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AVOIDING EMITTANCE DEGRADATION WHEN TRANSFERRING THE BEAM FROM AND TO A PLASMA-WAKEFIELD STAGE *

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Abstract

The plasma-wakefield acceleration technique is known to provide a very strong accelerating gradient (GV/m), up to three orders of magnitude higher than the conventional RF acceleration technique. The drawback is a relatively higher energy spread and especially a huge beam divergence at the plasma exit, leading to an irremediable and strong emittance degradation right after beam extraction from the plasma. In this article, we first derive the expressions showing the parameters governing the emittance growth in a transport line, which allows to recommend a strategy for mitigating it. The application to a typical configuration of the EuPRAXIA project at 5 GeV is then discussed. It turns out that with optimized beam injection, acceleration, extraction, and transport to the end user, the total emittance growth can be contained to less than 22%. Especially for injection to, or extraction from a plasma acceleration stage, optimizing the ramp length, whatever its shape, is particularly efficient, by minimizing the Twiss parameter γ of the beam.

INTRODUCTION

The huge electric field in a plasma excited by either a powerful laser beam or a particle beam, can accelerate electron beams to the multi-GeV energy range within a few tens of cm long plasma. This principle of particle acceleration by plasma wakefields is now well proven theoretically and experimentally [1]. Its maturity allows to go beyond a physics experiment to plan an accelerator facility capable of delivering a high-energy and good-quality beam to users [2]. For that, extracting the beam from the plasma stage and transferring it to the end users or to the next plasma acceleration stage are the key points.

It is indeed well known that emittance can grow very strongly, by a factor up to 10 or more, when the beam abruptly leaves a plasma wakefield area with very strong focusing to enter into free space. Although this is a very well-known phenomenon and although many theoretical studies [3-7] have been dedicated to study improvements through smooth, adiabatic plasma density transitions, neither consistent description nor efficient remedy exists yet to fully understand the emittance growth and avoid it for a practical case. It is not clear which emittance, trace or phase emittance, increases the most, in the free drifts or in the focusing elements of transport lines. Additionally, the adiabatic transitions were studied without beam loading effects, which are not applicable to high-charge beams.

For these reasons, a thorough study has been performed to understand emittance growth and to mitigate it in the general case. The details are published in [8] and the main conclusions are reported here. We first derive the expressions governing the emittance behavior in a transport line, exhibiting all the parameters involved in its growth mechanism. From that, a strategy for mitigating emittance growth can be recommended and its application to a specific case of EuPRAXIA [9] is then shown as an example.

TRACE AND PHASE EMITTANCE

Let's recall that the trace emittance ε_{tr} and the normalized trace emittance $\varepsilon_{tr,n}$ are defined as:

$$\varepsilon_{tr} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}, \tag{1}$$

$$\varepsilon_{tr,n} = \beta_r\gamma_r\varepsilon_{tr} \tag{2}$$

where x, x' are the particle position and momentum angle, β_r, γ_r are the relativistic coefficients (not to be confused with the Twiss parameters below noted as α, β, γ), and $\langle\ \ \rangle$ denotes the statistic variance.

Equivalently, the phase emittance and the normalized phase emittance are defined as:

$$\varepsilon_{ph} = \sqrt{\langle x^2\rangle\langle p_x^2\rangle - \langle xp_x\rangle^2}, \tag{3}$$

$$\varepsilon_{ph,n} = \frac{\varepsilon_{ph}}{m_0 c} \tag{4}$$

where p_x is the particle momentum, m_0 the electron rest mass and c the speed of light.

If transverse and longitudinal distributions are independent, which is generally the case, algebraic calculations show that

$$\varepsilon_{ph,n}^2 = \varepsilon_{tr,n}^2\left(\frac{\overline{p_z^2}}{p_0^2} + \alpha^2\frac{\sigma_p^2}{p_0^2}\right) \tag{5}$$

where p_z is the longitudinal momentum, σ_p^2 its variance, and $\overline{p_z^2}$ denotes the average.

When in addition $\alpha = 0$, i.e. at a beam waist, where the beam changes from divergent to convergent and vice versa, the two normalized emittances are equal:

$$\varepsilon_{ph,n} = \varepsilon_{tr,n} \quad when\ \alpha = 0. \tag{6}$$

This is often the case at a focusing element and it is recommended to perform emittance measurement immediately after those locations in order to get relevant results [5]. It is worth noting that in the general case, $\varepsilon_{ph,n}^2$ and $\varepsilon_{tr,n}^2$ differ by the term $\alpha^2\sigma_p^2$, meaning that the two emittances are even more different from each other for a higher energy spread and when the beam is more divergent or else

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convergent. More generally speaking, Eq. (5) and (6) show that phase and trace emittances are linked, and thus the growth of both should be mitigated.

EMITTANCE EVOLUTION IN A TRANSPORT LINE

Let us study now the evolution of the two emittances in a transport line. The latter being a combination of drifts and focusing elements, their behavior can be completely described by examining the transport through a free drift where there is no slope change and that through a thin lens where there is no position change.

Some algebraic operations allow to show that through a free drift of length l, the trace emittance remains constant whereas the phase emittance varies (subscript 0 corresponds to the drift entrance and no subscript to the drift exit):

$$\varepsilon_{tr,n}^2 - \varepsilon_{tr0,n}^2 = 0, \tag{7}$$

$$\varepsilon_{ph,n}^2 - \varepsilon_{ph0,n}^2 = \varepsilon_{tr0,n}^2 \left(\frac{\sigma_p}{p_0}\right)^2 \gamma_0 l(\gamma_0 l - 2\alpha_0). \tag{8}$$

Inversely, through a thin lens of integrated normalized gradient k, the phase emittance remains constant whereas the trace emittance varies (subscript 0 corresponds to the lens entrance and no subscript to the lens exit):

$$\frac{\varepsilon_{tr,n}^2 - \varepsilon_{tr0,n}^2}{\varepsilon_{tr0,n}^2} = \beta_0^2 k^2 \left(\frac{\sigma_p}{p_0}\right)^2, \tag{9}$$

$$\varepsilon_{ph,n}^2 - \varepsilon_{ph0,n}^2 = 0. \tag{10}$$

Equation (8) points out that the phase emittance variation in a drift is higher when the initial trace emittance, or the energy spread, or the Twiss parameter γ_0 is bigger. Those parameters are known to be particularly big in wakefield acceleration, such that it is often concluded that significant emittance growth is unavoidable when transferring the accelerated beam to a user. But we propose nevertheless to take advantage of this equation by suggesting to use the plasma downramp for minimizing γ_0 (Twiss parameter at drift entrance) in order to minimize the emittance growth.

Equation (9) exhibits the well-known chromaticity effect. Due to energy spread, there is a jump of trace emittance when crossing a focusing element. To limit this jump, k_0^2 should be as low as possible (smoothest focusing) and β_0^2 (or the beam size) should be as small as possible. Notice that the latter condition can be met by minimizing the Twiss parameter γ in the drift preceding the lens. Taking into account the same result highlighted just above, minimizing γ at the plasma exit is the key point: it is doubly beneficial, for minimizing phase emittance growth when going through a free drift and also for minimizing trace emittance growth when crossing a thin lens.

The particle tracking code TraceWin [10] has been used to check all these emittance behaviors (Fig. 1):

- The trace emittance is constant in the drifts and experiences an abrupt jump in the quadrupoles, a jump that is bigger for bigger integrated gradient Gl.

- The phase emittance increases in the drifts but only when the beam size is varying, whether it is convergent or divergent, and varies very little in the quadrupole.

- The two emittances are equal only and wherever $\alpha = 0$. This occurs at two precise locations in the case of strong focusing with two beam waists (top graph), or on a long distance where the beam envelope is parallel (middle graph), whereas the two emittance curves never cross when there is no beam waist (bottom graph).

Those behaviors are in total agreement with the above formulas. Quantitatively, discrepancies between analytical formulas and numerical tracking are less than 15% [8] although the weak dependence between transverse and longitudinal coordinates, and the finite length quadrupole, both of which being neglected in the analytical model.

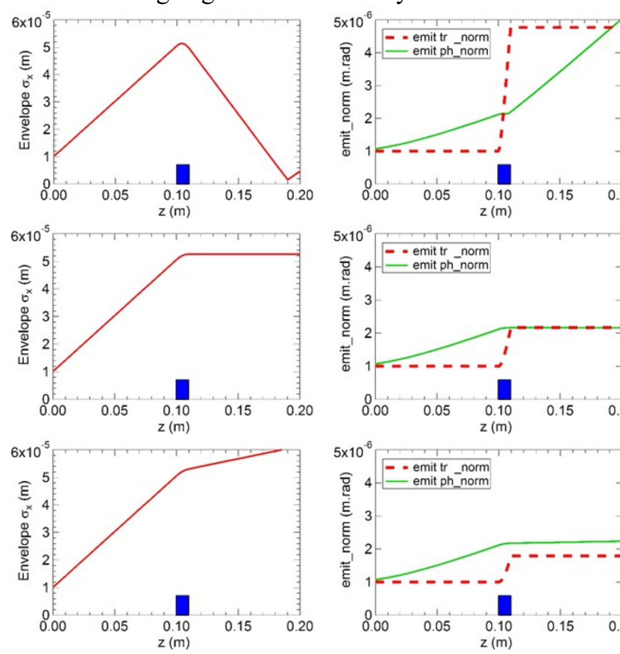

Figure 1: Variation of beam size (left), normalized phase and trace emittances (right) along a structure consisting of a 0.1 m drift, followed by a 0.01 m quadrupole (blue rectangle) and a 2nd drift. Three cases of quadrupole strengths are shown from top to bottom: $Gl = 330, 130, 100\,T$, the second one being chosen to obtain a parallel beam after the quadrupole.

MINIMIZING EMITTANCE GROWTH

The above studies show precisely the relation between the two emittances, which emittance grows in which context, and all the parameters governing these growths. Equations (8) and (9) enable furthermore to distribute different roles to each stage of a multistage accelerator in the emittance preservation task:

1. Minimizing emittance and energy spread during acceleration, therefore this should be the exclusive role of the acceleration part.

2. Minimizing the Twiss parameter γ_0 at the transfer line entrance, therefore this should be the exclusive role of the plasma downramp, with the reservation that the latter would not itself induce significant emittance growth.

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3. Minimizing the total length and the integrated focusing strength in the transfer line, therefore this should be the exclusive role of the focusing elements in the transfer line.

It is important here to insist on the exclusive role of each stage. Once the best is achieved at each of those three components as recommended, the emittance growth is ensured to be minimal. The advantage is that the optimization can be done separately at each stage without minding about the nature or the capacity of the next stage. On the contrary, if it is not done correctly at a given stage, it cannot be compensated elsewhere.

APPLICATION TO EuPRAXIA

The EuPRAXIA project aims at designing a plasma-based accelerator to 5 GeV with ambitious requirements of high beam charge and quality [9]. Minimizing emittance growth during beam injection, acceleration, extraction, and transport to the end user is essential.

Minimizing emittance growth [11] and energy spread [12] in the acceleration plasma stage has already been carried out with great care. But, as usual at the plasma exit, the Twiss parameter $\gamma = 400\ \mathrm{m}^{-1}$ is still too big, i.e. its divergence is too big which, combined with the 1% energy spread, makes that it cannot be transported without significant emittance degradation. According to the above recommendations, a density down ramp with different types of profiles and lengths have been considered. It turns out that regardless of the profile type, it is enough to roughly tune the length of the profile to drastically decrease γ while deteriorating only marginally the emittance [8]. Figure 2 shows the effect of ramp length on γ and emittance. For example, an exponential shape with 7 mm characteristic length allows decreasing γ from 400 down to 80 m^{-1}. Symmetrically on the up ramp side, optimizing its length will allow to relax the required beam size at injection, by up to a factor of 10 from the tiny beam size (~1 µm) needed in the acceleration part. Simulations within the plasma stage have been done with the 3D-PIC code WARP in the boosted frame [13].

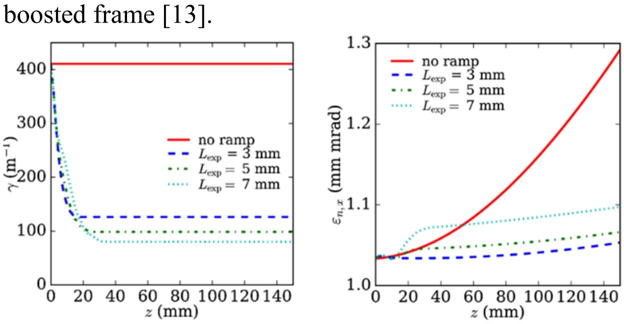

Figure 2: Evolution of the Twiss parameter γ and the emittance $\varepsilon_{ph,n}$ along the downramp (35 mm) and the following drift, for different exponential density profile lengths, compared to the case without ramp. $z = 0$ mm is the downramp entrance.

All of these aspects will greatly help to lower the needed focusing strengths of the transport lines, as at both of the plasma ends, the beam size will be the largest and the beam divergence the lowest. However, the transport line must also be correctly optimized and designed in order to fully benefit from these advantages. For such a line, the inputs are the beam parameters at entrance and the constraints are the beam parameters at exit required either by the next plasma stage or the end user. In addition, as said above, the transport line should be as short as possible and should provide the smoothest focusing possible to limit the emittance growth. Gathering all that, three constraints in each transverse direction must be achieved at the end of the transfer line: the RMS beam size, divergence and emittance. That means six constraints in total and therefore the transfer line should be designed with six quadrupoles. Using more quadrupoles is not recommended [8], unless a longer line is asked for including diagnostics and chicanes.

For the injection transfer line at 150 MeV linking two plasma stages that have beam sizes and divergences in the same order of magnitude, the best is to use two triplets with antisymmetric polarities. As the divergence is rather high, short and strong magnets are needed. Two triplets including six permanent magnets should be used. A section of 0.7 m is enough, with a place reserved to diagnostics in between the triplets, where beam envelopes are parallel.

For the extraction transfer line at 5 GeV, as the required beam parameters on the user side, in particular for FEL operation, are very different from those on the plasma side - the beam size is much bigger for the former whereas the beam divergence is much bigger for the second - a triplet of permanent magnets will be used to capture the beam followed by a triplet of electromagnets. This offers the needed flexibility to shape the beam according to various user demands. A section of 4 m is enough, but an 8 m section with one more electromagnet is adopted in order to include a C-chicane for escaping the laser beam, and the place for steerers and diagnostics.

As a result, for a relatively high charge beam up to 30 pC, the emittance growth in the up ramp is about ~4%, during acceleration ~3%, in the down ramp ~4% and in the transfer line ~10%. Therefore the overall emittance growth through all the sections is ~22% of that at plasma entrance.

CONCLUSION

To go beyond a physics experiment of plasma-based acceleration in order to design a plasma-based accelerator, extracting the accelerated beam from the plasma and transporting it toward either a next plasma stage or the end user are the key points. The consistent formalism derived in this article allowed to exhibit exhaustively the parameters involved in the emittance growth mechanism within a transport line. A clear strategy for mitigating emittance growth can be deduced, assigning exclusive roles to the plasma stage, the plasma ramps, and the transport lines. Especially for the plasma ramp, minimizing the Twiss parameter γ by tuning its length, whatever its density profile, is very efficient for minimizing the emittance growth. The application of this strategy to the 5 GeV plasma stage of the EuPRAXIA project showed that the total emittance growth through injection, acceleration, extraction and transport to the FEL application can be contained within less than 22%.

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