A TWO-MODE MODEL TO STUDY THE EFFECT OF SPACE CHARGE ON TMCI IN THE "LONG-BUNCH" REGIME

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Abstract

Using a two-mode approach for the Transverse Mode-Coupling Instability (TMCI) in the "short-bunch" regime (where the mode-coupling takes place between the modes 0 and – 1, such as in the CERN LHC), both a reactive damper (ReaD) and Space Charge (SC) are expected to be beneficial: the ReaD would shift the mode 0 up while SC would shift the mode – 1 down, but in both cases the coupling (and related instability) would occur at higher intensities. However, the situation is more involved in the "long-bunch" regime (where the mode-coupling takes place between higher-order modes, such as in the CERN SPS). As the ReaD modifies only the (main) mode 0 and not the others, it is expected to have no effect for the main mode-coupling. As concerns SC, it modifies all the modes except the mode 0, and the result has been a subject of discussion for two decades. A two-mode approach is discussed in detail in this contribution for the case of a single bunch interacting with a broad-band resonator impedance in the "long-bunch" regime. This model reveals in particular that in the presence of SC, the intensity threshold can only be similar to or lower than the no-SC case.

INTRODUCTION

A fast vertical single-bunch instability has been observed for many years in the SPS above a certain intensity threshold when the chromaticity is corrected [1]. It has been studied in detail and many aspects of this instability (which looked like a TMCI) could be reproduced by simulations, first without taking into account SC and then taking also into account SC with different models. A threshold close to the no-SC case was found and therefore no (significant) stabilising effect from SC seemed to be observed, as opposed to several theoretical predictions [2-6]. Using a simple model, which is the subject of this paper, no beneficial effect was expected, as mentioned in Ref. [7]. The fact that something seemed to be missing in the currently most advanced theories was stressed again at the recent workshop at FNAL in Spring 2018 [8] and since then a new destabilising effect of SC was revealed [9]. New simulations were then performed and analysed in detail, as well as new measurements in the SPS, which confirmed the destabilising effect of SC [1]. With Ref. [9], a major step has been certainly achieved in the understanding of the intricate effect of SC on transverse instabilities. The purpose of this paper is to present the simple two-mode model, which was used in the past to predict no beneficial effect of SC in the (very) long-bunch regime, and to discuss it for the case of any SC parameter. This paper is structured as follows: in the first

section, the case of SC only is reviewed. Then, the "short-bunch" regime is discussed to show that it is completely different from the one of the "long-bunch" regime, which is the main subject of this contribution. The final result of this study is revealed in Fig. 4.

SC ONLY

SC only cannot lead to an instability but it shifts all the head-tail modes downward, except the mode 0 (as the SC force is expressed with respect to the centre of mass of the bunch). In 1998, it was shown in Ref. [2], using an Air-Bag Square well (ABS) model that the shift of the head-tail modes due to SC is given by

$$\frac{\Delta Q}{Q_s} = -q_{sc} \pm \sqrt{q_{sc}^2 + m^2} \ , \tag{1}$$

where $q_{sc} = \Delta Q_{sc} / (2\,Q_s)$ is the SC parameter (with ΔQ_{sc} the absolute value of the SC tune shift and Q_s the synchrotron tune) and where the + sign is used for $m \geq 0$ and the – sign for $m < 0$ (m being the azimuthal head-tail mode). Figure 1 shows the first head-tail modes as a function of the SC parameter, where it can be clearly seen that a huge (beneficial) effect can be expected in the case of an instability involving a coupling between both modes 0 and – 1, as SC pushes apart these two modes (see also next section). When higher-order modes are involved, the situation is more complex, and SC in this case is bringing the modes closer to each other: a destabilising effect could then already be anticipated.
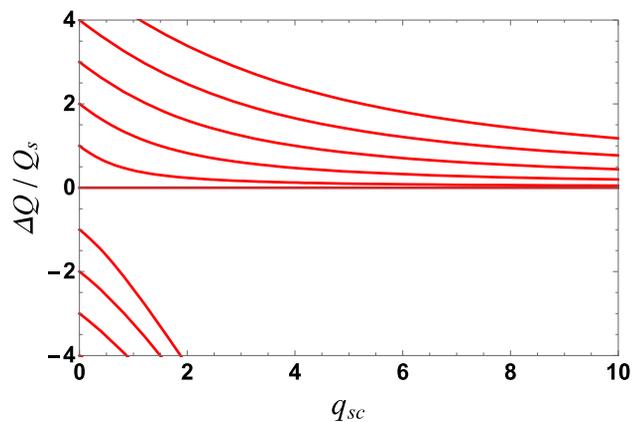

Figure 1: Normalised mode-frequency shifts vs. the SC parameter q_{sc} for the case of the ABS model [2].

"SHORT-BUNCH" REGIME WITHOUT AND WITH READ OR SC

As discussed in Ref. [10], in the "short-bunch" regime where the mode-coupling takes place between modes 0

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and -1, a ReaD can increase the intensity threshold. In the case of SC, instead of a ReaD, the simple model/example used in Ref. [10] for the mode-coupling between modes 0 and -1 can be extended and the 2×2 matrix to be diagonalised can be written as, in the general case of the presence of SC and/or ReaD,

$$\begin{pmatrix} F_{sc}\,x - \sqrt{1 + (F_{sc}\,x)^2} & -0.23\,j\,x \\ -0.55\,j\,x & -0.92\,x + F_D \end{pmatrix}, \quad (2)$$

where x is a normalised parameter which is proportional to the bunch intensity [10], F_D is the damper term (equal to 0.48 in Fig. 2, to use the same value as in Ref. [10] but with a reactive damper instead of a resistive one) and F_{sc} is the SC term, using Eq. (1) (equal to -0.48 to use a term of similar amplitude as for the ReaD case). The results are shown in Fig. 2, where the beneficial effect of both ReaD and SC is clearly revealed, which is due to the fact that both ReaD and SC are pushing apart the modes 0 and -1. It is worth mentioning that a potential destabilising effect in this case comes from Landau damping [11,12].
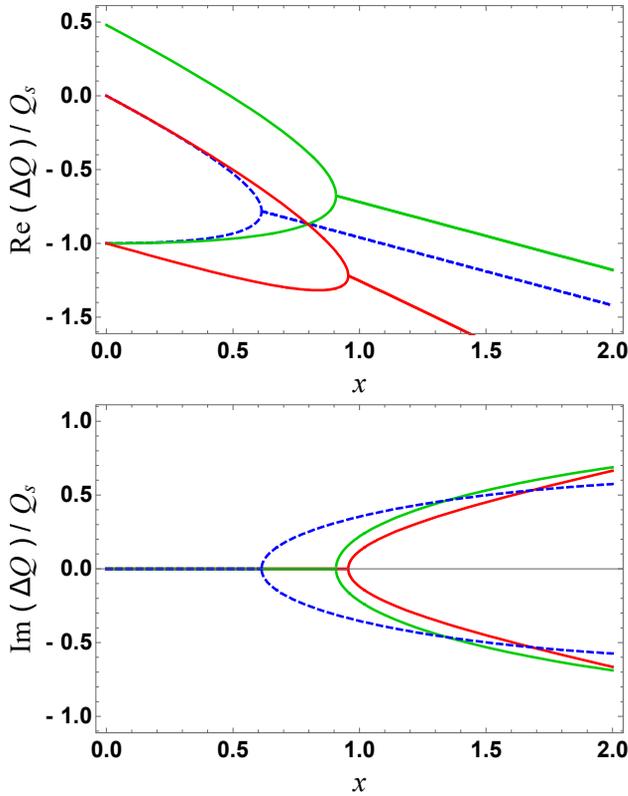

Figure 2: Simplified model/example of Ref. [10], describing the mode-coupling in the "short-bunch" regime, i.e. the mode-coupling between modes 0 and -1, extended here to take into account also SC, using the parameters mentioned above: (dashed blue) with impedance only, (green) with impedance and ReaD, (red) with impedance and SC.

"LONG-BUNCH" REGIME WITH NEITHER READ NOR SC

In the case of the "long-bunch" regime, where higher-order modes couple, the situation is more involved already with neither ReaD nor SC and the solution can be obtained from a Vlasov solver like GALACTIC [10] (others are discussed in Ref. [10]). However, it was shown in the past that a simple formula could be obtained in the case of a broad-band resonator [13,14], as it was confirmed by HEADTAIL macroparticle tracking simulations [15] and as it was recently checked with the NHT Vlasov Solver [16]. A 2×2 matrix as in Eq. (2) is obtained, this time not involving the modes 0 and -1, but the modes m and $m+1$ overlapping the peak of the real part of the impedance, as shown in Fig. 3.
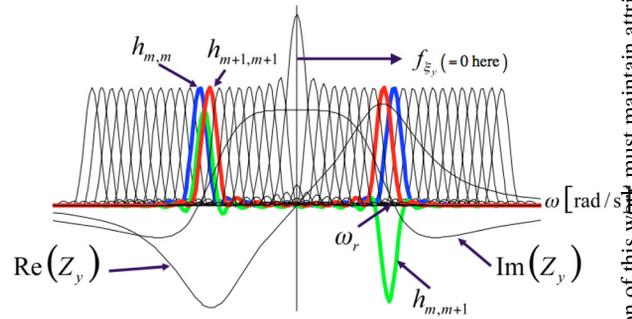

Figure 3: Real and imaginary parts of the broad-band impedance together with the bunch spectrum (assuming here sinusoidal modes [14]) with the 2 modes (m and $m+1$) overlapping the peak of the real part of the impedance.

The corresponding 2×2 matrix to be diagonalised, e.g. in the vertical plane, is given by [14]

$$\begin{pmatrix} Q - Q_y - m\,Q_s - \Delta Q_m & -\Delta Q_{m,m+1} \\ -\Delta Q_{m+1,m} & Q - Q_y - (m+1)\,Q_s - \Delta Q_{m+1} \end{pmatrix}, \quad (3)$$

where the coherent tune Q needs to be found, with Q_y the vertical unperturbed (low-intensity) tune, ΔQ_m the tune shift (from impedance) of mode m, ΔQ_{m+1} the tune shift of mode $m+1$, $\Delta Q_{m,m+1}$ the tune shift from mode $m+1$ on mode m, and $\Delta Q_{m+1,m}$ the tune shift from mode m on mode $m+1$. The solution of Eq. (3) is discussed in detail in Ref. [14] and the TMCI intensity threshold is reached when

$$|Q_s + \Delta Q_{m+1} - \Delta Q_m| = 2\,|\Delta Q_{m,m+1}|. \quad (4)$$

For our particular case of interest here, i.e. in the "long-bunch" regime depicted in Fig. 3, $\Delta Q_m \approx 0 \approx \Delta Q_{m+1}$ and therefore the intensity threshold is reached when

$$Q_s \approx 2\,|\Delta Q_{m,m+1}|, \quad (5)$$

which leads to the threshold number of charges [14]

$$N_b^{th} \propto |\eta| \, \varepsilon_l \, Q_y \, , \qquad (6)$$

with η the slip factor and ε_l the longitudinal emittance [13,14]. It can be seen in particular from Eq. (6) that, as discussed in Refs. [13,14], the TMCI intensity threshold can be increased by increasing the slip factor, i.e. going further away from transition. This method was implemented successfully in the SPS (by moving from the Q26 to the Q20 optics, where the number refers to the integer part of the transverse tune) and this instability is not a performance limitation anymore [15, 1].

"LONG-BUNCH" REGIME WITH ReaD

As a ReaD modifies only the (main) mode 0 and not the others, it is expected to have no effect for the main mode-coupling: the same Eq. (3) is obtained as it does not involve the mode 0. This was confirmed using the Vlasov solver GALACTIC [10], as can be seen in Fig. 1 of Ref. [10].

"LONG-BUNCH" REGIME WITH SC

As SC does not modify the mode 0 but it modifies all the others (i.e. it modifies ΔQ_m and ΔQ_{m+1}), SC could potentially play a role if/when $\Delta Q_{m+1} - \Delta Q_m$ starts to be non-negligible compared to Q_s, as can be seen from Eq. (4). In the presence of SC, Eq. (4) becomes

$$Q_s \left[\sqrt{q_{sc}^2 + (m+1)^2} - \sqrt{q_{SC}^2 + m^2} \right] = 2 \left| \Delta Q_{m,m+1} \right| . \quad (7)$$

This means that the same intensity threshold as the no-SC case is obtained, i.e. it is the same as Eq. (5) or (6), except that Q_s is now multiplied by the term $\sqrt{q_{sc}^2 + (m+1)^2} - \sqrt{q_{SC}^2 + m^2}$, which is equal to 1 when $q_{sc} \ll |m|$ (which corresponds to the "small SC" case, or very long bunch as considered in the past [7]) and which is equal to 0 when $q_{sc} \gg |m|$ (which corresponds to the "strong SC" case, with the absence of TMCI threshold and therefore to the "Beam Break-Up" type of instabilities). It should be noted that in the analysis performed above, what is important is the radial mode number $q = |m| + 2\,k$ (with $0 \le k \le +\infty$) as this is the one defining the bunch spectrum (overlapping the maximum of the real part of the impedance) and this is why m was replaced by q in Ref. [1]. The evolution of the reduction factor from SC, $\left[\sqrt{q_{sc}^2 + (q+1)^2} - \sqrt{q_{SC}^2 + q^2} \right]$, is depicted in Fig. 4. As can be seen from Fig. 4, this model reveals that in the presence of SC, the intensity threshold can only be similar to or lower than the no-SC case. In particular, applying it to the case of the CERN SPS TMCI (where a mode-coupling between modes -3 and -2 is revealed without SC), the intensity threshold is predicted to be reduced with SC by a factor ~ 10 for the Q26 optics (for which $q_{sc} / q = 27/2 = 13.5$) and by a factor ~ 2.2 for the Q20 optics (for which $q_{sc} / q = 5/2 = 2.5$).
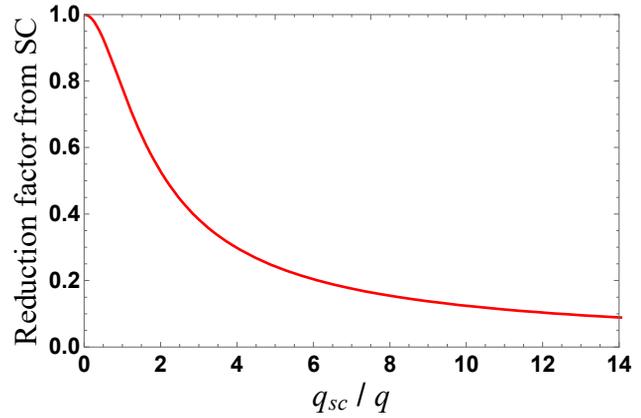

Figure 4: Reduction factor, from SC, of the intensity threshold, as a function of the ratio between the SC parameter q_{sc} and the radial mode number q.

CONCLUSION

The (simple) two-mode approach (with a mode-coupling between two consecutive modes m and $m+1$ overlapping the peak of the real part of the impedance), which was used in the past in the case of the (very) "long-bunch" bunch regime to reveal almost no effect of SC on TMCI [7,17,18], has been presented here in the general case of any SC parameter. It leads to the same intensity threshold as the no-SC case, except that the synchrotron tune Q_s is now multiplied by a reduction factor from SC, which is depicted in Fig. 4. Applied to the case of the CERN SPS (when the impedance is modelled as a broad-band resonator), it predicts a reduction of the intensity threshold by a factor ~ 10 for the Q26 optics and ~ 2.2 for the Q20 optics. These results should be carefully compared to simulation results [1,19], which also reveal an intensity threshold much lower than the no-SC case for the "strong SC" regime of the Q26 optics. However, more involved analyses are required due to the fact that the results depend also on the initial conditions for instabilities of the "Beam Break-Up" type (in the "strong SC" regime). Therefore, the instability with SC needs to be fully characterized first and then we will still need to understand why an intensity threshold similar to the no-SC case seems to be observed in the SPS, by looking in particular carefully at the effect of the machine nonlinearities.

ACKNOWLEDGEMENTS

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