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ELECTROMAGNETIC FIELD OF A CHARGE MOVING THROUGH A CHANNEL IN MAGNETIZED PLASMA*

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Abstract

The electromagnetic field of a point charge moving along a cylindrical channel axis in a boundless magnetized plasma is considered. A model of cold plasma in an external infinitely strong magnetic field is used. The main focus is on the analysis of the field near the charge trajectory. A deceleration force acting on the charge from the generated field is investigated. The obtained results are compared with those in the case of a charge moving along the channel inside a cold isotropic plasma. It is shown that there is a range of charge velocities where the decelerating force is less in the case of magnetized plasma.

INTRODUCTION

In recent years, beam-plasma interactions are of essential interest mainly due to their prospective applications in beam-driven plasma wakefield acceleration. For example, one can expect accelerating fields up to tens of GV/m which is at least an order higher compared to conventional rf cavity technology. To now, probably the most promising plasma-based accelerating scheme is the hollow plasma channel scheme where recent success with two proton bunches has been achieved [1]. Various aspects of this technique are extensively studied, including transverse wakes effects [2], beam-breakup instability [3] and processes connected with self-modulation of long proton driver bunches [4–7]. In this context, it is useful to investigate possibilities for additional tuning the structure of the accelerating field by the external magnetic field applied to the plasma. In particular, such a possibility has been shown for a charge traversing the homogeneous plasma [8]. In this report, we take into account a hollow channel in plasma and consider the specific case of strongly magnetized plasma.

ANALYTICAL RESULTS

We consider the boundless cold magnetized plasma with the vacuum cylindrical channel of radius a. The cylindrical coordinate system having z-axis directed along the channel axis is used. A point charge q moves with constant velocity $\vec{v} = c\beta\vec{e}_z$ (c is the speed of light in a vacuum) along the channel axis. It is assumed that external magnetic field is infinitely large thus the plasma permittivity tensor contains only diagonal elements

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix},$$

$$\varepsilon_\perp = 1, \quad \varepsilon_\parallel = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)},$$

where $\omega_p^2 = 4\pi N e^2/m$ is the plasma frequency (N means the electron density, e and m are the electron charge and mass respectively), ν is the effective collision frequency. We will consider the case of collisionless plasma, however an infinitely small value of parameter ν will be used in further analytical investigation to determine the relative position of the integration path and functions singularities. It is also assumed the plasma is described by magnetic permeability $\mu = 1$.

The electromagnetic field of a charge (E_r, E_z, H_φ) in the plasma with channel can be written in the following form [9]:

$$\left(\vec{E}, \vec{H}\right) = \begin{cases} \left(\vec{E}^{(i)}, \vec{H}^{(i)}\right) + \left(\vec{E}^{(r)}, \vec{H}^{(r)}\right) & \text{for } r \le a, \\ \left(\vec{E}^{(t)}, \vec{H}^{(t)}\right) & \text{for } r > a, \end{cases}$$

where $\vec{E}^{(i)}$, $\vec{H}^{(i)}$ is the "incident" field (the field of the charge in the unbounded vacuum), $\vec{E}^{(r)}$, $\vec{H}^{(r)}$ is the "reflected" field conditioned by the channel boundary at $r = a$ and the $\vec{E}^{(t)}$, $\vec{H}^{(t)}$ is the "transmitted" field in plasma. For the sake of brevity, we give there only the longitudinal field component:

$$E_z = \int_{-\infty}^{+\infty} E_{z\omega} e^{-i\omega t} d\omega = -\frac{iq}{\pi} \int_{-\infty}^{+\infty} d\omega \frac{k^2}{\omega} \exp\left(\frac{i\omega\zeta}{v}\right) \times \begin{cases} \left(K_0(kr) - R(\omega) I_0(kr)\right) & \text{for } r \le a \\ \dfrac{\pi s}{2k\varepsilon_\parallel} T(\omega) H_0^{(1)}(sr) & \text{for } r > a \end{cases} \qquad (1)$$

Here $\zeta = z - vt$, $k^2(\omega) = \omega^2(1-\beta^2)/v^2$, $s^2(\omega) = \omega^2\varepsilon_\parallel(1-\beta^2)/v^2$, $k(\omega) = \sqrt{k^2(\omega)}$ ($\mathrm{Re}\,(k(\omega)) > 0$ for $\omega \in \mathbb{R}$), $s(\omega) = \sqrt{s^2(\omega)}$ ($\mathrm{Im}\,(s(\omega)) \ge 0$ for $\omega \in \mathbb{R}$); $I_0(x)$, $K_0(x)$ are modified Bessel functions, $H_0^{(1)}(x)$ is the Hankel function. The reflection and transmission coefficients $R(\omega)$, $T(\omega)$ are determined by the continuity conditions for the tangential field components at the boundary $r = a$:

$$R(\omega) = \frac{\varepsilon_\parallel k K_0(ka) H_1^{(1)}(sa) + s H_0^{(1)}(sa) K_1(ka)}{\varepsilon_\parallel k I_0(ka) H_1^{(1)}(sa) - s H_0^{(1)}(sa) I_1(ka)},$$

$$T(\omega) = -\frac{2\varepsilon_\parallel}{\pi a \left(\varepsilon_\parallel k I_0(ka) H_1^{(1)}(sa) - s H_0^{(1)}(sa) I_1(ka)\right)}.$$

The main attention further will be focused on the analysis of decelerating force $F_z = q\, E_z|_{\zeta=r=0}$ acting on the charge. Therefore, it is necessary to determine the integrand of the expression (1) on the complex plane ω to calculate correctly

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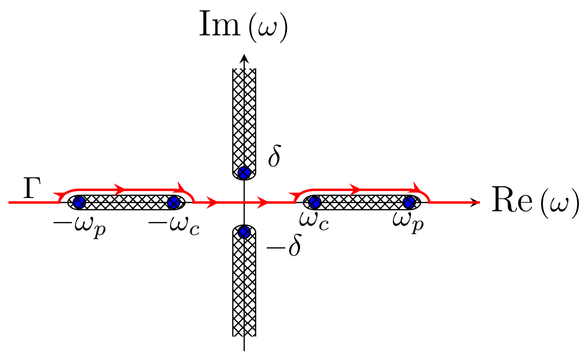

Figure 1: The location of the integration path Γ, the branch points $\pm\delta$, $\pm\omega_{c,p}$ on the complex plane ω in the case of magnetized plasma. The hatchings denote cuts on the complex plane.

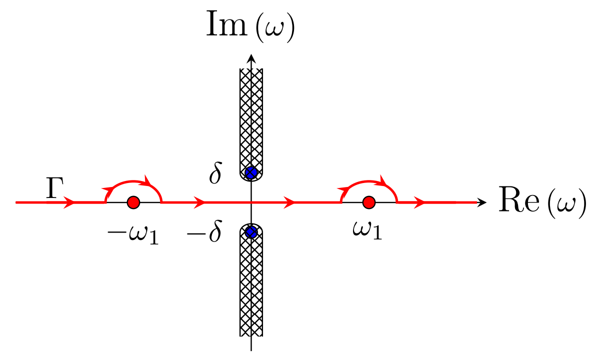

Figure 2: The location of the integration path Γ, the branch points $\pm\delta$ and the poles $\pm\omega_1$ on the complex plane ω in the case of isotropic plasma. The hatchings denote cuts on the complex plane.

the longitudinal field component on the charge. Figure 1 represents the location of the integration path Γ on the complex plane ω and the location of the integrand singularities in the expression (1): the branch point $\pm\delta$, $\pm\omega_{c,p}$ of functions $k(\omega)$ and $s(\omega)$. It should be noted that in the case under consideration parameters δ and ω_c tend to zero. Based on the properties of function $E_{z\omega}|_{\zeta=0}$, it can be shown that only two segments of all integration path Γ passing along the upper edge of the cuts make a contribution to the integral (1)

$$E_z|_{\zeta=0} = \int\limits_{C_1,C_2} E_{z\omega}|_{\zeta=0} \exp\left(-i\omega t\right) d\omega, \tag{2}$$

where C_1 means the integration segment $\left[-\omega_c, -\omega_p\right]$ and C_2 means the integration segment $\left[\omega_c, \omega_p\right]$. We next use the property $E_{z\omega}(-\bar{\omega}) = \bar{E}_{z\omega}(\omega)$ (the bar means the complex conjugation) and write the expression (2) in the form

$$E_z|_{\zeta=0} = 2\mathrm{Re} \int\limits_{C_1} E_{z\omega}|_{\zeta=0} \exp\left(-i\omega t\right) d\omega.$$

The result is

$$E_z|_{\zeta=r=0} = -\frac{2q}{\pi} \int\limits_{C_1} \frac{k^2(\omega)}{\omega} I_0\left(k(\omega) r\right) \mathrm{Im}\left(R(\omega)\right) d\omega. \tag{3}$$

The same analysis can be conducted for the point charge moving along the vacuum channel in the cold isotropic plasma which is described by dielectric permittivity $\varepsilon = \varepsilon_\parallel$ and magnetic permeability $\mu = 1$. The longitudinal component of the electromagnetic field in cold plasma with channel has the form coinciding with (1). Functions $k(\omega)$, $R(\omega)$ and $T(\omega)$ are the same as it was described above. Function $s(\omega)$, however, differs. Now $s^2(\omega) = \omega^2\left(\varepsilon_\parallel\beta^2 - 1\right)/v^2$, $s(\omega) = \sqrt{s^2(\omega)}$, $\mathrm{Im}(s(\omega)) \geq 0$ for $\omega \in \mathbb{R}$. This leads to a significant changes at the complex plane ω. Function $E_{z\omega}$ has two poles $\pm\omega_1$ determined by the dispersion equation $F_{disp} \equiv \varepsilon_\parallel k I_0(ka) H_1^{(1)}(sa) - s H_0^{(1)}(sa) I_1(ka) = 0$ and two branch points $\pm\delta(\delta \to 0)$ (see Fig. 2). Taking into account the infinitely small effective collision frequency ν we obtaind that the integration path Γ passes above the poles. Now only two semi-circles $C_{1,2}$ make a contribution to the longitudinal field component on the charge:

$$E_z|_{\zeta=0} = \oint\limits_{C_{1,2}} E_{z\omega}|_{\zeta=0} \exp\left(-i\omega t\right) d\omega. \tag{4}$$

Calculating integrals (4) using the residue theorem we finally obtain:

$$E_z|_{\zeta=r=0} = -2q\left[\frac{k^2}{\omega}\left(\frac{dF_{disp}}{d\omega}\right)^{-1}\left(\varepsilon_\parallel k K_0(ka) H_1^{(1)}(sa) + s H_0^{(1)}(sa) K_1(ka)\right)\right]\Bigg|_{\omega=\omega_1}, \tag{5}$$

$$\frac{dF_{disp}}{d\omega} = \frac{2\omega_p^2 k^3}{\omega^3 s\left(1-\beta^2\right)} I_0(ka) K_1(sa) + \frac{a k^3 \omega_p^2}{\omega^3 s\left(1-\beta^2\right)} \times I_0(ka) K_0(sa) - \frac{a k^2 \omega_p^2}{\omega^3} I_1(ka) K_1(sa).$$

NUMERICAL RESULTS

Figure 3 shows the dependence of the decelerating force $|F_z|$ (in units $q^2\omega_p^2/c^2$) on the relative charge velocity β. The decelerating force is calculating on the basis of analytical results (3) and (5). Red line corresponds to the charge moving in magnetized plasma, blue line – to the charge in isotropic plasma. As can be seen, the decelerating force is less in magnetized plasma for small channel radius. The channel widening leads to the appearance of the velocity range in which the decelerating force is less in the isotropic plasma. In this connection, it is interesting to note that ultra-relativistic charge does not experience deceleration in the magnetized plasma.

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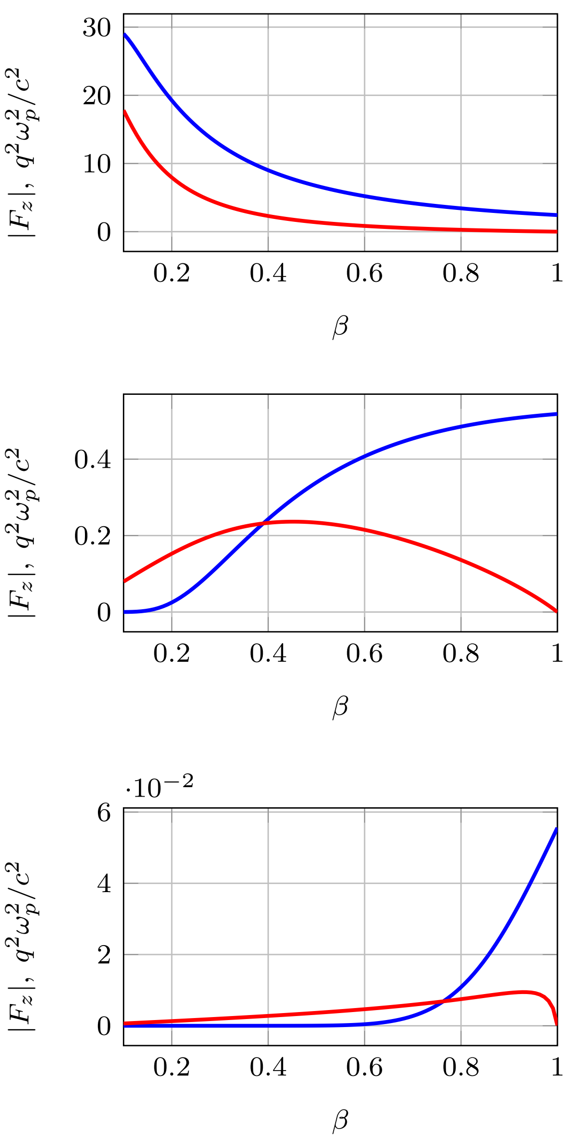

Figure 3: The dependence of decelerating force $|F_z|$ (in units $q^2\omega_p^2/c^2$) on the relative charge velocity β for different channel radius (top: $a = 0.1$, middle: $a = 1$, bottom: $a = 5$ in units c/ω_p). Red line denotes to the case of magnetized plasma, blue line denotes the isotropic plasma.

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