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ITERATIVE TRAJECTORY-CORRECTION SCHEME FOR THE EARLY COMMISSIONING OF DIFFRACTION-LIMITED LIGHT SOURCES

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Abstract

The commissioning of diffraction-limited light sources will be significantly affected by the fact that typical lattice designs rely on very strong focussing elements in order to achieve the small emittance goals. Especially in the early-commissioning phase this can render procedures successfully used in the commissioning of existing third-generation light sources ill-suited for the application to these new machines. In this contribution we discuss an iterative approach to the early trajectory correction, based on the well-known pseudo-inversion of a trajectory-response matrix. Measuring this matrix during early commissioning can be cumbersome, so that an algorithm working with the model response matrix of the lattice is desirable. We discuss the stability of the iteration in the presence of lattice errors, resulting in differences between the actual and the model response matrix. Further, Tikhonov regularization is investigated as a means to trade off the RMS trajectory variation against the strength of the required corrector kicks.

INTRODUCTION

Trajectory control based on the inversion of the so-called *response matrix* (RM) is a standard tool, widely used in the operation of particle accelerators. In this paper we will investigate a problem which arises when the same approach is to be applied to a machine that is not yet in nominal operation but is just in the early stages of its commissioning. In this uncorrected state the machine can not be expected to provide full transmission, let alone a beam life time long enough to allow for the time efficient measurement of the orbit response matrix. Any initial trajectory correction scheme therefore has to employ an idealized response matrix, calculated from the lattice model. Even if the lattice does not contain non-linear magnetic elements, the beam trajectory will still exhibit a non-linear behavior with respect to the corrector kicks, as they are limited in the maximum kick they can provide. As a result, the solution obtained by a naive inversion of the response matrix might not be physically realizable as it might involve corrector settings that exceed this limit. Therefore, regularization of the inverse response matrix is in order. Another type of non-linearity to consider is driven by the finite physical aperture of the beam pipe. If the beam gets lost due to insufficient trajectory correction it will not reach BPMs further downstream, which then can not produce a meaningful reading. These non-linearities together with noise and the inevitable differences between this model response matrix and the actual behavior of the machine drive the need for an iterative correction scheme. In the past, such schemes typically employed an empirically chosen subset of corrector magnets and matrix regularization, both varying between correction steps [1,2]. In contrast, we present a less intricate correction scheme, based on the Tikhonov regularized pseudo-inverse of the complete response matrix.

Iterative adjustment of corrector kicks based on a flawed response matrix can, however, generally not be expected to be stable. The stability of such an iteration is analysed in the linear case, taking into account a generalized regularization of the inverse response matrix.

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FORMALISM

Let $\vec{R}: \mathbb{R}^C \to \mathbb{R}^B$ be the *response function* relating the settings $\vec{\phi}$ of the $C \in \mathbb{N}_{>0}$ corrector magnets (CMs) to the readings $\vec{r}$ of the $B \in \mathbb{N}_{>0}$ beam position monitors (BPMs): $\vec{r} = \vec{R}(\vec{\phi})$, ordered with respect to their longitudinal position in the lattice. Time-dependent effects (such as noise or the variation of experimental parameters other than the settings of the corrector magnets) are not treated at this point. The task is to find a *feedback function* $\vec{\Phi}: \mathbb{R}^B \to \mathbb{R}^C$ so that the iteration

$$\vec{\phi}_{n+1} = \vec{\phi}_n - \vec{\Phi}(\vec{r}_n) \tag{1}$$

$$\vec{r}_n = \vec{R}(\vec{\phi}_n), \tag{2}$$

or, equivalently, with $\vec{T} := \mathrm{Id} - \vec{\Phi} \circ \vec{R}$

$$\vec{\phi}_{n+1} = \vec{T}(\vec{\phi}_n), \tag{3}$$

converges to a corrector setting $\vec{\phi}_*$ which yields full transmission through the machine, small variations in $\vec{r}_*$, while preferably using small corrector kicks.

Non-linearities from limitations on the maximum corrector kicks $\vec{\phi}_{\max}$ and the possibility of beam loss, can be included by defining componentwise

$$\vec{T}(\vec{\phi}_n)|_{\vec{\phi}_{\max}} := \begin{cases} \vec{T}(\vec{\phi}_n)_i & \text{if } |\vec{T}(\vec{\phi}_n)_i| \leq \vec{\phi}_{\max,i} \\ \vec{\phi}_{n,i} & \text{else} \end{cases} \tag{4}$$

and replacing $\vec{R}$ in $\vec{T}$ by

$$\vec{R}|_{\vec{r}_{\max}}(\vec{\phi}_n) := \begin{cases} \vec{R}(\vec{\phi}_n)_i & \text{if } |\vec{R}(\vec{\phi}_n)_j| \leq \vec{r}_{\max,j} \quad \forall j \leq i \\ 0 & \text{else} \end{cases}. \tag{5}$$

Equations (4) and (5) formalize the problem of threading a beam through a beam pipe with limited physical aperture using only a limited amount of corrector kicks, where beam loss is modelled to occur if the trajectory exceeds a threshold at one of the BPMs.

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Our approach is to choose a linear feedback function $\vec{\Phi}(\vec{r}) = \underline{M}^+\vec{r}$, where $\underline{M}^+$ is a regularized pseudo-inverse of the response matrix $\underline{M} = \partial\vec{R}/\partial\vec{\phi}|_{\vec{\phi}=0}$. This matrix is determined once and is then used throughout the whole correction process. In numerical commissioning studies of the ALS-U storage- and accumulator ring [3] we observed that this global correction scheme produces solutions requiring weaker corrector kicks and converges significantly faster than the previously implemented method, which basically constituted a trial-and-error approach trying to find a suitable regularization and subset of corrector magnets that would improve the RMS BPM reading [2].

A feedback matrix can be found based on the singular-value decomposition (SVD) of the response matrix

$$\underline{M} = \underline{U}\,\underline{\Sigma}\,\underline{V}^T = \underline{U}\ \mathrm{diag}_{B,C}(\sigma_i)\,\underline{V}^T \qquad (6)$$

where $\sigma_{i=1,\dots,\min(B,C)} \in \mathbb{R}^+$ are the singular values of $\underline{M}$ and $\underline{U} \in \mathrm{O}(B)$, $\underline{V} \in \mathrm{O}(C)$. From this a regularized pseudo-inverse can be calculated via

$$\underline{M}^+ = \underline{V}\,\underline{\Sigma}^+\underline{U}^T = \underline{V}\ \mathrm{diag}_{C,B}(\sigma_i^+)\underline{U}^T \qquad (7)$$

with $\sigma^+_{i=1,\dots,\min(B,C)} \in \mathbb{R}^+$. For the purposes of this paper we will use the term *regularization* to refer to the process of constructing suitable σ_i^+ from the singular values σ_i.

A well-known regularization approach is the *truncated singular value decomposition* (TSVD) method. Here, the regularized singular values are chosen to be $\sigma_i^+ = 1/\sigma_i$ if σ_i is above an arbitrary threshold value and $\sigma_i^+ = 0$ otherwise. Conventionally, singular values are ordered by magnitude $\sigma_i \geq \sigma_{i+1}$, so that this scheme can be written as

$$\sigma_i^+(\nu) = \begin{cases} 1/\sigma_i & i \leq \nu \\ 0 & i > \nu \end{cases} \quad \nu \in \mathbb{N}_{>0}. \qquad (8)$$

In our numerical studies, however, a different method called *Tikhonov regularization* [4] has shown itself to result in more a desirable behavior of the iteration, as described in the last section. Here, the σ_i^+ are constructed via

$$\sigma_i^+(\alpha) = \sigma_i/\left[\sigma_i^2 + \alpha^2\right], \qquad (9)$$

where $\alpha \in \mathbb{R}$ is a free regularization parameter. The resulting regularized pseudo-inverse matrix $\underline{M}^+_\alpha := \underline{V}\,\mathrm{diag}_{C,B}\left(\sigma_i^+(\alpha)\right)\underline{U}^T$ can be shown to minimize the expression

$$||\underline{M}\,\underline{M}^+_\alpha\vec{r} - \vec{r}||_2^2 + ||\alpha\underline{M}^+_\alpha\vec{r}||_2^2, \qquad (10)$$

for a given $\underline{M}$, α, and $\vec{r}$, so that the regularization parameter α effectively provides a means to trade off the accuracy of a correction step against the required change in the RMS strength of the corrector magnets, $\Delta\vec{\phi}_n = \underline{M}^+_\alpha\vec{r}_n$.

In the following the effect of a generalized regularization on the fixed point of (3) is investigated in the linear case, as well as the stability of the iteration in presence of lattice errors.

FIXED POINT ANALYSIS

In the linear case described above, Equation (3) takes the form

$$\vec{\phi}_{n+1} = \underline{T}\vec{\phi}_n - \vec{\kappa}, \qquad (11)$$

with $\underline{T} = \underline{1} - \underline{M}^+\underline{M}$ and $\vec{\kappa} = \underline{M}^+\vec{R}_0$. By induction it can be seen that

$$\vec{\phi}_{n>0} = -\left[\sum_{j=0}^{n-1}\underline{T}^j\right]\vec{\kappa}, \qquad (12)$$

where we have w.l.o.g. set $\vec{\phi}_0 = 0$. Plugging in Equations (6) and (7), $\underline{T}$ can be written in diagonalized form $\underline{T} = \underline{V}\ \mathrm{diag}_{C,C}(1 - \sigma_i^+\sigma_i)\underline{V}^T$, where we have exploited the orthogonality of $\underline{V}$ to see $\underline{1} = \underline{V}\,\underline{1}\,\underline{V}^T = \underline{V}\,\mathrm{diag}_{C,C}(1)\underline{V}^T$. This diagonal form allows to write the sum in Equation (12) in terms of geometric sequences in the diagonal elements

$$\vec{\phi}_{n>0} = -\underline{V}\,\mathrm{diag}_{C,C}\underbrace{\left(\sum_{j=0}^{n-1}\left[1 - \sigma_i^+\sigma_i\right]^j\sigma_i^+\right)}_{\begin{cases}\frac{1-\left[1-\sigma_i^+\sigma_i\right]^n}{\sigma_i}, & \sigma_i \neq 0\\ n\,\sigma_i^+, & \text{else}\end{cases}}\underline{U}^T\vec{R}_0 \qquad (13)$$

where additionally $\vec{\kappa}$ has been restored. By taking the limit $n \to \infty$ the fixed point $\vec{\phi}_*$ of this iteration can, if it exists, be found to be $\vec{\phi}_* = \lim_{n\to\infty}\vec{\phi}_{n>0} = -\underline{M}_*\vec{R}_0$, with

$$\underline{M}_* = \underline{V}\,\mathrm{diag}_{C,C}\left(\begin{cases}1/\sigma_i, & \sigma_i^+\sigma_i \in (0,2)\\ 0, & \sigma_i^+ = 0\\ \infty, & \text{else}\end{cases}\right)\underline{U}^T. \qquad (14)$$

Some interesting conclusions can be drawn from this result: If the response matrix has vanishing singular values the iteration will diverge linearly, unless the corresponding σ_i^+ are also chosen to be 0. If, however, any of the σ_i is non-zero and the corresponding regularized singular value is ill-chosen so that $\sigma_i^+\sigma_i \notin (0,2)$ the system will exhibit exponential divergence instead. Most importantly, we see that if the iteration converges, its fixed point does not depend on the value of the non-zero σ_i^+ and $\underline{M}_*$ can be seen to be the well-known Moore-Penrose pseudo-inverse [5] of $\underline{M}$; completely independent on the regularization scheme. In case $\sigma_i^+\sigma_i \in (0,2)$ holds for all singular values the iteration converges to the actual inverse of the response matrix $\underline{M}_* = \underline{M}^{-1}$. This last result can also be obtained by assuming the boundedness $\underline{T}$ and recognizing Equation (12) as a Neumann series.

LATTICE ERRORS

Due to time constraints in the early phases of commissioning a precise measurement of the response matrix is not feasible so that the initial trajectory correction will have to work with an approximation $\underline{M}$ (calculated numerically from the ideal lattice model) of the physically realized response matrix $\underline{\hat{M}} = \underline{M} + \underline{\epsilon}$, meaning that the feedback matrix is

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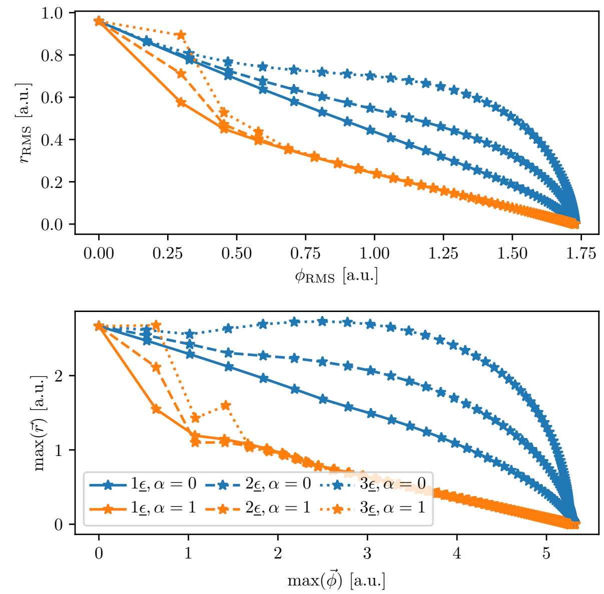

Figure 1: Typical evolution of the root mean square (RMS) (left) and maximum values (right) of the BPM readings $\vec{r}$ and corrector strengths $\vec{\phi}$ for two different Tikhonov regularization parameters α and different response matrix error magnitudes $\underline{\epsilon} = \hat{\underline{M}} - \underline{M}$. In the unregularized cases the feedback matrix was scaled by a factor 0.1 , showing that the beneficial effect of Tikhonov regularization can not be reproduced by including a simple "gain factor".

determined based on $\underline{M}$, while the physical system reacts according to $\hat{\underline{M}}$. Substituting $\underline{M} \to \hat{\underline{M}}$ in $\underline{T}$ yields

$$\underline{T} \to \hat{\underline{T}} = \underline{1} - \underline{M}^{+}\underline{M} - \underline{M}^{+}\underline{\epsilon} \tag{15}$$
$$= \underline{T} - \underline{M}^{+}\underline{\epsilon}. \tag{16}$$

In general, $\hat{\underline{T}}$ is no longer diagonalizable so that in contrast to the undisturbed case Equation (12) can not be evaluated directly. However, a criterion for the convergence of $\sum_{j=0}^{\infty} \hat{\underline{T}}^j$ can be deduced by invoking the Banach fixed point theorem on Equation (11), showing that this iteration allows a unique fixed point, if $\hat{\underline{T}}$ defines a contraction: $||\hat{\underline{T}}||_{\mathrm{op}} < 1$, where $||\cdot||_{\mathrm{op}}$ is an operator norm. By virtue of the subadditivity and submultiplicativity of the operator norm, we see

$$||\hat{\underline{T}}||_{\mathrm{op}} \leq ||\underline{T}||_{\mathrm{op}} + ||\underline{M}^{+}\underline{\epsilon}||_{\mathrm{op}} \tag{17}$$
$$\leq ||\underline{T}||_{\mathrm{op}} + ||\underline{M}^{+}||_{\mathrm{op}}||\underline{\epsilon}||_{\mathrm{op}}. \tag{18}$$

Choosing the l^2 operator norm $||\cdot||_{\mathrm{op},2} = \max \circ \vec{\sigma}$ this becomes

$$||\hat{\underline{T}}||_{\mathrm{op},2} \leq \max(|1 - \sigma_i^{+}\sigma_i|) + \max(\sigma_i^{+})\,||\underline{\epsilon}||_{\mathrm{op},2}, \tag{19}$$

so that a sufficient condition for the convergence of $\sum_{j=0}^{\infty} \hat{\underline{T}}^j$ is

$$\frac{1 - \max(|1 - \sigma_i^{+}\sigma_i|)}{\max(\sigma_i^{+})} > ||\underline{\epsilon}||_{\mathrm{op},2}. \tag{20}$$

Choosing the σ_i^{+} based on the Tikhonov regularization method $\sigma_i^{+} = \sigma_i/(\sigma_i^2 + \alpha^2)$ it can be seen that $\max(\sigma_i^{+}) \leq 1/(2\alpha)$ and $1 - \max(|1 - \sigma_i^{+}\sigma_i|) = \sigma_{\mathrm{min}}^2/(\sigma_{\mathrm{min}}^2 + \alpha^2)$, with $\sigma_{\mathrm{min}} := \min(\sigma_i)$. With this, we see that Equation (20) is fulfilled if

$$\frac{2\alpha\sigma_{\mathrm{min}}^2}{\sigma_{\mathrm{min}}^2 + \alpha^2} > ||\underline{\epsilon}||_{\mathrm{op},2}, \tag{21}$$

which, for the case $||\underline{\epsilon}||_{\mathrm{op},2} \leq \sigma_{\mathrm{min}}$, yields a range for the free regularization parameter α which guarantees the convergence of $\sum_{j=0}^{\infty} \hat{\underline{T}}^j$

$$\frac{\sigma_{\mathrm{min}}^2 - \mu}{||\underline{\epsilon}||_{\mathrm{op},2}} < \alpha < \frac{\sigma_{\mathrm{min}}^2 + \mu}{||\underline{\epsilon}||_{\mathrm{op},2}} \tag{22}$$

with $\mu = \sigma_{\mathrm{min}}\sqrt{\sigma_{\mathrm{min}}^2 - ||\underline{\epsilon}||_{\mathrm{op},2}^2}$. We stress again that this presents a sufficient condition for the stability, not a necessary one.

ITERATION BEHAVIOR

The presented trajectory correction scheme has been implemented in the *Toolkit for Simulated Commissioning* (SC) [6] – an AT based toolbox allowing the realistic simulation of commissioning procedures for storage-ring light sources. During the numerical commissioning studies of the storage- and accumulator ring of the Advanced Light Source Upgrade (ALS-U) [7], it became apparent that employing Tikhonov-regularized feedback matrices generally has a beneficial effect on the correction iteration, as illustrated in Figure 1. It shows the evolution of $\vec{\phi}_{n+1} = \vec{\phi}_n - \underline{M}^{+}(\alpha)\vec{r}_n$ and $\vec{r}_n = \hat{\underline{M}}\vec{\phi}_n + \vec{\kappa}$ where $\underline{M}^{+}$ is calculated from the ideal response matrix of the ALS-U Accumulator Ring lattice [3] $\underline{M}$ and $\hat{\underline{M}}$ is its disturbed counterpart as calculated by SC, taking into account a set of realistic lattice errors. $\vec{\kappa}$ was chosen at random. While the exact behavior of the iteration naturally depends on the actual manifestation of $\vec{\kappa}$ and $\hat{\underline{M}}$, the fundamental advantages of the regularized case over the unregularized case indeed have proven to be characteristical: During the first correction steps, before the iteration has reached its fixed point, the regularized case reliably yields smaller BPM readings, while using significantly smaller corrector kicks, in terms of both the RMS and maximum value. Further, Tikhonov regularization makes the correction procedure less sensitive to to lattice errors; after a few steps both the RMS and maximum values are nearly identical, independent on the magnitude of the response matrix errors. The unregularized case shows increasingly large excursions with increasing error magnitudes. As predicted by Equation (14) the fixed point is not affected by the regularization.

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