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MEASUREMENT AND ANALYSIS OF BEAM PHASE-SPACE DISTRIBUTIONS FOR THE FERMILAB MUON CAMPUS ACCELERATOR COMPLEX*

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Abstract

The Muon g-2 experiment at Fermilab is tasked with measuring the muon's anomalous magnetic moment with high precision. Since the experiment requires large amounts of muons, it is imperative to systematically study the behavior of the beam along the transport line. Unfortunately, the available diagnostics only provide beam information in X-Y space. For a complete evaluation, information of the phase-space is required. This paper demonstrates a technique to measure the beam phase-direction distribution by using a set of beam profiles. First, we establish the theoretical framework that describes the principle of the technique. Next, we apply the technique at four different locations along the accelerator delivery line. Finally, we compare our findings to predictions from tracking simulations.

INTRODUCTION

The Fermilab Muon g-2 experiment is probing the anomalous magnetic moment of the muon with an unprecedent precision of 140 ppb [1]. To achieve such high precision, the experiment requires an abundance of muons to measure within in its storage ring. Previous research [2], as seen in Fig. 1, indicates a set of Twiss (Courant-Snyder) parameters at injection which maximizes the percentage of muon storage. Thus, it is important to have a technique to measure the properties, or Twiss parameters, of the beamline to monitor and improve performance.

In this paper, we will discuss one technique accelerator physicists use and utilize it to determine the Twiss parameters of the beam transport to aid in the optimization of the beam injection for the Fermilab Muon g-2 experiment.

THEORY

In linear beam dynamics [3], particles at any point along a beamline can be defined in six-dimensional phase space with the following coordinates: (X, X', Y, Y', Z, Pz). Z is parallel to the beamline with Pz as its respective momentum. X and Y are positions transverse to Z; X' and Y' are the angular displacements of momentum in X/Z and Y/Z planes. We can then plot the distribution of (X, X') and (Y, Y') points on a phase plot, and fit them with an ellipse contour using the following equation

$$\frac{1+\alpha^2}{\beta}x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon, \qquad (1)$$

where the coefficients α, β, ε are the Twiss parameters. α describes the tilt of the beam phase-space which could be positive (beam converging) or negative (beam diverging). β de-scribes the shape and size of the beam, while ε, the emittance, describes the ellipse area occupied by the particles.

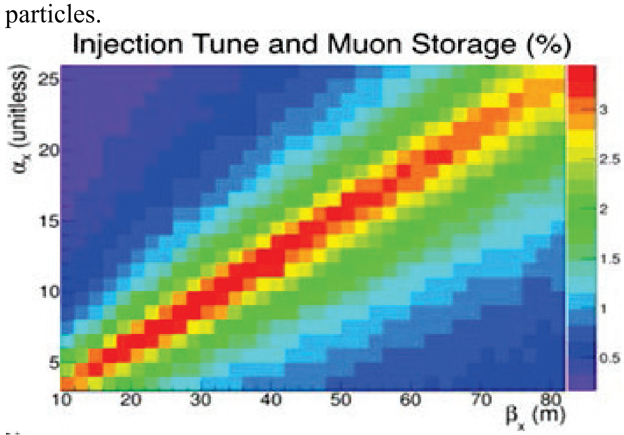

Figure 1: Example of Twiss parameters at injection and the corresponding muon storage. Injection is most sensitive to horizontal Twiss parameters since the narrowest aperture is Δx = ±9mm.

THE QUAD-SCAN METHOD

The quad-scan technique allows us to use profile monitors along an accelerator lattice to extract beam intensity pro-files as a function of position both in the X and Y direction. We can then use these profiles to derive our Twiss parameters upstream of a quadrupole. This method involves applying linear optics to the beam transport. First, we employ a quadrupole magnet and a profile monitor separated from the quad by a drift distance, d, as shown in Fig. 2.

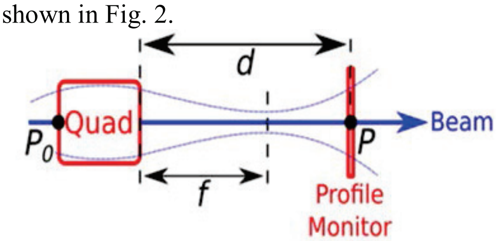

Figure 2: Diagram of the quad-scan setup. d is known as the drift distance and f is the focal length of the quadrupole. The Twiss parameters calculated from the measured beam widths describe the phase space at P_0.

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Using profile monitors positioned downstream of the quadrupole of interest we can extract the beam width within the X and Y directions which we define as σ_X and σ_Y. For this analysis, we utilized Proportional Wire Chambers (PWC). We selected four locations upstream of the g-2 storage ring as indicated on Fig. 3. Two analysis frameworks were used in this study, the Thick lens analysis described by S. Setiniyaz et al [4] and the Thin Lens Approximation described by J. Bradley et al [5]. The thin lens framework is an approximation of the thick lens method. This framework assumes that the focal length is much larger than the length of the quadrupole thus simplifying the complexity of the analysis and mathematics. For both frameworks we assumed zero dispersion and no coupling between X and Y planes. Beam profiles of each quadrupole were extracted while varying the current input of each quadrupole. The data was then analyzed using Python to employ gaussian fits on each beam profile to extract our σ values (which we will refer to as the root mean square of the gaussian) and plotted them with their respective focal lengths (thin lens approximation). The σ plots were both fitted using the thick lens analysis and the thin lens approximation which are parabolic fits. Once we extracted the fit coefficients, we derived our Twiss parameters and then used Eq. (1) to plot our ellipses.

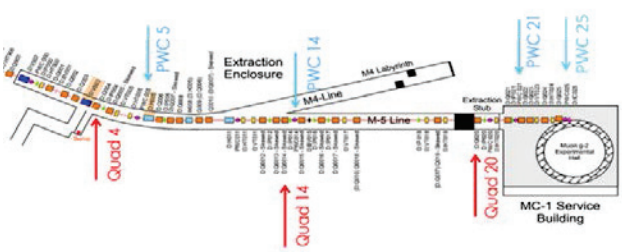

Figure 3: The M4 and M5 lines of the Fermilab Muon Campus. The quadrupoles marked are in red and the profile monitors are indicated by the blue arrows and any quadrupole in between the monitor and our quadrupole of interest is turned off. At the end of the M5 line is the g-2 storage ring in experimental hall MC-1.

To verify our measurements, we simulated the M4 and M5 lines [6, 7] using G4beamline [8], a particle tracking simulator based on Geant4. Because we can track the trajectories of each particle in G4beamline, we can use equations described by H. Wiedermann [3] to extract the simulated Twiss parameters and compare them with our experimental values.

RESULTS

During our study we observed a discrepancy between the measured and simulated Twiss parameters in Y direction. This is especially true for quadrupoles 4 and 14 since, due to technical difficulties, we were unable to switch the polarity of the quadrupoles in Y direction when measuring profiles. As a result, we could only extract half of the parabola (see Fig. 4) and therefore cannot reliably measure the Twiss parameters in the Y direction for quads Q004 and Q014.

Apart from this setback, we were able to extract information about the beamline in the X direction. Our measured Twiss parameters of the quadrupoles compared with the simulated ones are shown on Figs. 5 and 6. Parameters from the thin lens approximation are represented by the green contours for X and blue for Y. Twiss parameters from the thick lens analysis are marked in cyan for X and yellow in Y.

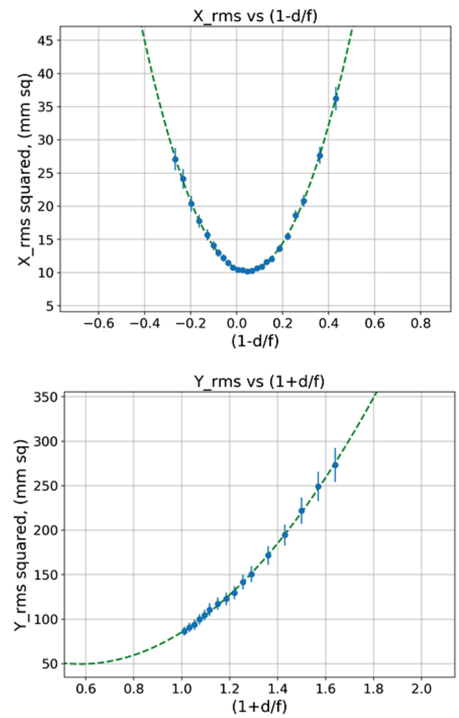

Figure 4: Thin lens parabolas for quadrupole 4 where the top plot (full parabola) goes through a waist in the X direction and the bottom plot (half parabola) does not have a waist in the Y direction.

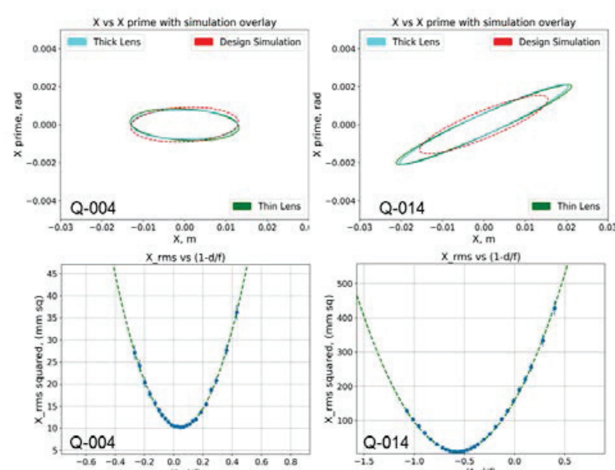

Figure 5: The phase-space and parabola plots for quadrupoles Q004 and Q014. In the phase-space plots, the dashed contours represent the phase plots from the simulation and the solid lines are from the measurement. The ellipses are generated from the Twiss parameters extracted from their respective parabola plot. Only horizontal plane is shown.

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From Figs. 5, 6 and Tables 1, 2, 3 and 4, we see reasonable agreement (on first-order) between the measured emittances and the predicted values from our simulations (dashed contours). The orientation (α) of each ellipse coincides with their simulated counterparts. The rms unormalized emittance as a function of distance seen on Fig. 7 shows that it is mostly conserved. However, the decrease in emittance from quad Q014 to Q020 may indicate beam loss along the beam line.

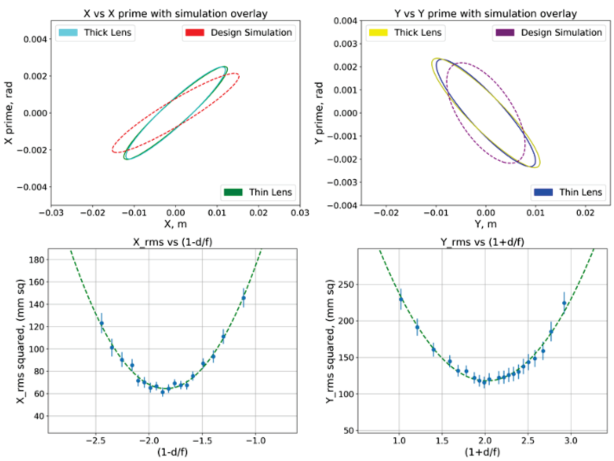

Figure 6: Same style plots as in Fig. 5 but now for quadrupole Q020. Both X (left) and Y (right) planes are shown.

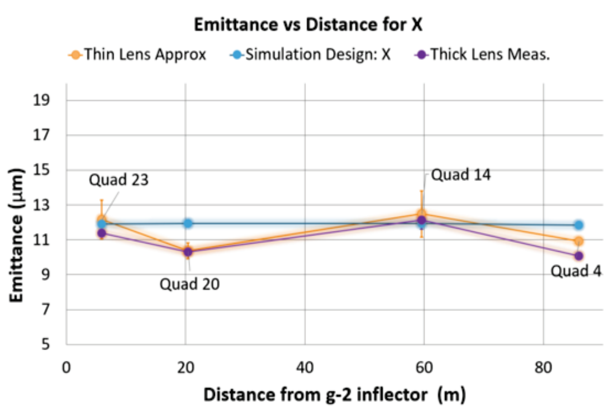

Figure 7: Emittance as a function of distance from the entrance of the storage ring of the Muon g-2 Experiment. We compared our measurements from both analysis frameworks with simulation. An extra emittance measurement was made for quad 23 using PWC25. Note that distance equal to zero points to the entrance of the storage ring.

Sources of error include noisy wires producing noisy data from the PWCs, numerical errors propagating in our analysis scripts and there may be some beam dispersion which we cannot account for with our current quad-scan technique. Despite these discrepancies, we can see that the beam emittance is conserved especially in X direction as expected from the simulations. Moreover, the Thick lens and the Thin lens approximation average emittance agree within 5% of each other in the X direction demonstrating that we can employ either one of the analysis frameworks for a decent measurement of the beam emittance. Thus, using the X direction average emittance data, we see an agreement between measurement and simulation within 4%. A future study in which the polarity of quads Q004 and Q014 can be flipped in the Y direction will be needed if one desires their respective Twiss parameters.

Table 1: Twiss Parameters of Q004 from both analysis frameworks and simulations. Only X direction

	Thin	Thick	Model
α	0.18 ±0.01	0.05 ±0.01	-0.14
β (m)	15.8±0.1	16.4±0.12	14.3
ε (µm)	10.9±0.1	10.1±0.1	11.8

Table 2: Twiss Parameters of Q014 from both analysis frameworks and simulations. Only X direction

	Thin	Thick	Model
α	-3.4 ± 0.4	-3.4 ± 0.1	-1.7
β (m)	36.2 ± 3.9	34.3 ± 1.5	20.1
ε (µm)	12.5 ± 1.3	12.1 ± 0.5	11.9

Table 3: Twiss Parameters of Q020 from both analysis frameworks and simulations. Only X direction

	Thin	Thick	Model
α	-2.9 ± 0.1	-2.7 ± 0.1	-2.6
β (m)	15.0 ± 0.6	14.1 ± 0.5	19.6
ε (µm)	10.4 ± 0.4	10.3 ±0.2	11.9

Table 4: Twiss Parameters of Q020 from both analysis frameworks and simulations. Only Y direction

	Thin	Thick	Model
α	2.3 ± 0.5	2.1 ± 0.1	0.81
β (m)	11.2 ± 2.5	10.7 ± 0.4	4.7
ε (µm)	12.1 ± 2.3	11.1 ± 0.2	13.4

CONCLUSION

Overall, we see agreement between our experimental and simulated results. The predicted and measured emittances in X and Y both agree and are mostly conserved along the accelerator beamline. Both frameworks produced an average X emittance 5% of each other. Thus, the quad-scan allows us to extract decent measurements of the Twiss parameters of a beam. However, this technique does not account for beam dispersion so future studies will need to modify the technique to account for this. Moreover, using this technique we can survey and optimize beam injection into the Muon g-2 experiment storage ring which will provide the experiment with more muons to measure.

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