

# BEAM DYNAMICS USING SUPERCONDUCTING PASSIVE HARMONIC CAVITIES WITH HIGH CURRENT PER BUNCH

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## Abstract

In 4<sup>th</sup> generation synchrotron light sources, harmonic cavities (HCs) are critical components needed to achieve the required performance. They provide longer bunches, which helps to reduce statistical effects (intra-beam scattering and Touschek effect). In "timing" modes, where the bunch spacing is larger than in conventional modes and the number of particles per bunch is higher, this need is even greater. In this article, we present the beam dynamics in the high current per bunch regime and how it interacts with the single bunch collective effects. In particular, a dipole-quadrupole instability is observed above the microwave threshold and a coupling between the dipole and cavity modes is shown to limit bunch lengthening at low current. The effective gain from the use of HCs in terms of lifetime, emittance, and energy spread is also discussed.

## INTRODUCTION

In this paper, we explore the beam dynamics of a double RF system with harmonic cavities (HCs) in the context of high currents per bunch. The parameters of the SOLEIL II project [1–3], which aims to replace the existing SOLEIL storage ring by a 4<sup>th</sup> generation synchrotron light source are used throughout the article.

### Double RF System

For a passive unloaded cavity at the  $k^{\text{th}}$  harmonic of the RF frequency  $f_{RF}$ , the only knob is the cavity tuning angle  $\psi$  or equivalently the cavity detuning  $\Delta f$ :

$$\Delta f = f_r - k f_{RF}, \quad (1)$$

$$\tan \psi = Q \left( \frac{f_r}{k f_{RF}} - \frac{k f_{RF}}{f_r} \right) \approx 2Q \frac{\Delta f}{f_r}, \quad (2)$$

where  $f_r$  is the cavity resonance frequency and  $Q$  is the cavity quality factor (equal to the unloaded quality factor  $Q_0$  in that case). The beam-induced voltage  $V_2$  in the harmonic cavity (neglecting form factors) is given by [4]:

$$V_2 = 2I_0 R_s \cos \psi \approx I_0 \frac{R_s}{Q} \frac{k f_{RF}}{\Delta f} \sin \psi, \quad (3)$$

where  $I_0$  is the total current,  $R_s$  is the cavity shunt impedance (using the circuit ohm definition). For a main cavity voltage  $V_1$ , an approximate voltage of  $V_2 \approx V_1/k$  is needed to get close to the near flat potential (NFP) conditions [5] and longer bunches. Using the beam voltage induced at

resonance, one can set a lower limit on the needed shunt impedance:

$$R_s \gg \frac{V_1}{2kI_0}. \quad (4)$$

For  $V_1 = 1.8$  MV, a 4<sup>th</sup> HC and the lowest beam current of 20 mA, the total shunt impedance of the HCs should be much larger than 11.25 M $\Omega$ . As the system should also be stable at higher currents, the total  $R_s/Q$  should not exceed 100  $\Omega$  to avoid instabilities [5, 6]. Both conditions are only possible with superconducting (SC) cavities, which typically have  $Q_0$  around  $10^8$ , while normal conducting (NC) cavities typically have  $Q_0$  of a few  $10^4$ . For these reasons, we consider in this article a double RF system using SC HCs as detailed in Table 1 and for such a SC passive system there is  $\psi \approx \pi/2$  and thus  $\sin \psi \approx 1$  in Eq. (3).

Table 1: RF Cavity Parameters

Parameter	NC MC	SC HC
Harmonic number $k$	1	3 or 4
Shunt impedance $R_s$ (per cavity)	5 M $\Omega$	4.5 G $\Omega$
Unloaded quality factor $Q$	35 700	$10^8$
$R_s/Q$ (per cavity)	140 $\Omega$	45 $\Omega$
Loaded quality factor	6 000	$10^8$
Cavity number	4	2

## BEAM DYNAMICS

In this part, the beam dynamics in 8-bunch mode at 100 mA and in single bunch mode at 20 mA is studied by tracking using mtrack2 [7]. Results for SOLEIL II main operation mode, uniform filling at 500 mA, using the same RF system but without short-range wake, were already discussed in Ref. [5].

### Short Range Wake

To be able to estimate accurately the beam dynamics at a high charge per bunch, a preliminary impedance model including the most important features of SOLEIL II (resistive wall with NEG coating, tapers, BPMs, ...) is included in the simulations [8]. Figure 1 shows the relative variation of the bunch length and energy spread when the single bunch current is varied from 0 mA to 20 mA. It can be observed that the micro-wave instability (MWI) threshold, characterized by an energy spread increase, is reached at 3 mA.

Even in uniform filling mode, where the current per bunch is limited to 1.2 mA, the short-range wake effect is important

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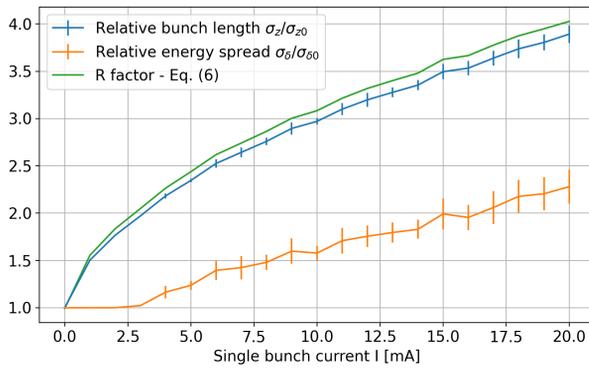


Figure 1: Relative variation of the bunch length  $\sigma_z$ , energy spread  $\sigma_\delta$  and R factor (Eq. (6)) versus single bunch current. Zero current RMS values:  $\sigma_{z0} = 8.6$  ps,  $\sigma_{\delta0} = 0.09$  %.

and allows to increase the maximum bunch length by 30 % compared to what was shown in Ref. [5].

### “Weak” Dipole-quadrupole Instability

In the 8-bunch mode at 100 mA, a “weak” dipole-quadrupole instability is observed in a given detuning range only when the short-range wake is included in the simulation. The instability is characterized by an in-phase oscillation of the beam center of mass and bunch length which does not lead to beam loss. The typical amplitude of the oscillations is about 20 ps for both the center of mass and the bunch length.

The coherent bunch spectrum, defined as the absolute value of the Fourier transform of the mean position of the bunch, is shown in Fig. 2 with and without the short range wake while the 3<sup>rd</sup> HC detuning is varied. In the left plot, one can see the dipole Robinson mode frequency decreasing from the unperturbed synchrotron frequency  $f = f_{s0}$  when the HC detuning decreases and the system is getting closer to the NFP conditions. In the right plot, in addition, there is the dipole synchrotron mode, from the inclusion of the short range wake, which is constant because of the (negative) incoherent tune shift compensated by the dynamic coherent frequency shift for mode  $m = 1$  [9]. When the dipole synchrotron mode is reached by what we believe to be the quadrupole Robinson mode around  $\Delta f \approx 20$  kHz, this “weak” dipole-quadrupole instability appears.

This instability seems remarkably similar to the fast mode coupling Robinson instability described in Ref. [10] but is driven by the dipole synchrotron mode excited by the MWI instead of the dipole Robinson mode. The instability is similarly observed for the 4<sup>th</sup> HC but closer to the NFP and in both cases it is possible to go beyond the detuning at which the modes couple and get back a stable beam.

### Mode Coupling Between The Cavity and Dipole Robinson Mode

The same “weak” dipole-quadrupole instability is observed in single bunch mode at 20 mA when the impedance

model is included. In addition, very fast beam losses are observed, even without short-range wake, which are driven by the mode coupling between the dipole Robinson mode originating from  $f = f_{s0}$  and the cavity mode from  $f = \Delta f$ .

The cavity (Robinson) mode has been introduced in Ref. [11], and recently studied in Refs. [12, 13], and is the consequence of the beam modulation at  $f_{RF} \pm \Delta f$  feeding back into the main cavity. Here, this effect only originates from the HC, i.e. not from the double RF system, and thus can be described by the point bunch Robinson theory applied to the HC [9]:

$$\Omega^2 = \omega_{s0}^2 + j \frac{eI_0\alpha_c}{E_0T_0} \times \sum_{p=-\infty}^{\infty} [p\omega_{RF}Z(p\omega_{RF}) - (p\omega_{RF} + \Omega)Z(p\omega_{RF} + \Omega)], \quad (5)$$

where  $\Omega$  is the coherent mode complex frequency,  $\omega_{s0} = 2\pi f_{s0}$ ,  $\omega_{RF} = 2\pi f_{RF}$ ,  $\alpha_c$  is the momentum compaction factor,  $E_0$  is the reference energy,  $T_0$  is the revolution period, and  $Z$  is the HC impedance.

Figure 3 shows the coherent bunch spectrum superimposed with the dipole Robinson mode and cavity mode computed numerically from Eq. (5). When the short-range wake is not included, there is a perfect agreement between the tracking and the point bunch theory. The addition of the impedance model strengthens the coupling between the modes, which lowers the instability threshold. In our case, the mode coupling is fully independent of the synchrotron damping time, differently from what is stated in Ref. [13].

This instability is a strong limitation for HC operation at low total current, the threshold is lower for lower harmonic HCs as the detuning to get the required voltage to reach NFP is lower. It explains why the 3<sup>rd</sup> HC can provide much less bunch lengthening than the 4<sup>th</sup> HC at 20 mA as shown in Table 2.

## IBS AND TOUSCHEK

The reference Touschek lifetime for the SOLEIL II TDR lattice [2] is 3.0 h without errors, calculated for 1.8 MV, 1.2 mA and the zero current RMS values: bunch length  $\sigma_{z0} = 8.6$  ps, energy spread  $\sigma_{\delta0} = 0.09$  %, horizontal emittance  $\epsilon_x = 84$  pm rad and vertical emittance  $\epsilon_y = 25$  pm rad exited by white noise.

To compute the Touschek lifetime  $\tau$  for non-Gaussian bunches including the IBS, we use the following method. Firstly, the equilibrium bunch length and energy spread are extracted from mbtrack2 tracking results including the HC and the impedance model. They are used to compute analytically, using the `ibsEmittance` command from elegant [14], new values for the equilibrium energy spread and emittances. Then the Touschek lifetime  $\tau_{\text{Piwinski}}$  is computed using the Piwinski formula [15] with the IBS equilibrium values and an arbitrary bunch length  $\sigma_{z0}$ . Finally the bunch profile  $\rho$  from tracking is used to compute the Touschek lifetime  $\tau$

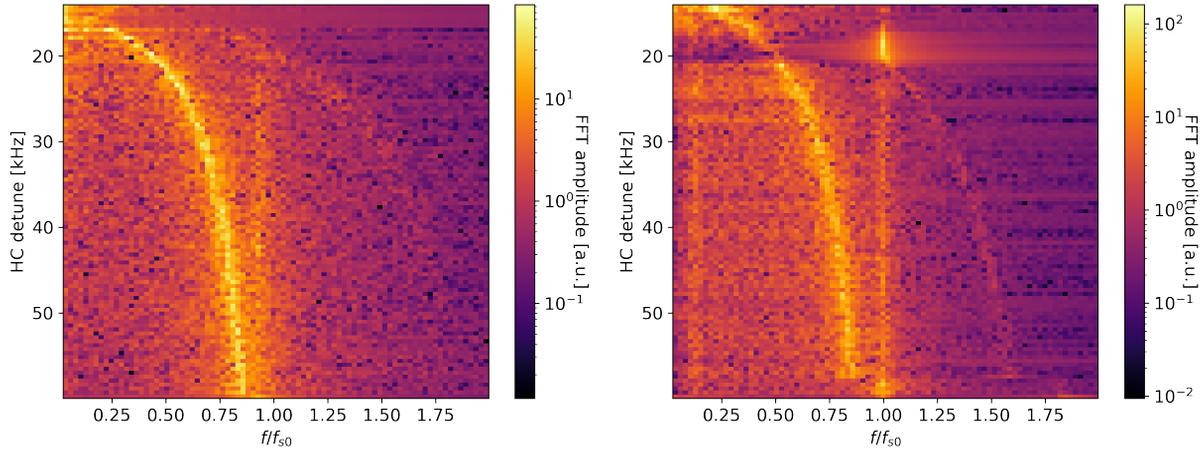


Figure 2: Coherent bunch spectrum from tracking data while the 3<sup>rd</sup> HC detuning is varied for the 8-bunch mode at 100 mA without (left) and with (right) short-range wake.

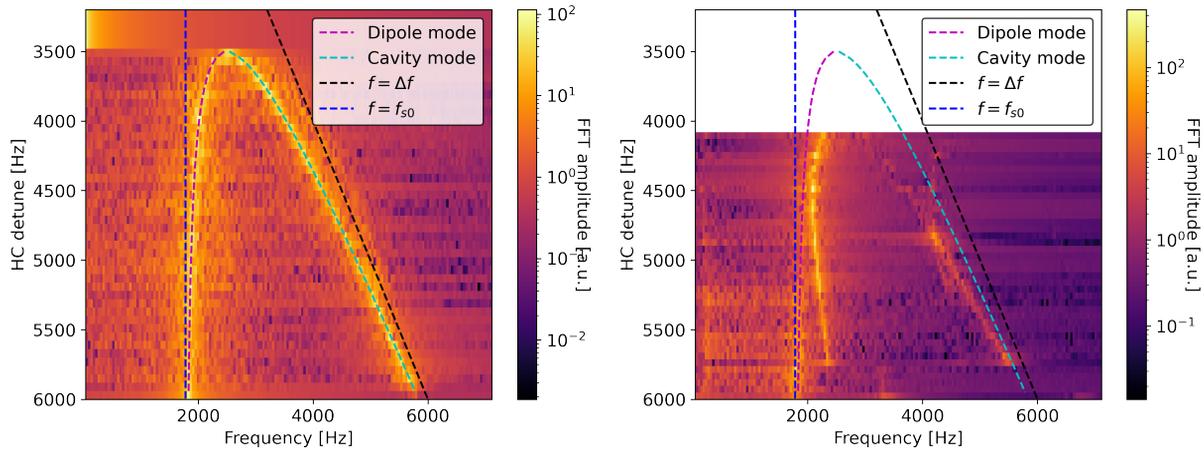


Figure 3: Coherent bunch spectrum from tracking data while the 3<sup>rd</sup> HC detuning is varied for the single bunch mode at 20 mA without (left) and with (right) short-range wake. The dipole and cavity modes shown in the dashed lines are the numerical solutions of Eq. (5).

using a scaling law [16]:

$$\tau = \tau_{\text{Piwinski}} \times \frac{\int \rho_0^2(z) dz}{\int \rho^2(z) dz} = \tau_{\text{Piwinski}} \times R, \quad (6)$$

where  $\rho_0$  is the longitudinal line density corresponding to a Gaussian bunch of  $\sigma = \sigma_{z_0}$  and  $\rho$  is an arbitrary longitudinal line density.

This approach is not fully self-consistent, as the elegant IBS calculation assumes Gaussian bunches, and any further bunch lengthening from the IBS is neglected in the  $R$  factor. The IBS would need to be included in the tracking simulation without assuming a Gaussian shape for a fully self-consistent calculation, but the result should give a good approximation of the combined effects.

The energy spread increase from the MWI plays a significant role for the IBS and Touschek at high bunch charge. Without HC, the Touschek lifetime is quasi-constant in the

range from 10 mA to 20 mA as the current per bunch increase is compensated by the energy spread blow-up (responsible for a 60 % increase at 20 mA), the emittance increase from IBS and the bunch lengthening.

The performances, including Touschek and IBS effects, resulting from the maximum bunch lengthening found by tracking are shown in Table 2. In uniform mode at 500 mA, the 3<sup>rd</sup> HC is more effective in lengthening the bunches than the 4<sup>th</sup> HC. In 8-bunch mode at 100 mA, both harmonics give similar results while for single bunch at 20 mA the 4<sup>th</sup> HC provides better performances than the 3<sup>rd</sup> one.

Overall, the HC provides a lower lifetime gain for the high current per bunch modes (factor 2) compared to what is obtained for the uniform mode (factor 4). A first reason is that the bunches are already quite long due to the impedance without HC, and the effective lifetime gain is only the ratio  $R_{\text{HC}}/R_{\text{noHC}}$ . The HC bunch lengthening also reduces the energy spread, if above the MWI threshold, and the emit-

Table 2: Performance in different operation modes with and without SC HC (RMS values for  $\sigma_z$  and  $\sigma_\delta$ ).

Operation Mode	HC	Bunch Length	Current per Bunch	Emittance H/V	R	Lifetime	Energy Spread
Uniform	Off	13 ps	1.2 mA	103/30 pm.rad	1.6	5.1 h	$1.06 \times 10^{-3}$
Uniform	3HC	64 ps	1.2 mA	90/27 pm.rad	7	22.4 h	$0.95 \times 10^{-3}$
Uniform	4HC	46 ps	1.2 mA	91/27 pm.rad	5	17.0 h	$0.97 \times 10^{-3}$
8 bunches	Off	27 ps	12.5 mA	126/38 pm.rad	3.4	1.5 h	$1.86 \times 10^{-3}$
8 bunches	3/4HC	77 ps	12.5 mA	112/34 pm.rad	8.7	3.6 h	$1.21 \times 10^{-3}$
1 bunch	Off	33 ps	20 mA	130/39 pm.rad	4	1.3 h	$2.31 \times 10^{-3}$
1 bunch	3HC	41 ps	20 mA	132/40 pm.rad	5	1.7 h	$1.88 \times 10^{-3}$
1 bunch	4HC	69 ps	20 mA	122/36 pm.rad	8.3	2.5 h	$1.55 \times 10^{-3}$

tance blow-up from IBS which tends to reduce the lifetime improvement.

## PERSPECTIVES

As shown in this paper, high current per bunch modes in ultra-low emittance rings are challenging for a double RF system, and the resulting Touschek lifetime can still be quite low despite large bunch lengthening factors. In addition, beamlines may require a limit on the maximum bunch length to maintain good resolution for “timing” mode experiments. A possible way to improve further the Touschek lifetime would be to go to a round beam, i.e.  $\epsilon_x = \epsilon_y$ , but it brings additional difficulties for lattice design and operation.

For the SOLEIL II project, it was decided to focus on the uniform mode at 500 mA and on a new 32-bunch mode, the other modes being kept but with lower priorities. As a consequence, we are now investigating an NC passive HC solution based on ESRF 4<sup>th</sup> HC design [17].

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