

WAKE-T: A FAST PARTICLE TRACKING CODE FOR PLASMA-BASED ACCELERATORS*

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Abstract

The design and study of plasma-based accelerators relies typically on costly 3D Particle-In-Cell (PIC) simulations due to the complexity of the laser-plasma and beam-plasma interactions. However, under certain assumptions, more efficient and simple models can be implemented to describe the dynamics of the accelerated beams. Wake-T (Wakefield particle Tracker) is a new code for analytical and numerical particle tracking in plasma-based accelerators which is orders of magnitude faster than conventional PIC codes. This allows for fast parameter scans and is well suited for the initial design and optimization of these novel accelerators.

INTRODUCTION

Plasma-based accelerators (PBAs) have demonstrated the capability of achieving GeV electron beams in only centimeter scales [1], orders of magnitude shorter than with conventional radiofrequency (RF) technology. PBAs thus offer an attractive solution towards highly compact and cost-effective accelerators with broad applications in areas such as photon science and high-energy physics.

The design of these novel devices is however highly non-trivial. This is due to the complex laser-plasma and beam-plasma interactions, which typically require the use of computationally demanding Particle-in-Cell (PIC) codes for accurate predictions. This implies that performing large parameter scans to, for example, optimize the performance and assess the robustness of PBAs (as routinely done for RF accelerators) is prohibitively expensive. However, in cases where certain complex processes such as electron self-injection [2,3] are not relevant, more computationally efficient numerical methods to obtain the evolution of the witness beam can be implemented by considering simplified models for the plasma wakefields. This is particularly relevant when the electron beams are externally injected, as the plasma-accelerating stages act only as energy boosters and can be characterized by their wakefields.

We present here the simulation tool Wake-T (Wakefield particle Tracker), a new particle tracking code specially designed for PBAs which offers orders of magnitude faster simulations than conventional PIC codes. Although based on simplified physical models, this tool allows for fast parameter scans and is well suited for the initial design of PBAs, including complex multi-stage setups [4], as well as

performing sensitivity and tolerance studies. Wake-T is an open-source project [5] and entirely written in Python.

PARTICLE TRACKING IMPLEMENTATION IN WAKE-T

The core idea behind Wake-T is to track the evolution of charged particles in the presence of externally prescribed electromagnetic fields, such as the plasma wakefields, which are determined through simplified models instead of full PIC plasma simulations. The 6D phase-space evolution of the beam particles is then obtained by means of analytical or numerical solutions of the equations of motion

$$\frac{d\mathbf{p}}{dt}(t) = -e[\mathbf{E}(\mathbf{r}, t) + \mathbf{v}(t) \times \mathbf{B}(\mathbf{r}, t)], \quad (1)$$

$$\frac{d\mathbf{r}}{dt}(t) = \frac{\mathbf{p}(t)}{m\gamma(t)}, \quad (2)$$

where $\mathbf{r} = (x, y, z)$, $\mathbf{v} = d\mathbf{r}/dt$ and $\mathbf{p} = m\gamma\mathbf{v}$ are the particle position, velocity and momentum, t is the time, e and m are the electron charge and mass and $\gamma = 1/\sqrt{1 - (v/c)^2}$ is the relativistic Lorentz factor of the particles, with c being the speed of light. The electric and magnetic fields experienced by the particles are given by $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$, respectively.

In the current state of development, the fields experienced by a particle beam in a plasma accelerating stage are determined by simple analytical models for the linear and blowout regimes [6,7]. These models do not currently include effects such as beam loading or driver evolution. Thus, as an alternative, it is also possible to externally specify the fields, which can be obtained from actual PIC simulations (using the VisualPIC libraries [8]), or by manually setting the values of E_z , the electric field slope $E'_z = \partial E_z / \partial z$ and the focusing gradient $g = \partial(E_x - cB_y) / c\partial x = \partial(E_y + cB_x) / c\partial y$ (for $v \simeq c$) around the particle beam.

After the fields have been specified, the particle evolution is obtained by solving the equations of motion either by using a fast analytical model or a more general numerical algorithm. The analytical model is based on the theory presented in [9], which takes into account the coupling of longitudinal and transverse particle dynamics. This set of analytical expressions allows a fast computation of the 6D particle distribution at any time $t = t_f$ simply from the initial conditions at $t = t_0$, and provides accurate solutions as long as the particle motion is relativistic and the fields have not significantly evolved between t_0 and t_f . More accurate solutions can be obtained if the numerical model is used, which solves the equations of motion with a Runge-Kutta

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method of 4th order (RK4) and is not limited by the approximations of the analytical model. A simple diagram of the basic implementation of this method can be seen in Fig. 1.

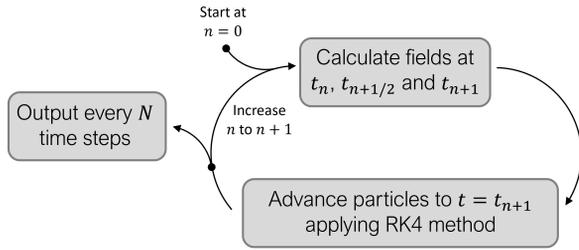


Figure 1: Simple schematic representation of the numerical algorithm based on the RK4 method. The time at the iteration n is defined as $t_n = t_0 + n\Delta t$, where t_0 is the initial time at the beginning of the computation and Δt is the time step.

In addition to the plasma acceleration stages, Wake-T can also simulate the beam evolution in other elements such as plasma up- and downramps (which are treated separately from the plasma stage itself), plasma lenses, drifts and conventional magnets, including dipoles, quadrupoles and sextupoles. This allows the simulation of complex beamlines for PBAs, which can include multiple plasma stages and beam transport elements. The particle tracking in dipoles, quadrupoles and sextupoles is performed through second order transport maps, whose implementation is based on the Ocelot code [10]. At the moment, no collective effects such as space-charge or coherent synchrotron radiation are included.

SAMPLE STUDIES AND VALIDATION AGAINST PIC SIMULATIONS

The simplified physical models in Wake-T allow for extremely fast simulations with respect to conventional PIC codes, with typical computation times on a standard PC ranging from a few seconds up to a few minutes depending on the number of beamline elements and particles. This code is therefore well suited for large parameter scans which can be used for the optimization and initial simulation of PBAs as well as for performing tolerance studies. In this regard, two relevant case studies are presented in this section and validated against PIC simulations performed with FBPIC [11]. The first case considers the optimization of the plasma upramp for matching an externally injected electron beam and minimizing its emittance growth during acceleration. The second case studies the sensitivity of the final beam emittance to transverse beam offsets at injection. Both cases consider the same electron beam, which corresponds to a simulated working point of the ARES linac at SINBAD [12,13]. The transverse phase-space of this beam at the beginning of the plasma can be seen in Fig. 2. This particle distribution has a total charge of 5.7 pC, an energy of 100 MeV with 0.5% spread, a duration $\tau_{FWHM} = 5.0$ fs, a peak current $I_p = 1.1$ kA, a normalized emittance $\epsilon_{n,x} = 0.52 \mu\text{m}$ and $\epsilon_{n,y} = 0.44 \mu\text{m}$, and Courant-

Snyder parameters $\beta_x = 1.7$ mm, $\beta_y = 1.9$ mm, $\alpha_x = 0.67$ and $\alpha_y = -0.72$. The beam is injected on-axis such that $\langle x \rangle = \langle y \rangle = \langle p_x \rangle = \langle p_y \rangle = 0$, where $\langle \rangle$ represents the average of the distribution.

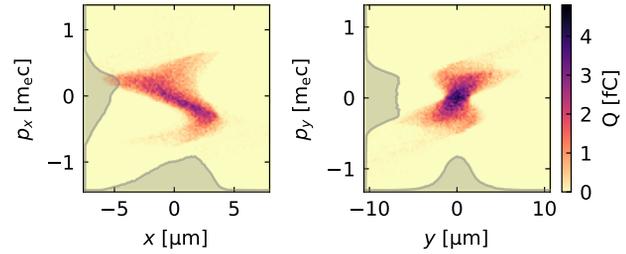


Figure 2: Transverse phase-space in the x and y planes of the electron beam at injection with $\langle x \rangle = \langle y \rangle = \langle p_x \rangle = \langle p_y \rangle = 0$.

Plasma Upramp Optimization

Properly tailoring the vacuum-to-plasma transition (or upramp) is essential for coupling an external electron beam into a plasma acceleration module without emittance degradation [14-16]. This upramp acts as a focusing element which, in an optimal case, matches the Courant-Snyder parameters of the beam to the focusing fields in the plasma stage [17,18]. Properly optimizing the length and shape of this ramp is essential for achieving this objective.

The sample case presented here consists on using Wake-T to find the optimal ramp length which minimizes the emittance growth during acceleration. The plasma stage has a plateau density $n_p = 10^{17} \text{ cm}^{-3}$ and an upramp of length L_{ramp} which, for simplicity, is assumed to have an initial density $n_{p,0} = 5 \times 10^{15} \text{ cm}^{-3}$ and grows exponentially as $n_{p,\text{up}}(z) = n_p e^{az/L_{\text{ramp}}}$, where $a = \ln(n_p/n_{p,0})$. The stage is driven by a 40 fs laser pulse with a peak normalized vector potential $a_0 = 3$, a beam waist $w_0 = 40 \mu\text{m}$ and a wavelength $\lambda_0 = 0.8 \mu\text{m}$, which corresponds to a peak power of 480 TW and a 20 J energy. This parameters are in the range of those considered in the EuPRAXIA project [19]. PIC simulations with FBPIC predict that such a laser generates a blowout in the plasma stage with focusing fields $g \approx 2.6 \text{ MT/m}$, which correspond to a matched beta function $\beta_m = 0.36$ mm. This is ~ 5 times smaller than that of the beam, so optimizing the upramp is therefore essential to mitigate the mismatch.

Using Wake-T, a set of 30 simulations have been carried out considering different ramp lengths ranging from 0 (no ramp) to 1 cm, as seen in Fig. 3a. In this simulations, which were performed with the numerical algorithm, it was assumed that the focusing fields along the upramp correspond to a blowout, i.e $g(z) = (e/2c^2\epsilon_0) n_{p,\text{up}}(z)$, and that the longitudinal fields can be neglected. Inside the plasma stage, it was simply assumed that $g = 2.6 \text{ MT/m}$, $E_z = -33 \text{ GV/m}$ and $E'_z = 1.4 \times 10^{15} \text{ V/m}^2$ at the beam position, as expected from PIC simulations. The results can be seen in Fig. 3b, where they are also validated against FBPIC simulations. In the FBPIC simulations the laser focus was set at the begin-

ning of the plateau and a transverse parabolic density profile is used to guide the laser. This can be seen schematically in Fig. 3a, where the laser envelope $L_{\text{env}}(z) = \pm w(z)$, with $w(z)$ being the beam radius, is represented. The emittance values shown in Fig. 3b are measured after a propagation distance of 2 cm inside the plateau. The agreement between Wake-T and FBPIC simulations is excellent, showing how this tool is well suited for fast parameter scans. As a comparison, the total computing time for each Wake-T simulation was less than 1 minute using 2 cores in a conventional PC, while each FBPIC simulation required ~ 4 hours in a dedicated GPU node in a high-performance computing cluster. The results also show that the emittance growth can be effectively mitigated by tuning the ramp length and that, in this case, it quickly converges to a minimum for $L_{\text{ramp}} \gtrsim 0.2$ cm.

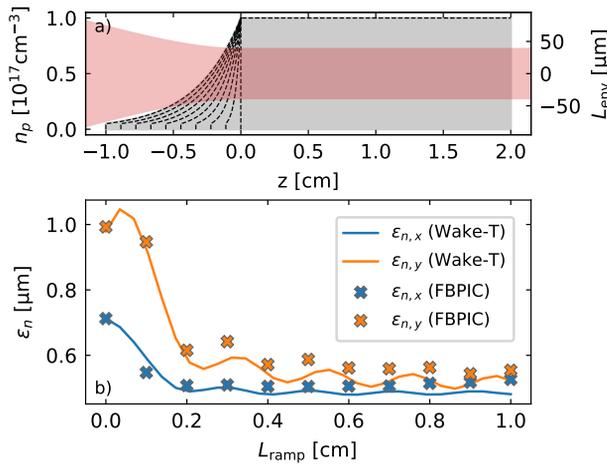


Figure 3: (a) On-axis plasma density profile showcasing the different ramps used. The approximate evolution of the laser envelope, L_{env} , in the FBPIC simulations is also included. (b) Normalized emittance of the beam in both transverse planes at $z = 2$ cm for different ramp lengths.

Analysis of Sensitivity to Injection Offsets

In addition to beam matching, another critical aspect of external injection into PBAs is that the beams should be ideally injected on-axis, i.e. $\langle x \rangle = \langle y \rangle = 0$, and with $\langle p_x \rangle = \langle p_y \rangle = 0$ in order to prevent, among other problems, emittance growth during acceleration. When designing a PBA it is therefore of key importance to determine the sensitivity to transverse injection offsets and establish tolerance limits. However, performing this kind of studies only with PIC simulations can be extremely costly.

In this section, the validity of Wake-T for performing sensitivity studies is tested. The same setup as in the previous case is used, considering an upramp length of 0.4 cm. In each simulation the initial centroid position of the beam is displaced by a certain offset, $\langle x \rangle = x_{\text{off}}$, and the final emittance in both planes is measured again at $z = 2$ cm. Injection

offsets between -10 and $10 \mu\text{m}$ have been tested. The results, displayed in Fig. 4, show again good agreement between the Wake-T and FBPIC simulations and predict a quadratic dependence of the final emittance on the initial transverse offset. It is also interesting to note that the minimum emittance is achieved for a non-zero offset. This is due to the asymmetric profile of the beam in the x plane, in which although $\langle x \rangle = \langle p_x \rangle = 0$ in the case with no offset, the peak of the distribution both in position and momentum is not centered, as seen in Fig. 2. Instead, an offset $x_{\text{off}} \approx 4 \mu\text{m}$ seems to minimize the misalignment of the peak and reduce the emittance growth.

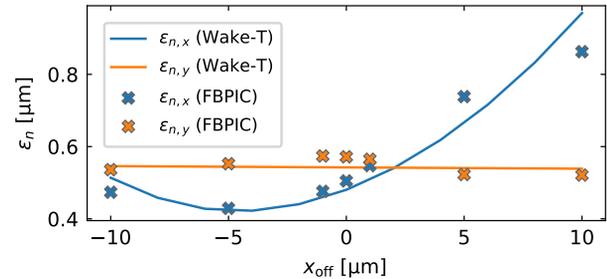


Figure 4: Normalized emittance of the beam in both transverse planes at $z = 2$ cm for different injection offsets.

CONCLUSION

The new fast particle tracking code Wake-T, specifically designed for PBAs, has been presented. The main working principle of the code and its basic functionality have been described. The results of two particular case studies, which showcase the potential of the code for fast parameter scans, have been shown and validated against full PIC simulations with the FBPIC code. The good agreement between Wake-T and FBPIC simulations, coupled with the vast reduction in computing time and resources (~ 1 min per Wake-T simulation in a regular PC versus ~ 4 hours per FBPIC simulation in a high-performance computing cluster) make this code an attractive option for fast parameter scans of PBAs. These scans, which would otherwise be prohibitively expensive with conventional PIC codes, can be used for the initial optimization of these devices as well as performing sensitivity and tolerance studies, among other applications. Current limitations of the code, such as the lack of beam loading effects, are planned to be improved in future releases with more accurate wakefield models. It would also in principle be possible to add support for other particle species such as positrons or protons.

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