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DIFFUSION MAP ANALYSIS IN HIGH ENERGY STORAGE RING BASED e+/e- COLLIDER

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Abstract

In a very high energy e+/e- storage ring collider, e.g. Circular Electron Positron Collider (CEPC), the dynamic aperture is limited by the strong synchrotron radiation especially in the vertical direction. Some tracking results also shows that the beam lifetime does not correspond well to the dynamic aperture. Here we develop a method called diffusion map analysis, aiming to describe the beam distribution diffusion in transverse amplitude space by tracking less turns. The diffusion may come from quantum fluctuation of SR, beamstrahlung effect and nonlinearity. Comparing cases with different configuration of sextupoles, the diffusion map analysis presents good consistency with beam lifetime that needs much more turns of tracking. Constraints based on the diffusion map is applied to our dynamic aperture optimization, which could help us achieve enough long beam lifetime.

INTRODUCTION

CEPC is a circular e^+e^-collider located in a 100-km circumference underground tunnel. Its centre-of-mass energy is 240 GeV, and at that collision energy will serve as a Higgs factory, generating more than one million Higgs particles. The main parameters for Higgs of CEPC are listed in Table 1 [1].

Table 1: Main Parameters of CEPC in CDR

Beam Energy (GeV)	12 0
Circumference (km)	100
Synchrotron radiation loss/turn (GeV)	1.73
Luminosity/IP $(\mathbf{10^{34} cm^{-2} s^{-1}})$	3
Number of IPs	2
Momentum compaction factor $(\mathbf{10^{-5}})$	1.11
Energy acceptance (%)	2.06
Emittance x/y (nm)	1.21/0.0024
Bunch length (mm)	4.4
$\boldsymbol{\beta}$ function at IP (m)	0.36/0.0015
Beam-beam parameter	0.018/0.109

At this energy, strong synchrotron radiation causes strong radiation damping which enlarges the dynamic aperture to some extent, especially for off-momentum particles. However, quantum fluctuations in the synchrotron radiation decrease the dynamic aperture, particularly in the vertical direction. This difference mainly comes from the radiation in the final focus quadrupoles in the interaction region. The results of dynamic aperture influenced by the effect of synchrotron radiation are shown in Fig 1. The SAD code [2] is used to do the optics calculation and dynamic aperture tracking.

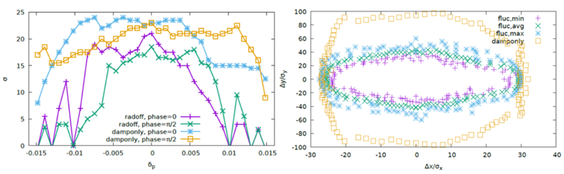

Figure 1: DA with damping/radiation off (left) and DA with damping/radiation fluctuation (right).

In this paper, we first present the inconsistency between dynamic aperture and beam lifetime. Then a method called diffusion map analysis is introduced to describe the beam distribution diffusion in amplitude space, which shows a close connection to beam lifetime. In the optimization of dynamic aperture, adding the constraints of diffusion rate can result in less large-amplitude particles in equilibrium distribution, as well as a longer lifetime.

BEAM LIFETIME

Beam lifetime is very critical for the feasibility of the collider. For a Gaussian distribution in phase space, the distribution of action J is

$$f(J) = \frac{1}{\epsilon_i} exp(-J/\epsilon_i), \text{ with } \int_0^\infty f(J)dJ = 1 \quad (1)$$

Lifetime near the boundary A is defined as [3, 4]

$$\tau \equiv \frac{N_0}{dN/dt} = \frac{\tau_i}{2Af(A)} = \frac{\tau_i}{2}\frac{\epsilon_i}{A} exp\frac{A}{\epsilon_i} \quad (2)$$

Here τ_i is the synchrotron radiation damping time. Through statistical analysis in the equilibrium with tracking data of many turns, we can get the distribution $f(A)$ to calculate the lifetime that contains the effects of radiation excitation, beamstrahlung and nonlinearity.

In some tracking results, beam lifetime does not correspond well to the dynamic aperture. (Fig.2)

A shorter lifetime means more large-amplitude particles exist in equilibrium distribution $f(A)$, which is a non-Gaussian distribution.

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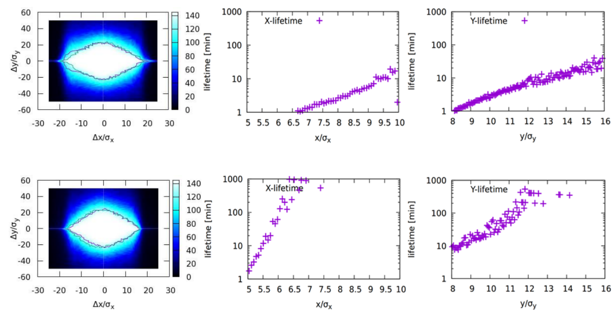

Figure 2: On-Momentum DA and beam lifetime in horizontal/vertical direction of two cases.

DIFFUSION

We try to describe the diffusion caused by complicated effects by using a method based on particle tracking. Hundreds of particles with same amplitude are tracked. As the number of turns grows, motions of the particles are different because of randomness. The variance of amplitude of particles in each turn is calculated to be presented as the rate of diffusion.

The amplitude of particle is defined as

$$ai \equiv \sqrt{\frac{2J_i}{\varepsilon_i}} \quad (i = x, y, z) \tag{3}$$

with their variance σ_{ai}.

To combine the three directions, we define the quantity

$$\sigma_a \stackrel{\text{def}}{=} \sqrt{\sigma_{ax}^2 + \sigma_{ay}^2 + \sigma_{az}^2} \tag{4}$$

Figure 3 gives the evolution of $\sigma_{ax}, \sigma_{ay}, \sigma_{az}$ and σ_a in different models with the initial amplitude $(ax, ay) = (3,11)$. Figure 4 shows the evolution of variances with two different initial amplitudes.

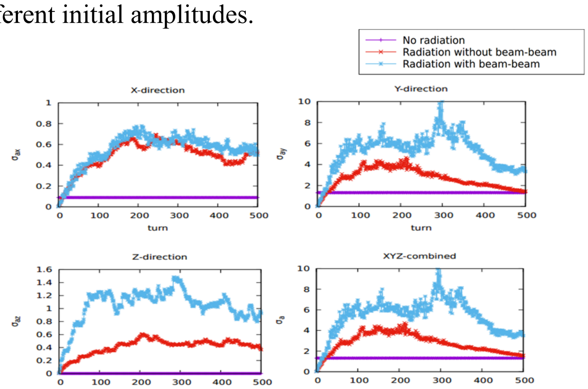

Figure 3: Evolution of variance in different models and different directions.

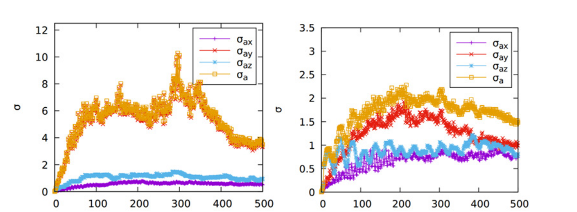

Figure 4: Evolution of variance in two different initial amplitudes. The left one is at (ax,ay)=(3,11) and the right one is at (ax,ay)=(8,1).

Evolution in the different models proves that the radiation fluctuation is the dominating effect of diffusion in amplitude of particles and σ_{ay} takes the largest proportion. It agrees well with the result of dynamic aperture decreasing in vertical direction caused by quantum excitation. In addition, as the two amplitude selected are far from the edge of dynamic aperture, damping effect dominates as the motion continues. Thus, we only care the first few turns when the diffusion increase and define a diffusion map based on this result.

DIFFUSION MAP ANALYSIS

Diffusion map is an image of beam distribution diffusion in transverse amplitude space. In a general case, a particle with large amplitude could have a large diffusion. If we see some singular points in this map, the corresponding amplitudes is not stable enough. We consider the information of less turns (25 turns) to reflect the increasing diffusion and save time. The map is developed as a test that whether particles with small amplitudes have large diffusion rate. The function is defined as

$$f \equiv \log_{10}\left(\sum_{turn} \sigma_a^2\right) = \log_{10}\left(\sum_{turn} \sigma_{ax}^2 + \sigma_{ay}^2 + \sigma_{az}^2\right) \tag{5}$$

Here we show the diffusion maps of four cases with different configuration of sextupoles which have almost the same dynamic aperture. (Fig.5 and Fig.6)

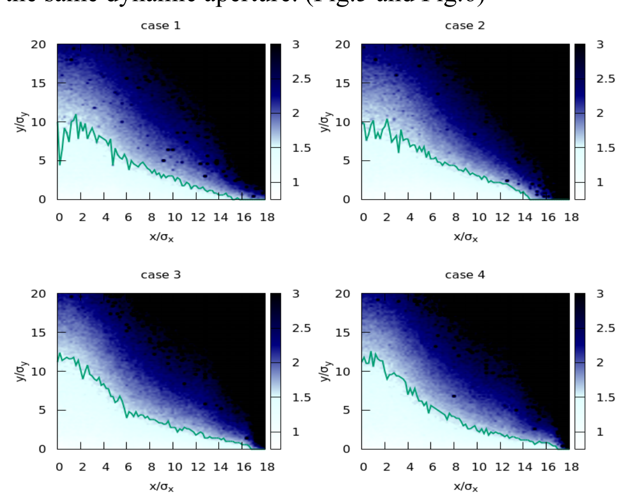

Figure 5: The diffusion maps of four cases. The green lines are corresponding to the value of the function f=1.52.

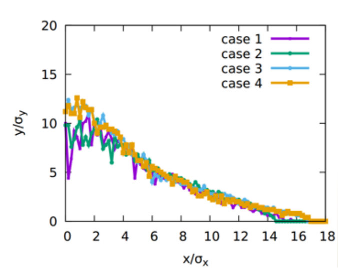

Figure 6: Lines of the four cases in Fig.5 are plotted in different colours.

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There is a clear difference of these lines in some space between first two cases and last two cases. Particles pass through this region have different diffusion in different cases. The lifetime in vertical direction of the four cases by tracking are showed below. (Fig.7)

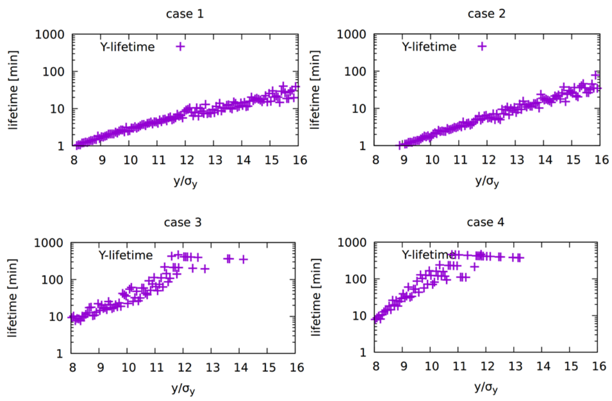

Figure 7: The vertical lifetime of 4 cases.

The diffusion map analysis presents good consistency with beam lifetime that needs much more turns of tracking. According to these results, we establish the relationship between beam lifetime and the function value in specific region. It can help us do the optimization in dynamic aperture.

OPTIMIZATION

A differential evolution algorithm based optimization code has been developed for CEPC, which is a multi-objective code called MODE [5]. Particle tracking is done in the SAD, where the random diffusion due to synchrotron radiation is implemented in each magnet.

Beam lifetime is not feasible to be an additional objective function in dynamic aperture optimization, as it costs so much time for tracking many turns. Based on our analysis above, the information of diffusion map can be a good alternative choice.

In our optimization strategy, minimizing the chromaticity is necessary and the dynamic aperture is our objective. Totally 50 variables are used (32 for sextupoles in arc, 10 for interaction region and 8 for phase advance in straight lines). Here we add one or more constraints of diffusion rate:

$$\begin{cases} con1 = \xi_x, \xi_y \ , \quad con2 = f(ax, ay) \\ \quad obj1 = DA_x \quad , obj2 = DA_y \end{cases} \tag{6}$$

Where ξ_x, ξ_y is the chromaticity, $f(ax, ay)$ is value of eq.(5). For example, $f(1,10)$ could be a constraint according to the difference of lines in Fig.5. More constraints at other amplitude can be added if necessary.

Several tests have been done to optimize the dynamic aperture using this constraint. Without focusing on the slight difference of DA, Figure 8 shows some tracking results of beam lifetime.

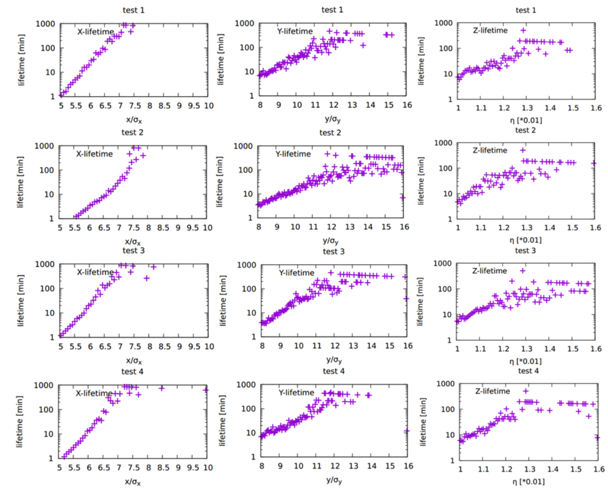

Figure 8: Lifetime in three directions of 4 selected optimized results (test 1 to test 4).

Almost all the results present longer lifetime than case 1 and case 2. Large diffusion rate in amplitude space may lead to halo particles distribution in long-turns tracking. By using these constraints in eq.(6), we can achieve enough long lifetime without tracking many turns.

CONCLUSION

In the collider of very high energy, the effect of radiation fluctuation reduce dynamic aperture and beam lifetime a lot. Tracking results show that a larger DA may not have a longer lifetime. We developed a method called diffusion map analysis to describe the influence of complicated effects in which is mainly radiation fluctuation. It presents good consistency with beam lifetime by tracking. Constraints of diffusion rate in amplitude space are applied in optimization of DA, solutions always present good results of beam lifetime.

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