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SYMPLECTIC AND EXACT TRACKING OF LOW ENERGY ^{197}Au^{78+} IONS IN THE RELATIVISTIC HEAVY ION COLLIDER*

Y. Luo†, W. Fischer, F. Méot, G. Robert-Demolaize,
Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

In the RHIC low energy run, the electron cooling technique is to be used to cool the 197Au79+ ions at energies between 3.85 GeV/nucleon to 5.75 GeV/nucleon. To overlap the electrons and the 197Au79+ ions in the cooling section for the best cooling efficiency, a monitor is to be installed downstream to detect the maximum flux of 197Au78+ ions which are generated in the cooling section. In the previous studies, we tracked 197Au78+ ions through the RHIC lattice defined with 197Au79+ ions with an equivalent momentum deviation 1/78. In the article, we explore different symplectic and exact ways to track 197Au78+ ions. We implemented these approaches, compared the tracking results, and they agreed very well.

INTRODUCTION

Normally a storage ring accelerator is designed to store a kind of particular charged particle with a particular particle energy. However, under some circumstances, same charged particles but with different energies, or isotope particles with different charges, or new particles with different energies and charges are generated in the ring. For example, when the stored particles hit on the collimation system, secondary particles will be generated. To track these particles in the storage ring and to calculate their loss maps are sometimes critical for the machine protection and other purposes.

In the RHIC low energy run [1], electron cooling technique is to be used to cool 197Au79+ ions. The electrons will travel with the 197Au79+ ions with a same velocity in the cooling section. The maximum cooling rate can be achieved when the electrons and the ions are perfectly overlapped in the cooling section. To monitor and optimize their overlapping, a monitor is to be installed downstream the cooling section to detect and maximize the flux of 197Au78+ ions, which are the by-product of the interaction between the electrons and 197Au79+ ions in the cooling section.

197Au78+ ions have the same energy as the 197Au79+ ions but with different charge state. To determine the best location to install the 197Au78+ monitor, we need to track the 197Au78+ ions exactly. Previously we tracked 197Au78+ ions approximately by tracking 197Au79+ ions with a larger $dp/p_0 = 1/78$, considering the magnetic forces on 197Au78+ ions are weaker than that on 197Au79+ ions.

In the following, we explore new methods to track 197Au78+ ions exactly with the lattice defined with 197Au79+ ions. We will first review the particle motion's Hamiltonian in a circular accelerator storage ring. Then we explore three approaches to track 197Au78+ ions exactly. We implement these approaches in a simulation code and compare the tracking results.

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† yluo@bnl.gov

HAMILTONIAN

In the planar curvilinear coordinate system, the Hamiltonian of a charged particle's motion in the external magnetic fields is given by [2]

$$H = -(1+\frac{x}{\rho})\sqrt{\frac{E^2}{c^2} - m_0 c^2 - p_x^2 - p_y^2} - qA_s, \quad (1)$$

where E is the particle's total energy, $m_0 c^2$ the particle's rest energy, (p_x, p_y) the particle's mechanical momenta, ρ is the radius of the reference coordinate system, A_s the magnetic vector potential. The canonical variables are $(x, p_x, y, p_y, t, -E)$. We further normalize momenta and Hamiltonian with a reference momentum p_0, which corresponds to a reference total energy E_0, $cp_0 = \beta_0 E_0$, $E^2 = m_0^2 c^4 + c^2 p^2$. Then the new Hamiltonian is

$$H = -(1+\frac{x}{\rho})\sqrt{(1+\delta)^2 - p_x^2 - p_y^2} - \frac{q}{p_0}A_s. \quad (2)$$

Here $\delta = \frac{\Delta p}{p_0}$.

It is convenient to use the longitudinal path length s as an independent variable in circular accelerators. Defining a generating function

$$F_2 = xp_x + yp_y + (\frac{s}{\beta_0} - ct)(p_t + \frac{E_0}{cp_0}), \quad (3)$$

we obtain a new Hamiltonian as

$$H = \frac{p_t}{\beta_0} - (1+\frac{x}{\rho})\sqrt{1 + 2\frac{p_t}{\beta_0} + p_t^2 - p_x^2 - p_y^2} - \frac{q}{p_0}A_s. \quad (4)$$

Here the canonical variables are $(x, p_x, y, p_y, -c\Delta t, p_t)$, $-c\Delta t = -c(t - t_0) = -c(t - \frac{s}{\beta_0 c})$, $p_t = \frac{E - E_0}{cp_0}$.

So far, in the above derivation, there is not direct connection between the reference coordinate system's radius ρ and the reference momentum p_0. However, during the accelerator design and in the most of accelerator design and simulation codes, we normally assume that a reference particle with the reference momentum p_0 will follow the reference closed orbit defined by ρ. Also the magnet strengths are defined with respect to the so-called magnetic rigidity, which is defined as $(B\rho) = p_0/q$. As we will see later, these connections give difficulties when we would like to track particles with different energy or different charge.

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To solve Hamiltonian Eq. (4), we adopt 4th order symplectic integration in our simulation code SimTrack [3]. We split the Hamiltonian into two solvable parts. One part is a drift or a simple sector bend. The other part is zero-length magnetic kicks, including quadrupole, sextupole, and so on. The Hamiltonians for drifts and sector bends are analytically solvable.

APPROACHES

For the RHIC low energy cooling run, the ring lattice is designed with the 197Au79+ reference ion. All the magnetic strengths and reference coordinate system are associated with the reference 197Au79+ ion too. To track 197Au78+ ions, we need to use 197Au78+ ion's own Hamiltonian, and define 197Au78+ ions' reference momentum, coordinate system's radius, and the values of the magnetic field strengths. As we will see, these quantities may or may not be the same as that of 197Au79+ reference ion.

Drift

In the drift, the particle motion's Hamiltonian is

$$H = \frac{p_t}{\beta_0} - \sqrt{1 + 2\frac{p_t}{\beta_0} + p_t^2 - p_x^2 - p_y^2}, \qquad (5)$$

which can be exactly solved.

In the drifts, we always use the same coordinate system (straight line) for 197Au78+ ions and 197Au79+ ions. The lengths of drifts are the same. However, we need to pay attention that p_x, p_y and $-c\Delta t$ are all defined with respect to the reference 197Au78+ ion's momentum p_{10}. In the following, we may choose different reference momentum p_{10} for the 197Au78+ ions other than the same reference momentum p_0 for the 197Au79+ ions.

Approach 1: Same Reference Momentum and Same Reference Orbit Radius

We assume that 197Au78+ ions are generated to have the same momentum and energy as the reference 197Au79+ ion, then we can choose the same reference momentum for the 197Au78+ ions as that for 197Au79+ ions, that is, $p_{10} = p_0$, where p_{10} and p_0 are the reference particles' momenta for 197Au78+ and 197Au79+ ions.

According to Eq. (4), the last term of Hamiltonian is $-\frac{q_1}{p_{10}}A_s$ for 197Au78+ ions and $-\frac{q_0}{p_0}A_s$ for 197Au79+ ions. Here q_0 and q_1 are the charges of197Au79+ and 197Au78+ ions. Since the magnetic strengths are originally defined with 197Au79+ ions, we need to scale them by $\frac{q_1}{q_0} = \frac{78}{79}$ when we track a 197Au78+ ion through a straight magnet, such as quadruple, sextupole, etc.

In the sector bends, the reference coordinate system's curvature, path length, and bending angles are all defined with the reference particle 197Au79+ ion. The bending radius of the 197Au78+ reference ion is actually larger than that of the 197Au79+ reference ion. In this approach, we choose to keep the radius of the reference coordinate system of 197Au79+ ions. Under such a situation, there is an exact solution for a pure sector bending magnet and is given in Ref [4].

Approach 2: Same Reference Orbit Radius but Different Reference Momentum

During particle tracking, it is not straightforward to change a magnet's strength on the fly. From Eq. (4), to keep the last term $-\frac{q_1}{p_{10}}A_s$ of 197Au78+ ion's Hamiltonian as that of 197Au79+ ion, we can choose a new reference momentum for 197Au78+ ions so that $\frac{q_1}{p_{10}}A_s = \frac{q_0}{p_0}A_s$. The new reference momentum of 197Au78+ ion is $p_{10} = p_0\frac{q_1 0}{p_0}$, which is slightly smaller than that of the reference 197Au79+ ion's. In the bending magnets, we choose the radius of the reference coordinate system for 197Au78+ ions as that for 197Au79+ ions in the bending magnets.

The advantage of this approach is that in the simulation code, we do not need to scale the magnet strengths and bending angles when we track 197Au78+ ions. We can call the same particle transfer functions with the same magnet strengths defined by 197Au79+ ions, even in the sector bending magnets. The disadvantage is that p_x, p_y are now normalized with the new reference momentum p_{10}. And the time-of-flight difference $-c\Delta t$ is also defined with respect to the new 197Au78+ ions' reference momentum p_{10}.

Approach 3: Same Reference Momentum but Different Reference Orbit Radius

Another approach is to use the same reference momentum of 197Au79+ ions for the reference 197Au78+ ion but adjust the new reference coordinate system's radius ρ locally in the bending magnets according to the 197Au78+ ion's charge state. In this approach, the radius of the reference coordinate system for 197Au78+ ions in the sector bends will be increased by $q_0/q_{10} = 79/78$, while the bending angle for the reference 197Au78+ ion is the same as that of the reference 197Au79+ ion.

In the approach, we also have to scale the magnetic strengths by q_1/q_0 as Approach 1 in the straight magnets, such as in quadrupoles, sextupoles, and so on. Also, for particle transfer in the sector bending magnets, we need to first shift x with respect to the new reference bending radius at the magnet entrance. And at the exit, we need to shift x back to the coordinate system defined by the 197Au79+ reference ion. Approach 3 needs more codings in its application and turns out to be less efficient.

IMPLEMENTATION AND COMPARISON

We implemented the above three approaches in SimTrack. Table 1 compares the tracking results after a test 197Au78+ ion goes through a quadrupole whose length and integrated strength are 11.44 m and 0.1010 m^{-1}. The quadrupole strength is defined with the reference 197Au79+ ion's momentum p_0. At the entrance, we set $x = 1.0 \times 10^{-3}$m and other coordinates zero. Table 1 shows $x, p_x, -c\Delta t$ from these three approaches. p_x and $-c\Delta t$ may be different among these

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Table 1: Comparison of Tracking Results After a Test 197Au78+ Ion Goes Through a Quadrupole

quantity	Approach 1	Approach 2	Approach 3
x	0.00048158672990719951	0.00048158672990713207	0.00048158672990713353
p_x	-8.1843050261955035e-05	-8.2892320137117301e-05	-8.184305026196341e-05
$-c\Delta t$	-1.5549716745558856e-08	0.0088947279495315591	-1.5549717071822395e-08
P_x/p_0	-8.1843050261955035e-05	-8.1843050261963911e-05	-8.184305026196341e-05
t	3.9327243214186826e-08	3.9327243214186826e-08	3.9327243214186826e-08

Table 2: Comparison of Tracking Results After a Test 197Au78+ Ion Goes Through a Sector Bend

quantity	Approach 1	Approach 2	Approach 3
x	0.0033226317865455712	0.0033226317864887278	0.0033226317865171495
p_x	0.00049198238851857925	0.00049828985503804824	0.00049198238851859703
$-c\Delta t$	-7.1502179466165217e-05	0.0072687262807296094	0.12466562921561852
P_x/p_0	0.00049198238851857925	0.00049198238851857925	0.00049198238851859703
t	3.2454344635300927e-08	3.2454344635300927e-08	3.2454344635300913e-08

Table 3: Comparison of Tracking Results After a Test 197Au78+ Ion Travels 10 Turns in RHIC

quantity	Approach 1	Approach 2
x	0.050147591765	0.050147591438
p_x	0.000539869456	0.000546790852
$-c\Delta t$	-0.732599208402	23.114302540194
Px/p_0	0.000539869456	0.000539869448
t	0.000105439200	0.000105439200

approaches due to different reference momentum and coordinate system. For comparison, we normalized the absolute horizontal mechanical momentum P_x by the reference momentum of the 197Au79+ reference ion's momentum p_0. From the table, the absolute values of x, Px/p_0 and the absolute time-of-flight t from these three approaches agreed very well.

Table 2 compares the tracking results after a test 197Au78+ ion goes through a sector bending magnet. Defined with the reference 197Au79+ ion and its reference coordinate system, the arc length is 9.44 m and the bending angle is 0.039 rad. At the entrance, we set $x = 1.0 \times 10^{-3}$m and other coordinates zero. At the exit, although the values of p_x and $-c\Delta t$ from different approaches may vary, the absolute values of x, Px/p_0 and t of the test 197Au78+ ion particle agreed very well.

Table 3 compares the tracking results of a test 197Au78+ ion after it travels 10 turns in RHIC. For this test, the RF cavities were turned off. Here we only implemented Aproach 1 and Approach 2 for multi-turn tracking. After 10 turns, the relative errors in the horizontal position x of the test 197Au78+ ion is less than 1×10^{-8}. The relative difference in the absolute Px/p_0 is less than 2×10^{-8}. The absolute time-of-flight t are exactly same up to 9 effective digits.

SUMMARY

In this article we discussed how to track 197Au78+ ions exactly in the RHIC low energy run where the magnet strengths and coordinate system are defined with 197Au79+ ion. Based on the Hamiltonian of 197Au78+ ions, we explored three approaches to transfer 197Au78+ ions exactly by appropriately choosing its reference momentum and the radius of coordinate system. We implemented and tested these methods in SimTrack. Although $p_x, -c\Delta t$ may differ due to different choices of reference momentum and radius of coordinate system, the absolute values of the horizontal position x, the normalized momentum Px/p_0, and the absolute time-of-flight t agreed very well in all our tests.

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