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CHANGE OF BEAM DISTRIBUTION DUE TO DECOHERENCE IN THE PRESENCE OF TRANSVERSE FEEDBACK

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Abstract

The effect of Landau damping is often calculated based on a Gaussian beam distribution in all degrees of freedom. The stability of the beam is however strongly dependent on the details of the distribution. The present study focuses on the change of bunch distributions caused by the decoherence of the excitation driven by an external source of noise, in the presence of both amplitude detuning and a transverse feedback. Both multiparticle tracking simulations and theoretical models show a similar change of the distribution. The possible loss of Landau damping driven by this change is discussed.

INTRODUCTION

In synchrotrons, the beam is kept stable partially by Landau damping due to the tune spread within each bunch. The stability diagram in plane $j \in \{x, y\}$ is calculated from [1]

$$\frac{1}{\Delta Q_{\mathrm{coh},j}} = -\int_0^\infty \mathrm{d}J_x \int_0^\infty \mathrm{d}J_y \, \frac{J_j \frac{\mathrm{d}\Psi(J_x,J_y)}{\mathrm{d}J_j}}{Q - Q_j(J_x,J_y)} \,, \tag{1}$$

where $\Delta Q_{\mathrm{coh},j}$, J_j and Q_j are the coherent tune shift, action and tune, respectively, in plane j, $Q \in (-\infty, \infty)$, and Ψ is the distribution. The stability can be changed significantly by a small change of the distribution [2–5]. In a recent experiment in the LHC, Landau damping was lost due to a noise driven diffusion [6]. Here we will introduce an analytical theory that explains how the distribution changes after an initial offset, due to the combined effects of a tune spread and a transverse feedback. The goal is to find how the distribution changes, and how the stability diagram evolves as a result.

THEORY

The calculation consists of 4 steps: (i) Derive an expression for the change of the action for each particle after a kick, taking into account the balance between the tune spread and the transverse feedback; (ii) Consider the change of action as a Wiener process with a drift, and derive the Fokker-Planck equation for the particle density distribution of the bunch [7]; (iii) Solve the Fokker-Planck equation to get the time evolution of the distribution; (iv) Calculate numerically the stability diagram with `PySSD` [8], as the distribution evolves. This approach has the advantage that it is modular, each step can be modified if necessary. Furthermore, the 4-step calculation may be applied to various sources of tune spread. Here we shall discuss the case when the tune spread is caused by Landau octupoles.

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Transverse Feedback and Decoherence

We apply normalized, canonical coordinates [9]

$$\begin{aligned} x &= \frac{1}{\sqrt{\beta\varepsilon_0}} X = \sqrt{2J}\cos(\phi) \,, \\ p &= -\frac{1}{\sqrt{\beta\varepsilon_0}} \left(\alpha X + \beta \frac{\mathrm{d}X}{\mathrm{d}s}\right) = -\sqrt{2J}\sin(\phi) \,, \end{aligned} \tag{2}$$

where X is the offset from the design orbit, s is the position in the beamline, α and β are Twiss parameters, ε_0 is the initial beam emittance and ϕ is the canonical conjugate of J.

If a bunch is kicked by $\Delta p = k$, the action changes to

$$J_k = J_0 + k\sqrt{2J_0}\sin(\phi_0) + \tfrac{1}{2}k^2 \,, \tag{3}$$

where J_0 and ϕ_0 are the action and phase of the particle prior to the kick. There exists an expression for the subsequent emittance growth, when there is both a transverse tune spread and a transverse feedback, and we will take a similar approach [10]. We refer to the centroid of the bunch as $z = \langle x \rangle + \mathrm{i}\langle p \rangle$, where the angle brackets signify the average over the distribution. The tune of the centroid is Q_c, and its transverse offset will each turn be reduced by a factor g, called the gain. Assuming a perfect, immediate feedback, the initial centroid offset $z_0 = \mathrm{i}k$ will after n turns be

$$z_n = z_0 \cdot \mathrm{e}^{-\mathrm{i}2\pi Q_c n} \cdot \left(1 - \tfrac{g}{2}\right)^n \xrightarrow{n\to\infty} z_0 \cdot \mathrm{e}^{-\mathrm{i}2\pi Q_c n} \cdot \mathrm{e}^{-\frac{g}{2}n} \,, \tag{4}$$

with a damping time of $\tau = 2/g$ turns. It is assumed that the reduction of the centroid amplitude due to the tune spread is negligible compared to that of the transverse feedback.

The position of an individual particle, with a constant tune of $Q_c + \Delta Q$, is referred to as $y = x + \mathrm{i}p$. After many turns, when the centroid tends to the origin in the limit $ng \gg 1$, the position will become

$$y_n = \mathrm{e}^{-\mathrm{i}2\pi(Q_c+\Delta Q)n} \left(r_0 + z_0 \cdot \frac{\left(1-\frac{g}{2}\right)\left(1 - \mathrm{e}^{\mathrm{i}2\pi\Delta Q}\right)}{1 - \left(1-\frac{g}{2}\right)\mathrm{e}^{\mathrm{i}2\pi\Delta Q}} \right) , \tag{5}$$

where $r_0 = x_0 + \mathrm{i}p_0$ is the position prior to the kick, and $y_0 = r_0 + z_0$ is the position just after the kick z_0. The change of the action in the limit $\Delta Q \ll 1$, $ng \gg 1$ is thus

$$\begin{aligned} \Delta J =& \frac{k^2}{2} \frac{\left(1-\frac{g}{2}\right)^2 4\pi^2\Delta Q^2}{\left(\frac{g}{2}\right)^2 + \left(1-\frac{g}{2}\right)4\pi^2\Delta Q^2} + k\sqrt{2J_0}\left(1-\tfrac{g}{2}\right) \\ &\times \frac{\cos(\phi_0)\left(\frac{g}{2}\right)2\pi\Delta Q + \sin(\phi_0)\left(1-\frac{g}{4}\right)4\pi^2\Delta Q^2}{\left(\frac{g}{2}\right)^2 + \left(1-\frac{g}{2}\right)4\pi^2\Delta Q^2} \\ =& \tfrac{1}{2}k^2L^2 + k\sqrt{2J_0}\left[M\cos(\phi_0) + N\sin(\phi_0)\right] \\ =& \tfrac{1}{2}k^2L^2 + k\sqrt{2J_0}\sqrt{M^2+N^2}\cos\left(\phi_0 - \mathrm{atan}\left(\tfrac{M}{N}\right)\right) , \end{aligned} \tag{6}$$

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where L, M and N are factors that depend on ΔQ and g. The first term of Eq. (6) is an average growth equal to the result in [10], while the second term is a spread based on the phase of the particle. Equation (6) simplifies to Eq. (3) minus J_0 in the limit $g \ll \Delta Q$, and to 0 in the limit $g \gg \Delta Q$.

Fokker-Planck Equation in Action

When one kick k becomes a coherent white noise source, the change of action in Eq. (6) can be considered a stochastic process, described by the Fokker-Planck equation [7]

$$\partial_t \Psi = -\partial_J (U\Psi) + \partial_J^2 (JD\Psi) \ , \tag{7}$$

with drift and diffusion coefficients

$$U(J,\Psi) = \int_{-\infty}^{\infty} \frac{\Delta}{\tau} \varphi(\Delta; J, \Psi) \mathrm{d}\Delta \ , \tag{8a}$$

$$D(J,\Psi) = \frac{1}{J} \int_{-\infty}^{\infty} \frac{\Delta^2}{2\tau} \varphi(\Delta; J, \Psi) \mathrm{d}\Delta \ , \tag{8b}$$

where τ is the time interval between each kick, and Δ is the change of action. The normalization of D by J is convenient in the following. This has been derived by Taylor expanding the Master equation [11, 12]. The details could not fit here.

The probability distribution for the change of action after a kick, derived from Eq. (6), can be written as

$$\varphi(\Delta; J, \Psi) = \frac{F(k)\mathrm{d}k}{\pi\sqrt{2Jk^2(M^2+N^2) - \left(\Delta - \frac{1}{2}k^2L^2\right)^2}} \ , \tag{9}$$

where $F(k)$ is the probability distribution of the kicks, with a standard deviation, σ_k, and an assumed mean of zero. The coefficients U and D are thus

$$U(J,\Psi) = U_0 = \frac{\sigma_k^2}{2\tau} \cdot \left\langle L^2 \right\rangle \ , \tag{10a}$$

$$D(J,\Psi) = \frac{\sigma_k^2}{2\tau} \cdot \left(M^2 + N^2\right) \ . \tag{10b}$$

In the limit $\Delta Q \ll 1$ one finds that $M^2 + N^2 = L^2$. The Fokker-Planck Equation takes the form

$$\partial_t \Psi = \partial_J \left[J \partial_J (D\Psi) \right] \ . \tag{11}$$

In the derivation of Eq. (11), the tune offset ΔQ was assumed constant for each particle individually. In general ΔQ depends on J, which is not constant. From considering the actual process, we postulate a time reversal symmetry at the microscopic level, that the probability of going from J_a to J_b is equal to the probability of going back, or $\varphi(J_b - J_a; J_a) = \varphi(J_a - J_b; J_b)$. By doing a Taylor expansion of φ as in [12], assuming small kicks k, the drift coefficient changes to

$$U(J,\Psi) = D + J\partial_J(D) \ . \tag{12}$$

The second term cancels a term in Eq. (11), which becomes the standard diffusion equation

$$\partial_t \Psi = \partial_J \left[JD\partial_J (\Psi) \right] \ . \tag{13}$$

For later reference and discussion, we combine Eq. (11) and Eq. (13), by use of a parameter $\alpha \in \{0, 1\}$, as

$$\partial_t \Psi = \partial_J \left[JD\partial_J (\Psi) \right] + (1-\alpha) \cdot \partial_J \left[J\partial_J(D)\Psi \right] \ . \tag{14}$$

Solving the Fokker-Planck Equation

The next step is to solve Eq. (14). If $g = 0$, this is the diffusion equation with a constant diffusivity $D_0 = \sigma_k^2/2\tau$. In another extreme limit, $g \gg \Delta Q$ and $\Delta Q \to 0$, $\partial_t \Psi = 0$, and the distribution will not change.

In the interesting regime, when there is a balance between the feedback and the detuning, we require a numerical solver. An original code has been written, implemented with the finite-volume-method to ensure mass conservation, and using `scipy.integrate.solve_ivp` to achieve the time integration [13]. The boundary at $J = 0$ has to be reflective. In the results that will be presented, the boundary at $J_{\mathrm{Max}} = 24.5$ is absorbing, representing an aperture. The centroid tune, Q_c, is kept constant.

Decoherence from Landau Octupoles

The Landau octupoles in the LHC cause a tune spread in both transverse planes, relative to the average, given by

$$\begin{aligned} \Delta Q_{x,y} &= a_{x,y} \cdot (J_{x,y} - \langle J_{x,y} \rangle) + b_{y,x} \cdot (J_{y,x} - \langle J_{y,x} \rangle), \\ a_{x,y} &= 520 \cdot I_{\mathrm{oct}} \cdot \varepsilon_{x,y,0} \ , \\ b_{x,y} &= -380 \cdot I_{\mathrm{oct}} \cdot \varepsilon_{y,x,0} \ , \end{aligned} \tag{15}$$

where a_j and b_j are detuning coefficients dependent on the octupole current, I_{oct}, and geometrical emittance [14]. In a simplified model when $b = 0$, L^2 takes the shape in Fig. 1.

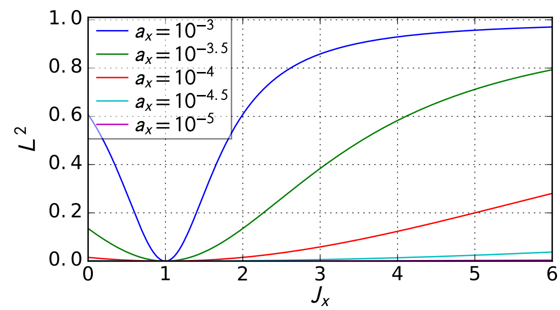

Figure 1: Action dependence of L^2 for a damper gain $g = 0.01$ and different values for the octupole detuning coefficient a_x, in the simplified case where $b_x = 0$.

RESULTS

We will repeatedly study a toy configuration with $a = 5 \times 10^{-3}$, $b = 0$ and $g = 0.2$. All values are in the horizontal plane. The subscript x has been omitted, since there is no dependence on vertical phase space. A macroparticle simulation has been run. Simulations of this process require $> 10^6$ macroparticles to reduce the numerical stochastic cooling, a small ratio σ_k^2/g to keep the centroid amplitude low, and $> 10^6$ turns for the distribution to change. The distribution is plotted as a function of $r = \sqrt{2J}$ in Fig. 2a after T turns such that $\sigma_k^2 \cdot T = [0, 1, ..., 8] \cdot 25/6$ turns. The time is scaled to hours of operation of the LHC, with a noise of $\sigma_k = 5.77 \times 10^{-4}$, comparable to the noise in a recent experiment in the LHC [6].

A stochastic process with kick strength solely dependent on the parameters before the kick, is modelled by Eq. (14)

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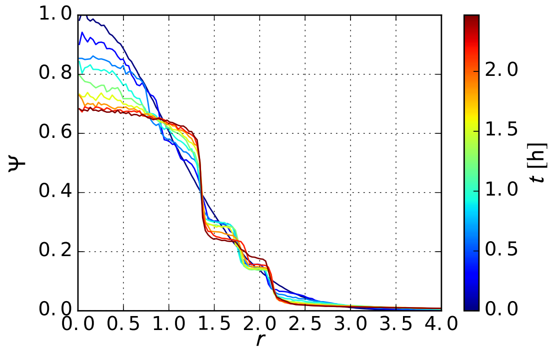
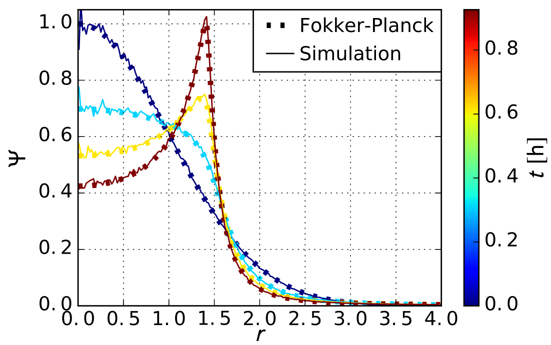
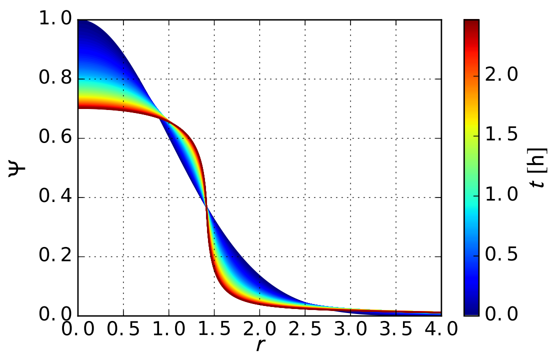


(a) Realistic macroparticle simulation.



(b) Solution of Eq. (14) with $\alpha = 0$, and a macroparticle simulation with incoherent noise of variance $\sigma_k^2 \cdot L^2$.



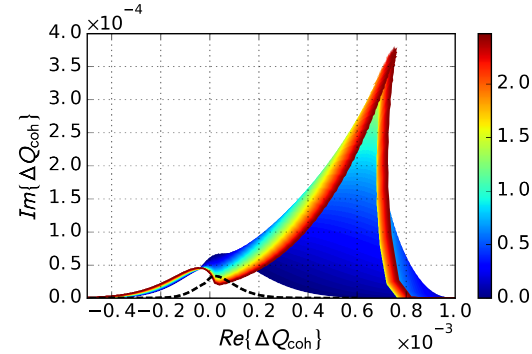
(c) Solution of Eq. (14) with $\alpha = 1$.

Figure 2: Distribution evolution with $a = 5 \times 10^{-3}$, $b = 0$, $g = 0.2$ and equivalent noise $\sigma_k = 5.77 \times 10^{-4}$.

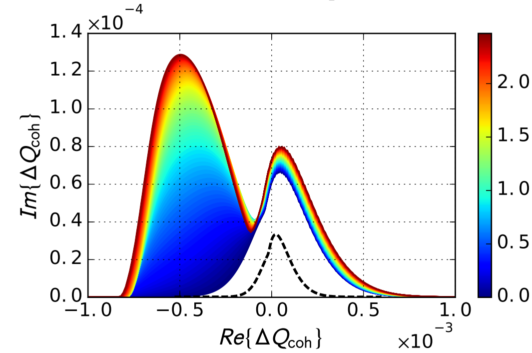
with $\alpha = 0$. A simulation was run with a centered incoherent noise of variance $\sigma_k^2 \cdot L^2$ over T turns. The distribution, predicted by the Fokker-Planck-solver and the simulation after $\sigma_k^2 \cdot T = [0, 1, 2, 3] \cdot 25/6$ turns, are shown to have a perfect agreement in Fig. 2b. These curves are significantly different from the first 4 curves in Fig. 2a, showing clearly that $\alpha = 0$ in Eq. (14) is not representing the beam dynamics well.

The distribution evolution calculated with $\alpha = 1$ is presented in Fig. 2c. This evolution is in comparison quite close to the macroparticle simulation. An edge develops at $r = \sqrt{2} \approx 1.4$, where $\Delta Q = 0$. The evolution of multiple edges in the simulation can be a numerical artefact. There is also a small nonzero diffusion across the edge due to the centroid oscillation, which is not included in the new theory.

The evolution of the stability diagram, corresponding to the distribution evolution in Fig. 2c, has been calculated



(a) Horizontal plane.



(b) Vertical plane.

Figure 3: Evolution of stability diagrams for $I_{oct} = 400$ A, corresponding to the evolution in Fig. 2c. The dashed black lines are the stability diagrams for $\Psi(t = 0)$ at $I_{oct} = 200$ A.

with PySSD, and is presented in Fig. 3. The tune spread is calculated with Eq. (15), using $I_{oct} = 400$ A and a normalized emittance of 2 µm at 6.5 TeV. Both diagrams show an increased stability at large real coherent tune shifts, due to the population of the tails at $J_x > 5$. The horizontal stability diagram eventually cuts into the stability diagram calculated with half the octupole current for the initial distribution, due to the increased gradient and curvature of Ψ at $J_x \sim 1$.

CONCLUSION

We have shown that a coherent white noise, combined with a transverse feedback and an amplitude dependent detuning, causes an amplitude dependent diffusion, which changes the distribution. The Fokker-Planck equation has here been used to model this process on long time scales. Macroparticle simulations can also be used, and have been run to compare to the new theory, but require high numbers of macroparticles and turns to study the relevant cases. With detuning due to Landau octupoles, a Gaussian distribution evolves towards a rectangular distribution. Simultaneously, the stability diagram changes. The example configuration studied in this paper show that this can allow instabilities to evolve at more than twice the predicted required octupole current, depending on the mode, over time scales of hours. The stability also increased for large real coherent tune shifts. To study the impact in the LHC, the model will be extended to include wakefields and tune dependence on the vertical action.

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