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EFFECT OF EMITTANCE CONSTRAINTS ON MONOCHROMATIZATION AT THE FUTURE CIRCULAR e^+e^- COLLIDER *

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Abstract

Direct s-channel Higgs production in e^+e^- collisions is of interest if the collision energy spread can be comparable to the natural width of the standard model Higgs boson. At the Future Circular e^+e^- Collider (FCC-ee) [1], a monochromatization scheme could be employed in order to reduce the collision energy spread to the target value. This may be achieved by introducing, at the interaction point (IP), a non-zero horizontal dispersion of opposite sign for the two colliding beams. In this case, the beamstrahlung increases the horizontal emittance in addition to energy spread and bunch length. The vertical emittance could either be tuned to a certain minimum value, possibly limited by the diagnostics resolution, or it could scale linearly with the horizontal emittance. For the FCC-ee at 62.5 GeV beam energy, we optimize the IP optics and beam parameters, considering these two assumptions for the vertical emittance. We derive the maximum achievable luminosity as a function of collision energy spread for either case.

INTRODUCTION

Monochromatization [2–9] could allow for direct Higgs production in the s channel, $e^+e^- \longrightarrow H$, at a beam energy E_b of 62.5 GeV, and also provide the energy resolution required to precisely measure the width of the Higgs particle. The monochromatic collision of electrons and positrons can be realized by introducing IP dispersion of opposite sign for the two colliding beams, so that the spread in the center-of-mass (c.o.m.) energy W, $(\sigma_w/W)_{\text{m.c.}} = \sigma_\delta/(\sqrt{2}\lambda)$, is reduced by the monochromatization (m.c.) factor $\lambda = \sqrt{D_x^{*2}\sigma_\delta^2/(\varepsilon_x\beta_x^*)+1}$, where $\sigma_\delta \equiv \sigma_{E_b}/E_b$ denotes the relative beam energy spread (which for ultra-relativistic beams is equal to the relative momentum spread), E_b the beam energy, β_x^* the horizontal beta function at the IP, D_x^* the horizontal IP dispersion function, and ε_x the horizontal emittance.

BEAMSTRAHLUNG

In present electron storage rings the equilibrium transverse emittances, energy spread and bunch length are determined by a balance of quantum excitation and radiation damping, both occuring in the accelerator bending magnets [10]. At future high-energy circular colliders, like FCC-ee [1] or CEPC [11], also the synchrotron radiation emitted during the collision in the electromagnetic field of the opposing beam becomes important. This additional radiation, which is called "beamstrahlung" [12–16], significantly increases the equilibrium bunch length and energy spread [17–19]. With non-zero dispersion at the IP, as required for monochromatized collisions [2], beamstrahlung also affects the transverse beam emittance [9, 19].

For all proposed high-energy circular colliders, the beamstrahlung can be described by classical radiation formulae [19]. In this case we can approximate the average number of photons per collision as [16]

$$n_\gamma \approx \frac{12}{\pi^{3/2}}\frac{\alpha r_e N_b}{\sigma_x^*+\sigma_y^*} \approx \frac{12}{\pi^{3/2}}\frac{\alpha r_e N_b}{\sigma_x^*}\,, \tag{1}$$

where α denotes the fine structure constant ($\approx 1/137$), $r_e \approx 2.8\times10^{-15}$ m the classical electron radius, N_b the bunch population, and $\sigma_{x(y)}^*$ the horizontal (vertical) rms IP beam size. The average relative energy loss, δ_B [20],

$$\delta_B \approx \frac{24}{3\sqrt{3}\pi^{3/2}}\frac{r_e^3\gamma N_b^2}{\sigma_z(\sigma_x^*+\sigma_y^*)^2} \approx \frac{24}{3\sqrt{3}\pi^{3/2}}\frac{r_e^3\gamma N_b^2}{\sigma_z\sigma_x^{*2}}\,, \tag{2}$$

depends on the rms bunch length σ_z. The average photon energy normalized to the beam energy, $\langle u\rangle$, is given by

$$\langle u\rangle = \frac{\delta_B}{n_\gamma} \approx \frac{2\sqrt{3}}{9}\frac{r_e^2 N_b\gamma}{\alpha\sigma_z\sigma_x^*}\,. \tag{3}$$

The quantum excitation of oscillations, which gives rise to energy spread and emittance, is the product of the mean square photon energy $\langle u^2\rangle$ and the mean rate [10]. In the case of beamstrahlung, the mean rate is simply given by n_γ divided by the average time interval between collisions (half the revolution period, with two interaction points).

In the classical radiation regime and for a constant bending radius ρ, the mean squared photon energy $\langle u^2\rangle$ is related to the average photon energy $\langle u\rangle$ via [10]

$$\langle u^2\rangle \approx \frac{25\times 11}{64}\langle u\rangle^2 \text{ (constant } \rho). \tag{4}$$

For a Gaussian bunch, with locally-varying bending radius, the relation between $\langle u\rangle$ and $\langle u^2\rangle$ is more complex [21]. In particular, in Ref. [21] we discussed the dependence of this relation on the transverse beam aspect ratio for the case of a head-on collision.

In general (4) must be modified as [21]

$$\langle u^2\rangle \approx Z_c\frac{25\times 11}{64}\langle u\rangle^2\,, \tag{5}$$

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where the correction Z_c is related to the variation of $1/\rho$ in time and space during the collision: $Z_c \equiv \langle 1/\rho^2 \rangle / (1/\langle\rho\rangle^2)$. For a typical ratio $\sigma_x^*/\sigma_y^* \sim 200$ we found $Z_c \sim 1.7$ [21].

SELF-CONSISTENT EMITTANCE

The beamstrahlung parameters (Υ, δ_B, $\langle u \rangle$ and ρ) strongly depend on the bunch length. The "total" (equilibrium) bunch length is related to the total energy spread via [10]

$$\sigma_{z,\mathrm{tot}} = \frac{\alpha_c C}{2\pi Q_s}\sigma_{\delta,\mathrm{tot}} \,, \tag{6}$$

where Q_s denotes the synchrotron tune, C the circumference, and α_c the momentum compaction.

In the presence of nonzero IP dispersion, the energy spread, the bunch length, and the horizontal emittance increase due to the beamstrahlung. Assuming $D_x^* \sigma_{\delta,\mathrm{tot}} \gg \sqrt{\beta_x^* \varepsilon_x}$ (i.e. monochromatization), and $\tau_x = 2\tau_E$, where τ_x (τ_E) denotes the horizontal (longitudinal) damping time due to arc synchrotron radiation, we have [19]

$$\sigma_{\delta,\mathrm{tot}}^2 = \sigma_{\delta,\mathrm{SR}}^2 + \frac{V}{D_x^{*3}\sigma_{\delta,\mathrm{tot}}^5} \,, \tag{7}$$

$$\varepsilon_{x,\mathrm{tot}} \approx \varepsilon_{x,\mathrm{SR}} + \frac{2V\mathcal{H}_x^*}{D_x^{*3}\sigma_{\delta,\mathrm{tot}}^5} \,, \tag{8}$$

where the subindex SR designates equilibrium parameters without beamstrahlung as determined by the arc synchrotron radiation, the coefficient

$$V \equiv 47\, Z_c \,\frac{n_{\mathrm{IP}}\tau_{E,\mathrm{SR}}}{T_{\mathrm{rev}}}\,\frac{r_e^5 N_b^3 \gamma^2}{(\alpha_c C/(2\pi Q_s))^2} \tag{9}$$

has the dimension of a volume, and the dispersion invariant $\mathcal{H}_x^*$ is defined as [10]

$$\mathcal{H}_x^* \equiv \frac{\left(\beta_x^* D_x'^* + \alpha_x^* D_x^*\right)^2 + D_x^{*2}}{\beta_x^*} \,, \tag{10}$$

where β_x^*, α_x^*, D_x^* and $D_x'^*$ denote optical beta and alpha function (Twiss parameters), the dispersion and slope of the dispersion at the IP, respectively.

PARAMETER OPTIMIZATION

Searching for an optimal point in parameter space, we adopt a fixed β_y^* value of 1 mm [22]. We then transform β_x^* and D_x^* with parameter S [23], so as to keep λ without beamstrahlung fixed, namely $D_x^* = S \times D_{x,0}^*$, starting from $D_{x,0}^* = 0.22$ m, and $\beta_x^* = S^2 \times \beta_{x,0}^*$, starting from $\beta_{x,0}^* = 1.0$ m. We introduce a second transformation with parameter T [23], which would lead to $L \propto T^{-1}$ in case of no beamstrahlung and no limit on the beam-beam tune shift, namely $n_b = n_{b,0} \times T$ and $N_b = N_{b,0}/T$, so that the total beam current is constant, where n_b and N_b refer to the number of bunches per beam and the bunch population, respectively, and the values with subindex 0 are the initial values for our optimization. The product $n_b N_b$ is held constant, as it is limited by the arc synchrotron radiation. The initial values correspond to parameters for which $\lambda \approx 10$ [23] (where λ is computed without the effect of beamstrahlung). Including the effects of beamstrahlung, the actual monochromatization factor is reduced and no longer constant in the (S, T) parameter space.

For the vertical emittance, we now consider two possibilities. As a first case, the minimum vertical emittance might be due to residual dispersion or be limited by the resolution limit of the available diagnostics, e.g. X-ray interferometer and BPM responses, and, therefore, independent of the horizontal emittance. This is the scenario we considered in [21]. In a second scenario, we assume that the vertical emittance is dominated by residual betatron coupling, and proportional to the horizontal emittance. So we have

$$\varepsilon_{y,\mathrm{tot}} = \text{constant} \quad (\text{case } 1) \tag{11}$$

$$\varepsilon_{y,\mathrm{tot}} = \kappa_\varepsilon \varepsilon_{x,\mathrm{tot}} \quad (\text{case } 2) \tag{12}$$

The effect of the first emittance constraint on the luminosity, as a function of IP dispersion D_x^* and the number of bunches n_b, is shown in Fig. 1; the effect of the second emittance constraint is presented in Fig. 2

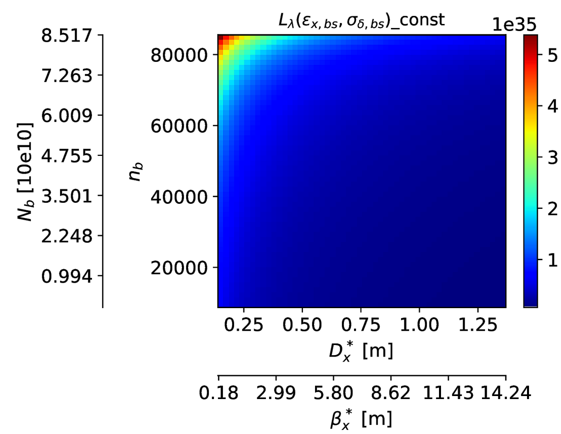

Figure 1: Luminosity including beamstrahlung effects in the S-T plane for a constant value of vertical emittance.

Considering either scenario, we have reoptimized the IP-optics and beam parameters for monochromatization at 125 GeV, following the recipe described in Ref. [23]. The updated dependence of the luminosity on the monochromatization factor λ, is shown in Fig. 3 for a constant vertical emittance ε_y, and in Fig. 4 for a constant vertical-to-horizontal emittance ratio $\varepsilon_y/\varepsilon_x$. For constant vertical emittance the peak luminosity decreases with increasing λ, whereas for constant vertical emittance ratio we obtain a maximum around $\lambda \approx 5$–6, close to our target value. In general, the luminosity is lower in the second scenario, where the vertical emittance blows up together with the horizontal emittance under the effect of beamstrahlung. For large values of λ ($\lambda \approx 10$), where the beamstrahlung becomes less important, the luminosity values for the two cases converge. Example

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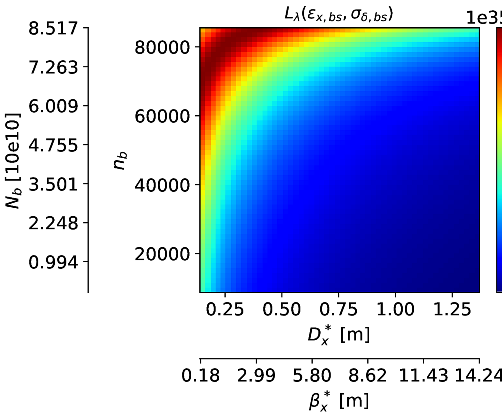

Figure 2: Luminosity including beamstrahlung effects in the *S*-*T* plane for a constant transverse emittance ratio.

parameter sets for both cases, corresponding to the same centre-of-mass energy spread $\sigma_W \approx 6$ MeV, are compiled in Table 1.

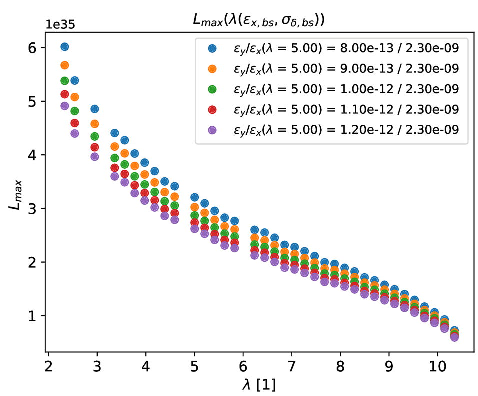

Figure 3: Optimal luminosity as a function of λ for constant vertical emittance.

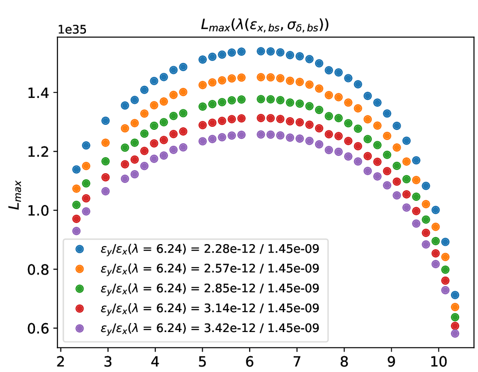

Figure 4: Optimal luminosity as a function of λ for a constant vertical-to-horizontal emittance ratio.

Table 1: Optimized monochromatization parameters for constant vertical emittance (case 1) or emittance ratio (case 2) at the same c.o.m. energy spread $\sigma_W \approx 6$ MeV.

parameter	case 1	case 2
E_b [GeV]	62.50	62.50
circumference C [km]	97.76	97.76
I_b [mA]	418	418
$n_{b,\mathrm{opt}}$	15950	8700
$N_{b,\mathrm{opt}}$ [10^{10}]	5.33	9.77
$\varepsilon_{x,\mathrm{SR}}$ [nm]	0.51	0.51
$\varepsilon_{x,\mathrm{opt}}$ [nm]	2.30	1.45
$\varepsilon_{y,\mathrm{SR}}$ [pm]	1.00	1.00
$\varepsilon_{y,\mathrm{opt}}$ [pm]	1.00	2.85
α_c [10^{-6}]	14.80	14.80
$\beta^*_{x,\mathrm{opt}}$ [m]	0.24	1.25
β^*_y [mm]	1.00	1.00
$D^*_{x,\mathrm{opt}}$ [m]	0.1624	0.3712
$\sigma_{x,\mathrm{opt}}$ [µm]	119.2	269.5
$\sigma_{y,\mathrm{opt}}$ [nm]	31.6	53.4
$\sigma_{z,\mathrm{SR}}$ [mm]	1.64	1.64
$\sigma_{z,\mathrm{opt}}$ [mm]	1.65	1.65
$\sigma_{\delta,\mathrm{SR}}$ [%]	0.0714	0.0714
$\sigma_{\delta,\mathrm{opt}}$ [%]	0.0720	0.0717
U_0 [GeV]	0.1254	0.1254
V_{rf} [GV]	2.0	2.0
Q_s	0.1002	0.1002
τ_E [ms]	162.5	162.5
L_{opt} [10^{35} cm^{-2}s^{-1}]	2.87	1.38
$\xi_{x,\mathrm{opt}}$	0.0033	0.0062
$\xi_{y,\mathrm{opt}}$	0.0518	0.0249
$\sigma_{w,\mathrm{opt}}$ [MeV]	6.03	6.00

CONCLUSIONS

Different assumptions on the vertical emittance behavior can greatly affect the estimated luminosity performance for monochromatized *s*-channel Higgs production at FCC-ee. Two updated parameter sets correspond to two different assumptions. At $\sigma_W \approx 6$ MeV the maximum luminosity is 2.9×10^{34} cm^{-2}s^{-1} for a constant vertical emittance of 1 pm, and 1.4×10^{34} cm^{-2}s^{-1} for a constant emittance ratio $\varepsilon_y/\varepsilon_x = 0.2\%$ ($\varepsilon_y = 2.85$ pm in this specific case). For the above examples, assuming a constant emittance ratio leads to roughly two times lower luminosity than a constant vertical emittance. These results highlight the importance of vertical emittance correction and precise emittance diagnostics for this mode of operation.

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