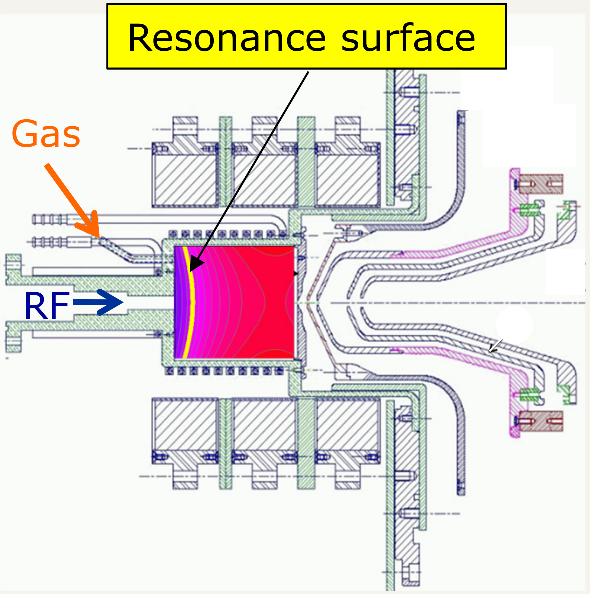
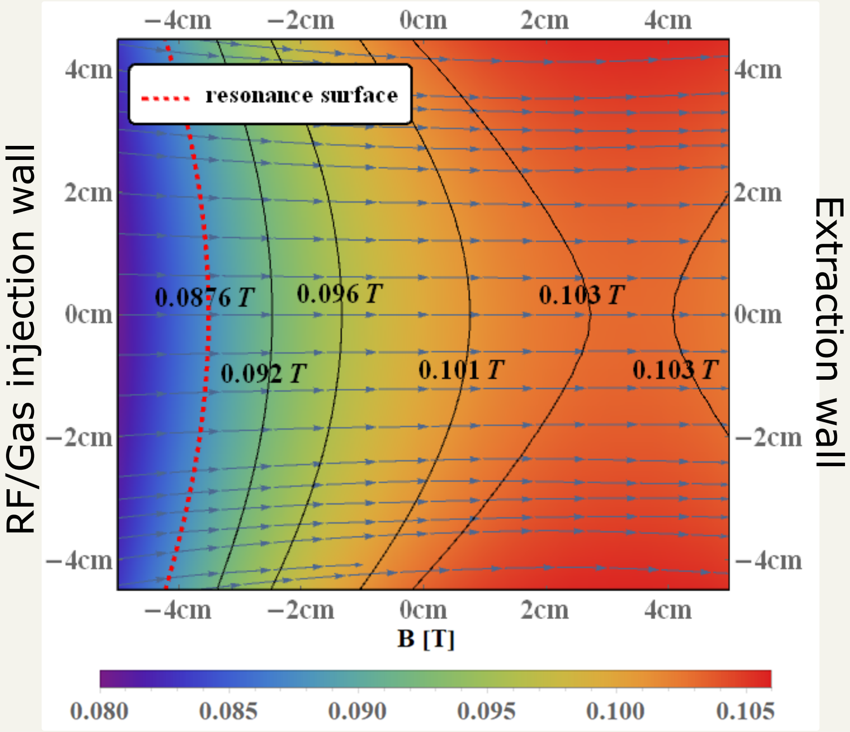


Figure: SILHI source with permanent magnets (CEA/IRFU/GANIL)



Trapped orbit in SILHI (one bounce)

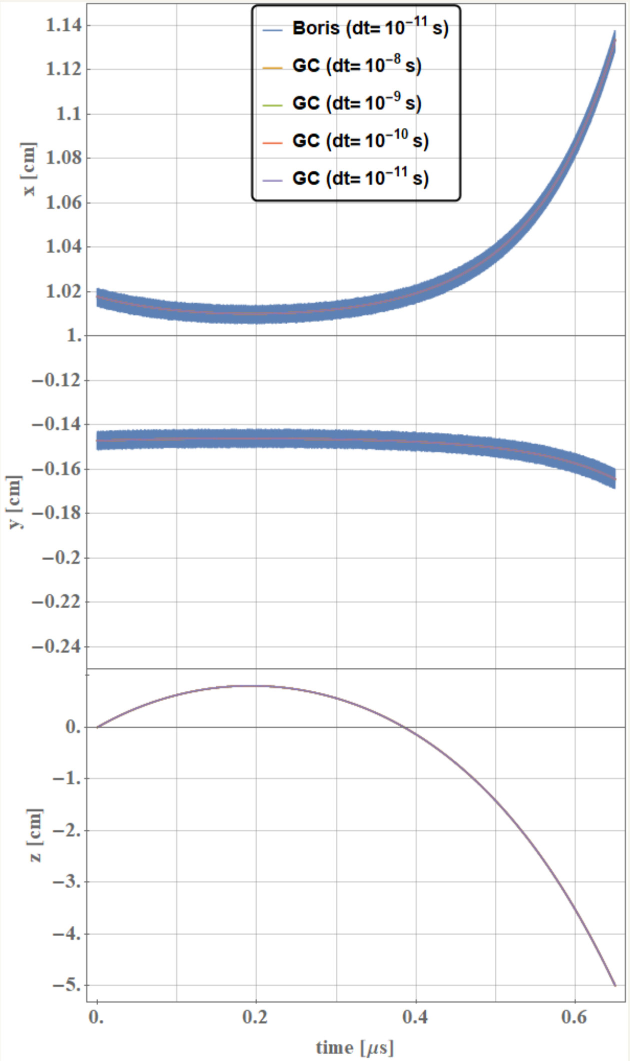


Figure: One bounce electron orbit components

- Very good qualitative agreement.
- A one bounce orbit chosen as it is one of the most challenging examples.
- A typical energy electron of ~1eV is confined for ~1μs. In the absence of interaction

Trapped orbit analysis

- GC algorithm ~5.7 times more computationally expensive per step.
- 10 times larger timestep rends it faster.
- The GC trajectories stable up to a 1000ps time-step, broken at a 10^4ps.
  - GC up to ~10 times faster when compared to Boris.

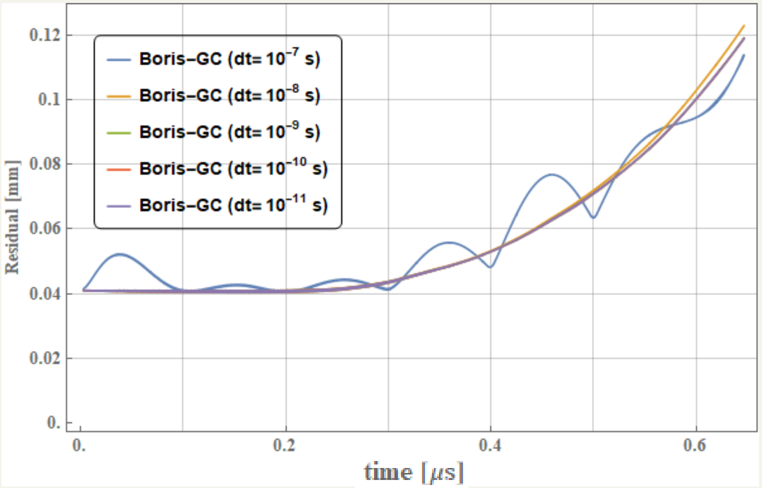
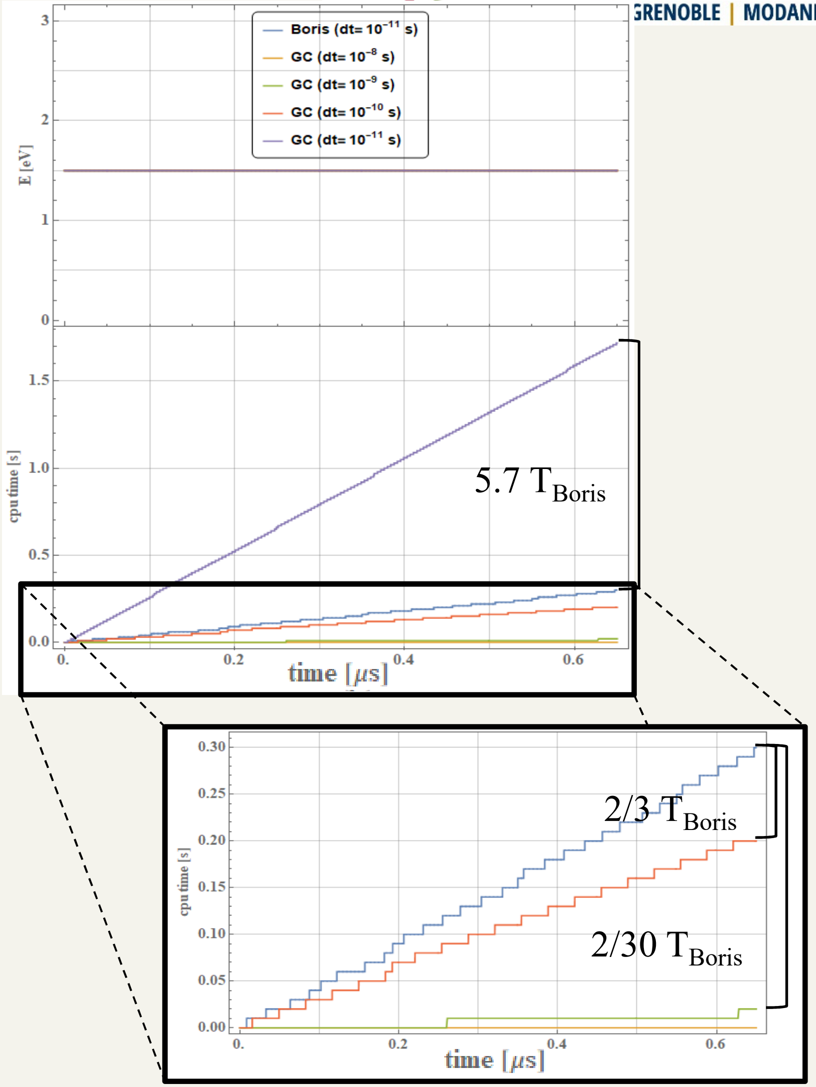


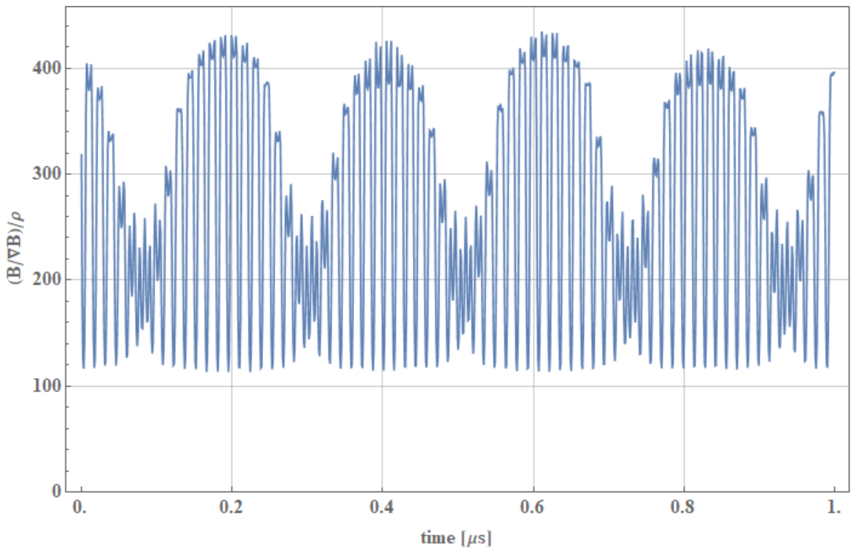
Figure: Boris-GC orbit residual at corresponding propagation times.



GC algorithm validity

$$\rho_L \ll \frac{B}{|\nabla B|} \Rightarrow \frac{(B/|\nabla B|)}{\rho_L} \gg 1$$

ECRIS (Phoenix V2)



MDIS (SILHI)

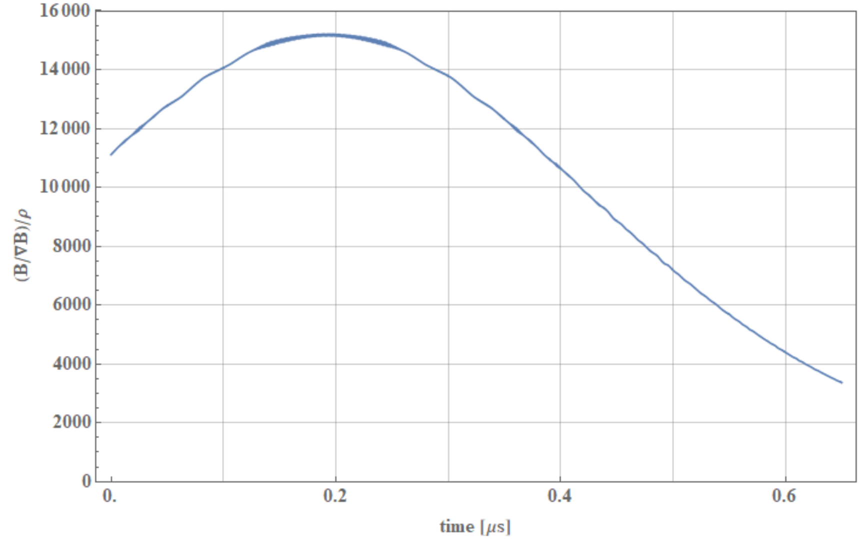


Figure: Validity condition for the GC algorithm for ECRIS and MDIS typical orbit

The GC algorithm is more suitable for a MDIS (SILHI type) source than for an ECRIS

Conclusions and prospects

- The GC algorithm can accurately reproduce electron trajectories in the domain of both studied ion sources
  - PHOENIX V2 ECRIS: B=1T, T_e~1keV, volume=0.6L
  - SILHI@GANIL: B=0.1T, T_e~1eV, volume=0.6L
- The GC algorithm can provide an advantage in terms of computation time for particle plasma simulations.
  - This advantage is greater in a flat field, as observed for SILHI. The time step can be increased by a factor of 10^3 with a computation time two orders of magnitude smaller with respect to Boris.
  - For the Phoenix V2 ECRIS the gains are more modest, with a time-step increased by a factor of 10^2 and one order of magnitude gain in computation time.
- A smart switcher for orbit integration could be implemented, where the GC approximation is used with a large time-step when valid and a high time resolution isn't required.

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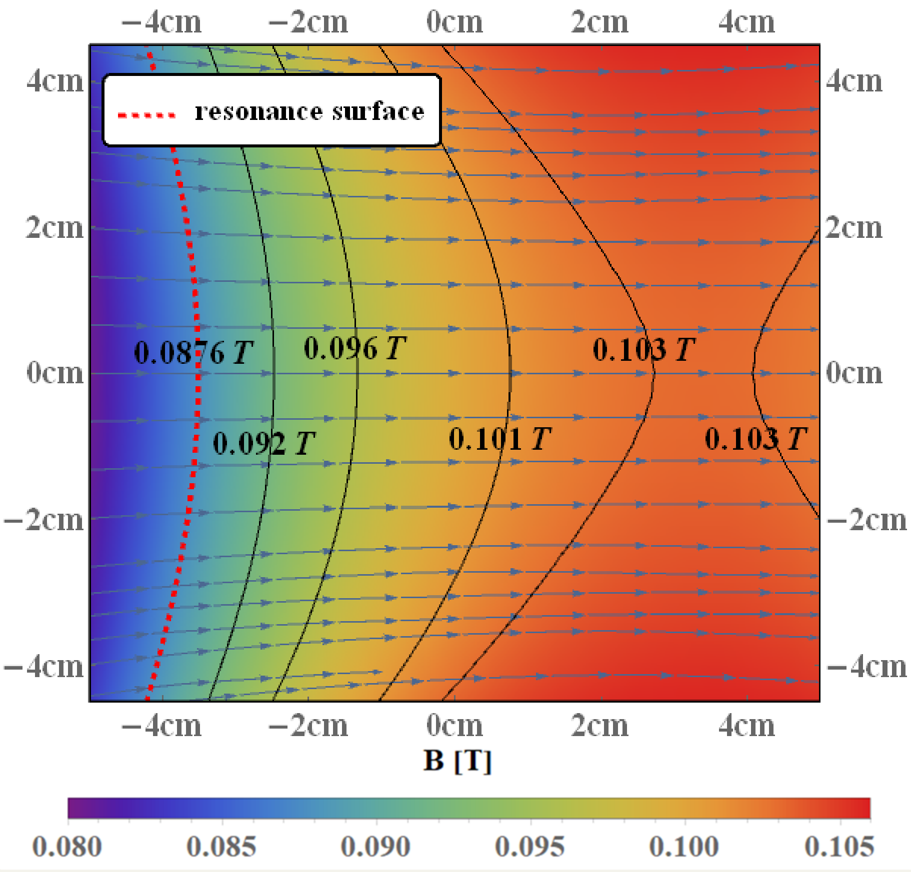
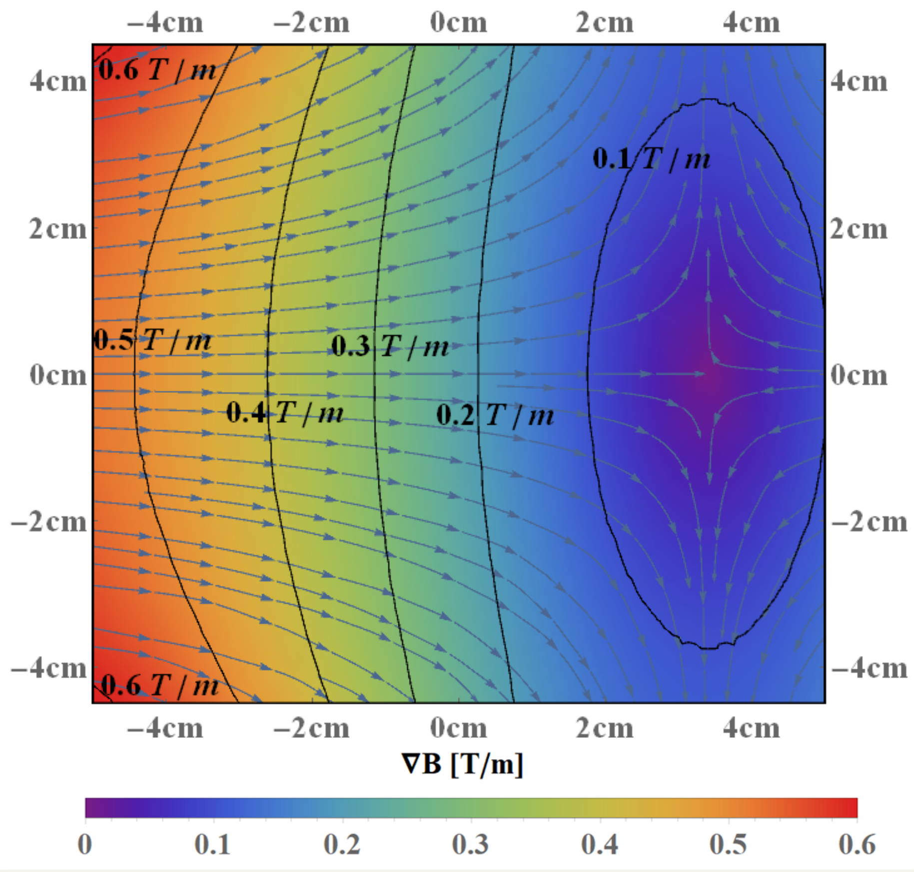
http://dx.doi.org/10.5120/1402-1891

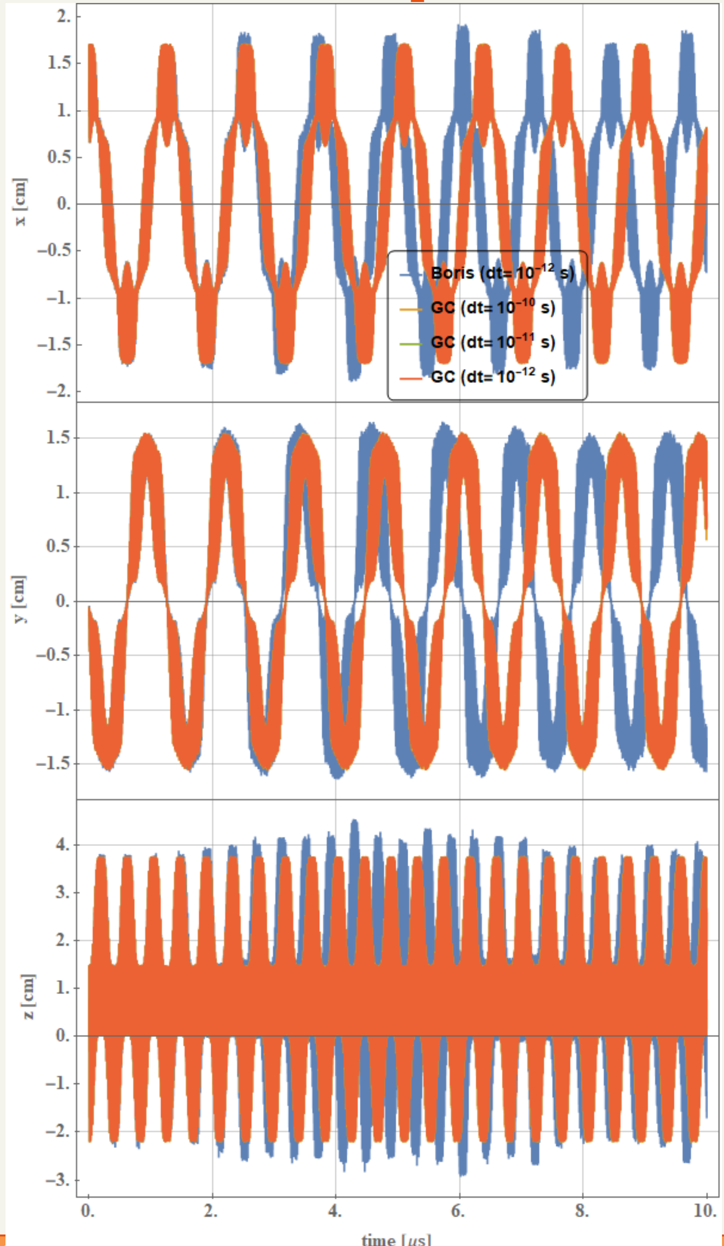
APPENDIX

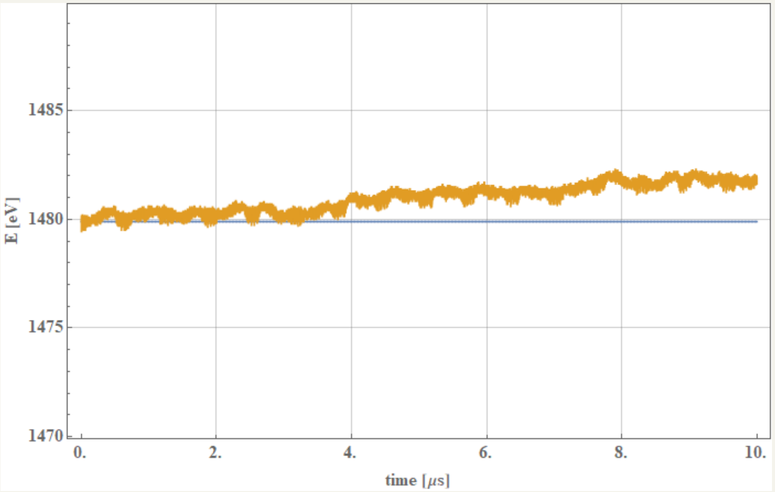
SILHI@GANIL B field and gradient

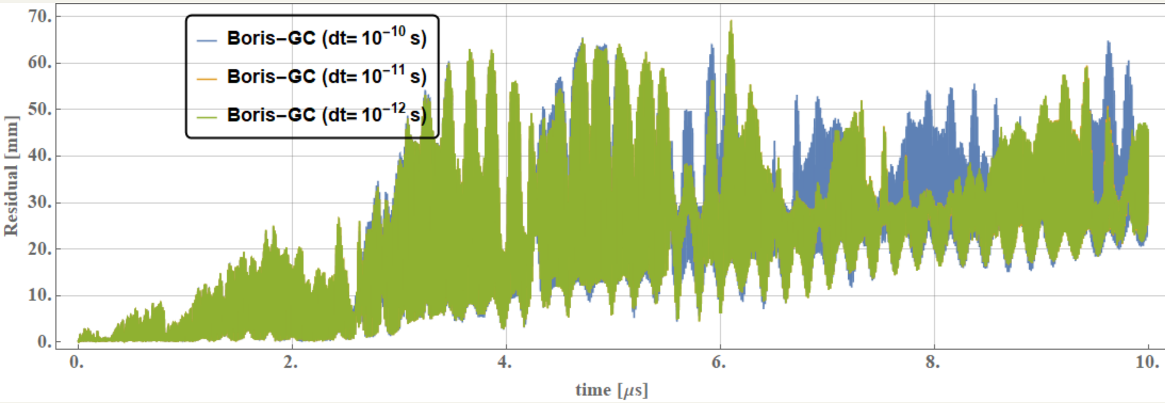




Disphasement FFA







GC approximation validity by electron's kinetic energy

T [eV]	γ	v [m/s]	~$v_\perp$ [m/s]	B [T]	∇B [T/m]	ρ [μm]	B/∇B	(B/∇B)/ρ
1	1.000002	593097	342425	0.64	27	3	0.024	7792
10	1.000020	1875511	1082827	0.64	27	10	0.024	2464
100	1.000196	5930105	3423748	0.64	27	30	0.024	779
1,000	1.001957	18727914	10812566	0.64	27	96	0.024	246
10,000	1.019570	58455268	33749165	0.64	27	306	0.024	78
100,000	1.195695	164352596	94889016	0.64	27	1008	0.024	24
1,000,000	2.956955	282128500	162886965	0.64	27	4279	0.024	6

Table: ECRIS near ECR region

T [eV]	γ	v [m/s]	~$v_\perp$ [m/s]	B [T]	∇B [T/m]	ρ [μm]	B/∇B	(B/∇B)/ρ
1	1.000002	593097	342425	0.102	0.3	19	0.340	17813
10	1.000020	1875511	1082827	0.102	0.3	60	0.340	5633
100	1.000196	5930105	3423748	0.102	0.3	191	0.340	1781
1,000	1.001957	18727914	10812566	0.102	0.3	604	0.340	563
10,000	1.019570	58455268	33749165	0.102	0.3	1918	0.340	177
100,000	1.195695	164352596	94889016	0.102	0.3	6324	0.340	54
1,000,000	2.956955	282128500	162886965	0.102	0.3	26848	0.340	13

Table: MDIS near plasma chamber centre