

# INVESTIGATION OF LONGITUDINAL BEAM DYNAMICS WITH HARMONIC CAVITIES BY USING THE CODE MBTRACK

N. Yamamoto\*, KEK, Tsukuba, Japan

A. Gamelin and R. Nagaoka, Synchrotron SOLEIL, Gif-sur-Yvette, France

## Abstract

The *mbtrack* code is a multi-bunch tracking code to investigate collective effects in circular accelerators. In the next generation light sources, the impact of harmonic cavity on the beam dynamics should be investigated. For this purpose, we extended the *mbtrack* code to be able to treat high-Q resonators in a more general manner as the previous versions. By using this extension, the impact of main/harmonic rf cavities and their parasitic resonant modes can be investigated with arbitrary bunch filling patterns. As a result of benchmarking tests, it is confirmed that well-known instabilities caused by cavity-impedances, such as AC/DC Robinson and coupled bunch instabilities could be calculated. In addition, consistent results for the bunch phase shift and the bunch length modulation due to transient beam loading were also obtained in case of the harmonic rf operation with unoccupied buckets.

## INTRODUCTION

In diffraction-limited light sources, the study of collective effects is essential. If harmonic cavities (HCs) are used for the purpose of mitigating intrabunch scattering, the study of HC induced instabilities [1, 2] becomes also important. With HCs, the “flat potential condition” can be achieved, lengthening the bunch by a factor of  $\sim 5$ . However, the effective radio-frequency (rf) voltage seen by the beam becomes sensitive to both positions and distributions of all bunches, as the beam-induced voltage of both HCs and main accelerating cavities (MCs) contribute.

The impact of HC impedances to collective effects was recently investigated analytically by M. Venturini [2]. In this work, the Vlasov equation for the longitudinal phase-space was linearized by dividing the rf potential into unperturbed and perturbed parts, where the perturbed part consists only of HC impedances, with the assumption of a uniform beam filling. Consequently, coupled bunch instabilities (CBIs), having coupled bunch modes 0 and 1, driven by the HC impedance were predicted in ALS-upgrade case.

With the unoccupied buckets in the beam filling, considerable variations of the rf voltage are caused by the transient beam loading, impacting the beam performance [3]. Here the use of analytical approaches is difficult. In addition, the contribution from MC impedances is also of interest for CBI mode 0 that is AC Robinson instability.

Then we introduced additional computation routines to be able to treat high-Q resonators driven by beams and external generators into the *mbtrack* code (*mbtrack*) [4]. Using these

procedures, various operating conditions with arbitrary fillings can be simulated, allowing to investigate CBIs induced by parasitic resonance modes of installed cavities, AC/DC Robinson instabilities and beam motions related to HCs.

In this paper, the detail of the code modification is reported after a short introduction of the original *mbtrack*. Then well-known instabilities described above were investigated by *mbtrack*, and compared with analytical results as benchmarking tests.

## MBTRACK CODE

The *mbtrack* is an extension of the *sbtrack* code (*sbtrack*) to multibunch. The *sbtrack* is a single bunch tracking code with wakefields in fully 6-dimensional phase space, developed earlier [5]. Its core module can treat different kinds of wake functions for short-range wakefields, particularly those computed numerically with impedance codes via decomposition into a series of fundamental wake functions.

The *mbtrack* was developed to be able to investigate multi-bunch stabilities under several typical beam fillings, by preserving internal bunch motions. Each bunch consists generally of a large number of macroparticles, allowing to simulate both intra- and interbunch motions.

In recent years, a computation procedure to handle HC beam-induced voltages was introduced to the original *mbtrack* by adding wakefields induced by all bunches passing through harmonic cavities by following its complex temporal evolution [4]. This modification allowed simulating beam motions in bunch lengthening operations with HCs. However, more realistic beam motions influenced by both MC and HC impedances could not be simulated since MC was not treated explicitly as impedance sources but merely with ideal sine-function voltages in this *mbtrack* version.

## CODE MODIFICATION

MC impedance is also important for beam stability investigations. The perturbation of the MC voltage from the ideal values, which is mainly caused by beam wakefields, sometimes perturbs the stable motion of the beams. Especially for the harmonic rf operation, bunch lengthening performances are strongly affected by the voltage fluctuations of both MCs and HCs. If the unoccupied buckets exist in the filling, the periodic voltage fluctuation appears, the so-called transient beam loading [3]. In this case, beam stability performances are also considered to be affected because the total rf potential seen by each bunch is modulated.

In order to investigate these effects, we conducted several modifications as follows to *mbtrack*;

\* naoto.yamamoto@kek.jp

- a) extension of the computation procedure for HC beam-induced voltages to universal high-Q resonators,
- b) adding an optional procedure to handle generator-induced voltages into a),
- c) adding cavity feedback procedures to keep each cavity voltage and phase constant during the simulation,
- d) allowance of an arbitrary filling pattern input.

The beam-induced voltages  $\tilde{V}_b^{(m)}$  seen by the  $m$ th particle is written by

$$\tilde{V}_b^{(m)} = \tilde{V}_b^{(m-1)} \exp \left[ \left( i\omega_r - \frac{\omega_r}{2Q} \right) (t_m - t_{m-1}) - \frac{\omega_r R_s}{2Q} q_m \right], \quad (1)$$

where  $\omega_r$ ,  $Q$ ,  $R_s$  and  $q$  are the resonant angular frequency, the quality factor, longitudinal shunt impedance and the charge of the particle. In the modification a),  $\tilde{V}_b^{(m)}$  is calculated for all inputted resonators.

The cavity voltage  $\tilde{V}_c^{(m)}$  is a superposition of the generator-induced voltage  $\tilde{V}_g$  and  $\tilde{V}_b^{(m)}$ ,

$$\tilde{V}_c^{(m)} = \tilde{V}_g + \tilde{V}_b^{(m)}. \quad (2)$$

In b), the  $\tilde{V}_g$  is characterized by an amplitude  $|\tilde{V}_g|$  and a phase angle  $\theta_g$  taken account in a tuning offset angle of the cavity.

When the cavity feedback c) is used, the two parameters of  $\tilde{V}_g$  and  $\omega_r$  are adjusted with certain gain factors by monitoring the amplitude and phase of  $\tilde{V}_c^{(m)}$ . In the code, the  $\omega_r$  value is controlled via a tuning angle of the cavity.

## BENCHMARKING

To validate the investigations for the collective effects, we benchmarked the solutions of the analytical calculations against *mbtrack* simulations. In the benchmarking for DC/AC Robinson and CBI instabilities, the uniform filling for KEK-LS storage ring without any HCs was assumed, and the operation parameters for the MC were changed according to the calculations. For all benchmarking reported here, the particle number for each bunch was set to be 10,000.

### DC Robinson Instability

For DC Robinson instability, both the coherent synchrotron frequencies and the growth rates of the instabilities were calculated with the KEK-LS parameters [3]. In the calculation, only the MC impedance was inputted as an impedance source, and the cavity coupling factor was set to unity instead of the design value of 3.5 in order to decrease the instability threshold for stored currents.

The analytical solutions for coherent synchrotron frequencies and growth rates are calculated as numerical solutions of the equations of motions for synchrotron oscillations and shown in Fig. 1 as solid lines. The synchrotron frequencies for the *mbtrack* result indicated by red circles are evaluated as peak frequencies of fast-Fourier transform spectra of the longitudinal beam motions.

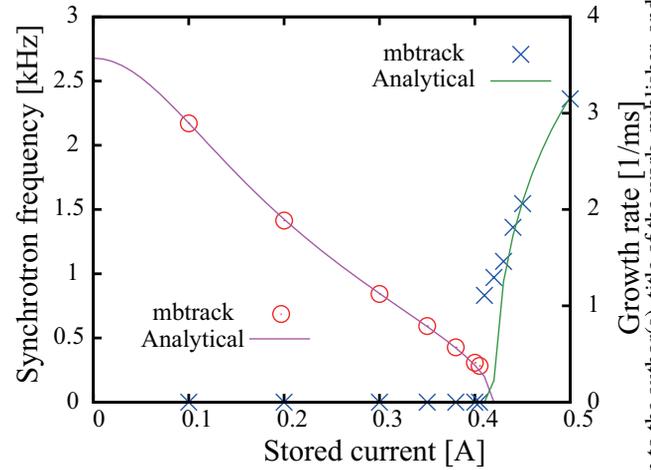


Figure 1: Coherent frequencies and growth rates as functions of the stored current in special parameter sets for the KEK-LS storage ring. The solid curves indicate the analytical solution of the equations of motions for synchrotron oscillations. The evaluated values by *mbtrack* investigations are plotted by circles and crosses.

The growth rates for *mbtrack* result indicated by blue crosses are estimated by exponential fitting to the amplitude invariant for bunch center motion in the longitudinal plane. In estimating the growth rates by *mbtrack*, very long damping time ( $\sim 100$  seconds) was set during the simulations to minimize the contribution from the radiation damping to particle motions. In Fig. 1 the analytical curves are qualitatively similar to the *mbtrack* result.

### AC Robinson

Three different positive offsets for the cavity tuning angle of KEK-LS MC were used for the benchmarking of the AC Robinson case. The obtained amplitude invariants for the tuning offset of 6, 56 and 106 kHz as functions of turn numbers are plotted in Fig. 2. The nominal synchrotron and revolution frequencies for the KEK-LS ring is 2.65 and 525 kHz respectively. In the investigation, the growth rates of the amplitude invariants were also estimated by numerical fittings.

With the assumption of the rigid bunch model, the analytical growth rates for the instabilities were calculated by using the quality factor and shunt impedance of the KEK-LS MC, and compared to the *mbtrack* result in Table 1. In Table 1, the fitting errors for the *mbtrack* result are also summarized.

Table 1: Comparison of Growth Rates Calculated by the *mbtrack* and Rigid Bunch Model

Detuning angle [kHz]	<i>mbtrack</i> [1/s]	Rigid bunch model [1/s]
+6	$524 \pm 21$	513
+56	$211 \pm 4$	216
+106	$41.7 \pm 0.6$	43.7

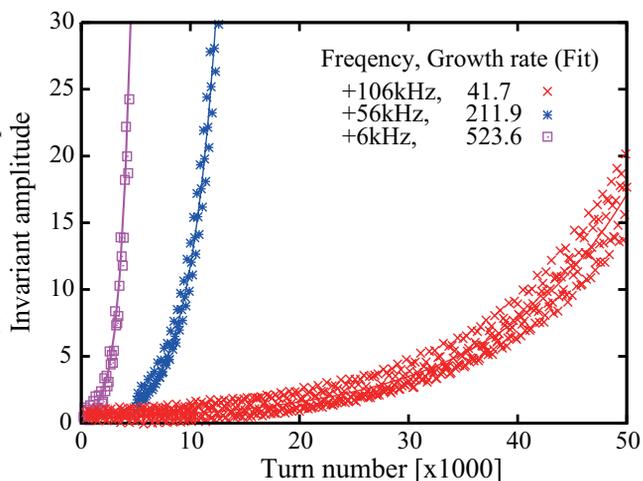


Figure 2: Amplitude invariants for longitudinal bunch center motions as functions for turn numbers with three different tunings offset for MC. The growth rates obtained by numerical fittings are also shown in this figure.

The estimated values for the *mbtrack* are consistent with the analytical result.

### Coupled Bunch Instability

The CBIs induced by parasitic mode of the MC were also benchmarked by using similar manners to AC Robinson. The difference is that additional impedances aside from MC are inputted for the simulations. As a result, it is confirmed that both of the instability growth rates and CBI mode numbers between the *mbtrack* result and analytical calculations are consistent.

### Beam Loading in Harmonic Operation

A serious beam loading effect was observed during the HC operation for Swiss Light Source (SLS) storage ring where a 1.5-GHz third-harmonic super-conducting cavity was used [6]. In their experiment, the bunch lengths were significantly affected by the transient beam-loading effect owing to the use of a long bunch gap of 280 ns. It was reported that the bunch positions shifted by 210 ps in peak-to-peak while the root-mean-square (rms) bunch lengths varied from 24 to 66 ps among the bunch train, at a beam current of 320 mA.

The *mbtrack* calculation with the similar parameter set was conducted, and the bunch positions shift of 189 ps and rms bunch lengths from 22.3 to 51.5 ps were obtained (see Fig. 3). These values are slightly small compared to the experimental values, but the change tendencies are consistent with the observed ones.

### SUMMARY

Conducting several calculations related to the rf cavities having high quality-factor, it is confirmed that the *mbtrack* results are consistent with the analytical calculations for well-known collective effects. In addition, the *mbtrack* code

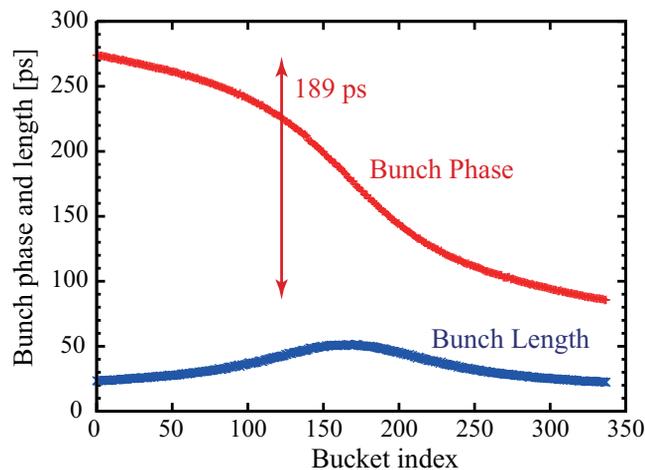


Figure 3: Bunch center phase and length along the bunch train with large bunch gaps for the SLS storage ring during the HC operation.

can be used for the investigation of transient loading-effect in the case of the harmonic rf operation.

Recently, we started to investigate realistic situations of extreme low-emittance storage rings with harmonic rf systems. Up to now, we confirmed that *mbtrack* can reproduce some of the collective beam motions reported by M. Venturini [2]. In addition, it is found that unstable beam motions could appear even in other low-emittance rings. These results shall be reported in forthcoming papers.

Finally, we believe that this modified *mbtrack* is useful for the investigation of collective effects and can be used for the design of light sources having very ambitious emittances.

### ACKNOWLEDGEMENTS

The authors would like to thank Patrick Marchand and Jörn Jacob for fruitful discussions about harmonic cavities and calculation results. We also thank all of the SOLEIL staff for their support.

This work was supported by JSPS KAKENHI Grant Number JP17K05131 and the KEK support program for long-term research abroad.

### REFERENCES

- [1] M. Venturini, *Phys. Rev. Accel. Beams*, vol. 21, p. 024402, 2 Feb. 2018.
- [2] M. Venturini, *Phys. Rev. Accel. Beams*, vol. 21, p. 114404, 11 Nov. 2018.
- [3] N. Yamamoto, T. Takahashi, and S. Sakanaka, *Phys. Rev. Accel. Beams*, vol. 21, 1 Jan. 2018, 012001.
- [4] G. Skripka, R. Nagaoka, M. Klein, F. Cullinan, and P. F. Tavares, *Nucl. Instr. Meth. A*, vol. 806, pp. 221–230, 2016.
- [5] R. Nagaoka, R. Bartolini, and J. Rowland, in *Proc. of PAC09*, Vancouver, BC, Canada, vol. 1, 2010, pp. 4637–4639.
- [6] M. Pedrozzi *et al.*, in *Proc. of 11th SRF workshop*, MOP25, 2003, pp. 91–94.