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SENSITIVITY ANALYSES OF ALL-ELECTRIC STORAGE RING DESIGNS

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Abstract

Future searches of electric dipole moments (EDMs) of fundamental particles can require electrostatic storage rings operating at the particle's "magic momentum" whereby spin precessions out of the plane of the particle motion would be governed in principle only by the presence of an EDM. An EDM search for the proton, for example, requires a momentum of approximately 700 MeV/c and thus implies a half-kilometer circumference, where relatively modest electric fields are assumed. As no all-electric ring on this scale has been constructed before, the ability to produce precise radial fields for establishing a central orbit and precise electrostatic focusing fields about that orbit requires attention. Results of initial investigations into the feasibility of designing a proper system and the sensitivities of such a system to placement, mis-powering errors and other requirements on realistic electrostatic elements will be presented.

STORAGE RING AND EDM MEASUREMENT

Spin Dynamics

Just as a particle's intrinsic magnetic dipole moment (MDM) would make the spin precess about a magnetic field in its rest frame, the EDM would make its spin precess about an electric field. For a particle bathed in $\vec{E}$ and $\vec{B}$ fields, with a velocity $\vec{\beta}$ (perpendicular to both $\vec{E}$ and $\vec{B}$), the spin precession frequency due to MDM and EDM (given by the Thomas-BMT equation [1]) respectively are,

$$\vec{\omega}_a = \frac{e}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] \tag{1}$$

$$\vec{\omega}_e = \frac{\eta e}{2m}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right) \tag{2}$$

where η is the intrinsic electric dipole moment, m is the mass of the particle, and $G = (g-2)/2$.

In an all-electric storage ring, $\vec{B} = 0$, and if we choose a 'magic momentum' ($\gamma_{\text{magic}} = 1.248107$) such that $G = 1/(\gamma^2 - 1)$, the precession ω_a due to MDM becomes zero, leaving us just with $\omega_e = \eta e E/(2mc)$. For an all-electric ring, the magic momentum for proton turns out to be about 0.7007 GeV/c.

The Ring

The ring proposed by the EDM collaboration [2] (see Fig. 1) was taken up for the purposes of our study. The idea of the experiment is to simultaneously run proton beams clockwise (CW) and counter-clockwise (CCW), with the radial electric field giving the centripetal force. Since the EDM would make the spin precess about $\vec{E}$, the spin of the proton would tip *out* of the plane, and the split in the CW and CCW beams, detected using SQUID-based magnetometer BPMs, gives us the vertical precession rate, and thus the ω_e [2].

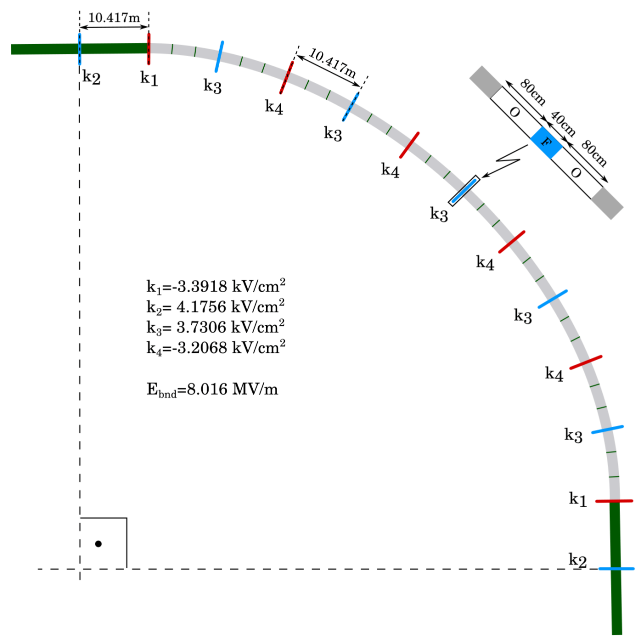

Figure 1: The lattice of a quadrant of the storage ring [2].

ELECTRIC FIELD DUE TO MISALIGNMENTS

Misalignments

In the real world, it is impossible to place any electrode plate infinitely precisely, and the finite offset/misplacement is going to result in the E-field deviating from the ideal value. Since this being a precision experiment, it is imperative to quantitatively gauge the distortion in fields coming from misalignments, and how the field deviations eventually end up contributing to the systematic errors of the experiment. But even more important is to study what tolerance of misalignment would give us a stable beam in the first place, which is the study of this paper.

Misalignments can come in random forms. For example, the plates (be it opposing quad plates or concentric bending cylindrical plates) may end up slanted at an angle to each other instead of being parallel, or they may be almost

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Table 1: Ring and Beam Parameters

Bending radius, R_0	52.3 m
Electrode spacing, d	3 cm
Electrode height	20 cm
Deflector shape	cylindrical
No. of bend sections	40
Radial E-field, E_0	8 MV/m
Momentum	0.7007 GeV/c
Horizontal Tune	2.42
Vertical Tune	0.44
Horizontal beta function, $\beta_{x,\max}$	47 m
Vertical beta function, $\beta_{y,\max}$	216 m

perfectly parallel but have a shearing offset due to longitudinal or vertical misplacement, or a shearing offset due to difference in plate lengths, or most likely a combination of all.

Field due to Misaligned Bend Element

To numerically compute the distortion in the static E-fields resulting from misalignments, the software *Finite Element Method Magnetics* (FEMM) [3] was used. Even though the concentric cylindrical bend element is 3-dimensional, a 2-D slice of the 3-D cylindrical plates was taken for the purpose of our study. Thus the reader is advised to bear in mind that the results borne by this paper are true to first-order, and a more precise actual 3-D distortion of the field shall be taken up in the future.

FEMM RESULTS

The Scheme

To get the E-field values, the spacing between the plates was chosen to be the design value of 30 mm and the thickness of the plates was chosen to be 1 mm. To quantify the E-field distortion due to misalignment, the scheme employed was to first get the E-field in between the plates for the *ideal* configuration, do the same for the *misaligned* configuration, and compute the difference in E-field values between the two configurations. The E-field was collected ±15 mm about the centre in x (and the same in y) at a step-size of 0.1 mm in both x and y (see Fig. 2).

E-field Difference

The misalignment chosen for the analyses is where one plate is slanted as shown in Fig. 2. Since the plate is slanted, we expect the field to weaken (or get stronger) linearly towards the slanted-out (or slanted-in) plate. In other words, there is going to be a field gradient:

$$E_{\text{actual}} = E_{\text{ideal}} \pm \delta E(x)$$

The ‘extra’ field $\delta E(x)$ is of the form $ax + b$, where a is the gradient of the field and b is the extra field (or lack thereof). To give the reader a feel for the field deviation, an inward slant of even just 2 mm adds about 0.3 MV/m to the ideal field of 8 MV/m (even though this may only be 3.75% deviation from the ideal value, this could send a particle astray as much as 20 to 30 mm from the ideal trajectory, which is non-trivial for any type of storage ring).

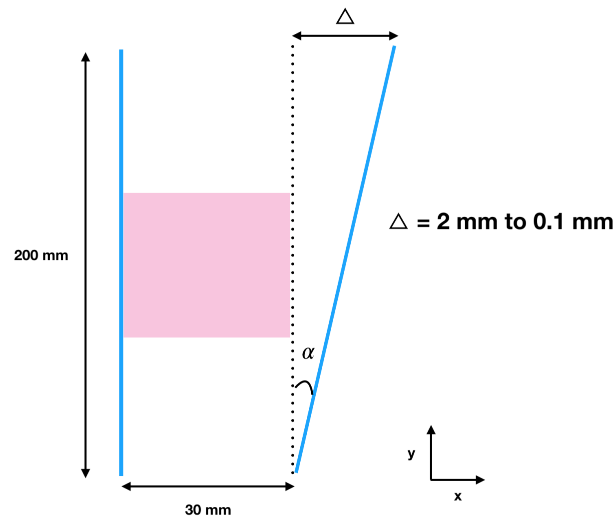

Figure 2: Shaded region is where E-field was collected for study (figure is not to scale).

To numerically get $\delta E(x)$, FEMM simulations were carried out for twenty different slant values (the Δ in Fig. 2), ranging from $\Delta = 0.1$ mm to 2 mm at a step-size of 0.1 mm. A sample $\delta E(x)$ for a misalignment of 0.1 mm is shown in Fig. 3, and the complete set of results are summarized in Fig. 4. With each $\delta E(x)$, a curve-fitting was done to get twenty pairs of a and b, and a curve-fitting was done for the (almost) linear correlation between a and b (so that one could generate random values of b for random misalignments and get the corresponding a’s). The wiggles in Fig. 3 are a numerical artefact resulting from the finite smallness of mesh-size.

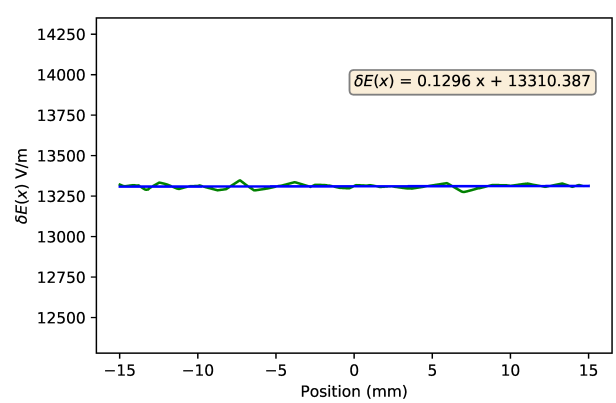

Figure 3: $\delta E(x)$ for an offset of 0.1 mm.

Since $\delta E(x)$ is a function of position, it can be incorporated into the transfer matrix, facilitating the analyses of closed orbit positions and tune shifts.

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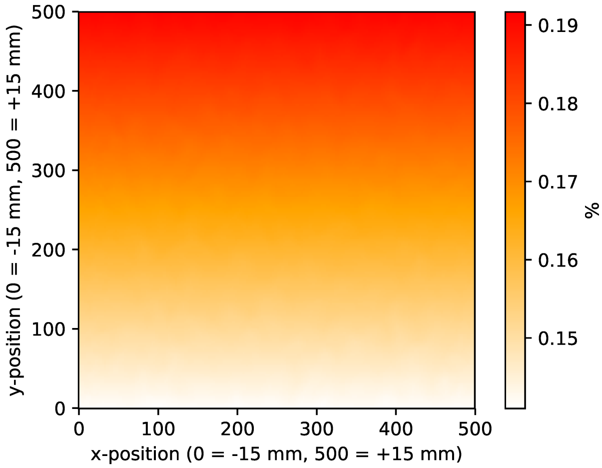

Figure 4: Map of $\delta E(x)/E(x)[\%]$ for a 0.1 mm offset.

TRANSFER MATRIX

Slanted Cylindrical Bend Element

For an ideal case where the concentric cylindrical plates are perfectly aligned parallel to each other and perpendicular to the orbit plane, the bend-field $\vec{E} = E_x\ \hat{x}$ (to a linear approximation) is [4] ,

$$E_x = -E_{x0}\frac{R_0}{R_0 + x} \approx -E_{x0}\left(1 - \frac{x}{R_0}\right) \tag{3}$$

But if the plates are slanted, there is the presence of a linear field $\delta E(x) = ax + b$ in addition to the cylindrical E-field. We then have the equation of motion for x to be,

$$\frac{\mathrm{d}^2 x}{\mathrm{d}s^2} + k_0 x = \frac{e}{pv}[ax + b] \tag{4}$$

whose solution, written in the form of 'misalignment incorporated' transfer matrix, is given by,

$$\begin{pmatrix} x_f \\ x_f' \\ 1 \end{pmatrix} = \begin{pmatrix} C(\sqrt{k}s) & \frac{1}{\sqrt{k}}S(\sqrt{k}s) & \frac{be}{kpv}(1 - C(\sqrt{k}s)) \\ -\sqrt{k}S(\sqrt{k}s) & C(\sqrt{k}s) & be/(\sqrt{k}pv) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ x_i' \\ 1 \end{pmatrix}$$

where s is the distance travelled by the particle in the bend-segment, $k = k_0 - ae/(pv)$, k_0 is $(3 - n)/R_0^2$ (n being 1 for cylindrical E-field), R_0 is the design radius, $C() = \cos()$ and $S() = \sin()$.

Closed Orbits

Since $\delta E(x)$ makes the field higher (or lower) than the ideal E-field, this makes the particle bend more (or lesser) than the bend-element ideally ought to. In other words, $\delta E(x)$ virtually acts like a dispersion. With misaligned plates, the particles are thus going to execute betatron oscillations not about the ideal trajectory but about a different trajectory (which would be the collection of closed orbits at each point along the ring). We know the order of magnitude (and the correlation) of a and b obtained from FEMM. Since the four corners of the plate can be slanted and displaced arbitrarily, the net $\delta E(x)$ is a superposition of 'a'-s and 'b'-s contributed by each corner. Hence the strategy is to pick random values for b from a probability distribution (and calculate the corresponding a) for each of the 40 bend elements. Since we can build and assemble plates in the real world to an order of $\Delta = \pm 0.1$ mm accuracy, the probability distribution of b was chosen to be a Gaussian of mean 0 and rms of $b_{0.1\text{ mm}} = 3 \times 10^4$ V/m.

In such fashion, random misalignments were given to each of the 40 bend elements a 10,000 times and the closed orbit (in x) at the injection point was computed for each of those 10,000 misaligned configurations (Fig. 5). We see that the closed orbit can be as big as 200 mm for a few cases, but for the far majority of the cases, the closed orbit is centred around zero, with a standard deviation of about ±50 mm. Though it maybe so, only about 18% of all closed orbits lie within the bending-plate aperture for a Gaussian distribution of slant offsets with mean = 0 and $\sigma_{\text{rms}} = 0.1$ mm. These wild closed-orbit deviations could be tackled using electrostatic correctors to make the beam stable, the incorporation of which is one of our future objectives.

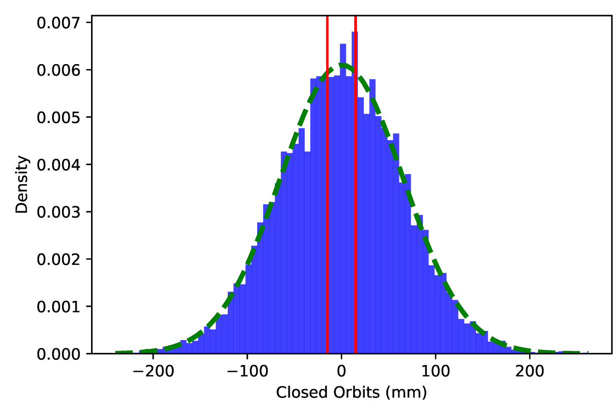

Figure 5: Histogram plot of closed orbits for 10,000 random misalignments (red vertical lines denote the bending-plate position).

Tune Shifts

Even though one might expect a tune-shift due to the gradient a, it turns out that the tune-shift is negligible. Since $\delta\nu$ for a change in gradient δk is given by,

$$\delta\nu = \frac{1}{4\pi}\beta_0 L_0\ \delta k \tag{5}$$

with $\delta k = ae/(pv) \approx 10^{-11}$ mm^{-2}, $\beta_{\max} = 44$ m and $L = 8.04$ m, this gives us a maximum tune-shift only on the order of $\delta\nu = 10^{-5}$.

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