

SEEDED SELF-MODULATION OF TRANSVERSELY ASYMMETRIC LONG PROTON BEAMS IN PLASMA

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Abstract

The AWAKE experiment at CERN recently demonstrated the world's first acceleration of electrons in a proton-driven plasma wakefield accelerator. Such accelerators show great promise for a new generation of linear e-p colliders using $\sim 1\text{-}10$ GV/m accelerating fields. Effectively driving a wakefield requires 100-fold self-modulation of the 12 cm Super Proton Synchrotron (SPS) proton beam using a plasma-driven process which must be carefully controlled to saturation. Previous works have modelled this process assuming azimuthal symmetry of the transverse spatial and momentum profiles. In this work, 3D particle-in-cell simulations are used to model the self-modulation of such non-round beams. Implications of such effects for efficiently sustaining resonant wakefields are examined.

INTRODUCTION

Proton-driven plasma wakefield acceleration (PDPWFA), has been proposed to overcome the problem of energy depletion of drivers in previous experiments, with the view of application towards a new generation of plasma-wakefield-based colliders for high energy physics research.

However, current high-energy-content bunches, such as those of the Super Proton Synchrotron (SPS) used in AWAKE are too long by two orders of magnitude to efficiently drive a wakefield in plasma of suitable density [1]. Therefore, the concept relies on the self-modulation of the long proton bunch in plasma due to an initial weak 'seed' wakefield driven by the unmodulated bunch which causes the bunch to compress and diverge at periodic intervals along its length. The resulting train of shorter micro-bunches, if formed so that they are positioned correctly within the wakefield, can then resonantly excite much stronger accelerating gradients in the plasma [1].

It has been shown by numerical investigations in previous works that the seeded self-modulation (SSM) process may be highly sensitive to beam parameters such as emittance and radial spot size [1, 2]. However, such works have almost consistently considered only transversely round bunches. Here we present preliminary results from 3D particle-in-cell (PIC) simulations to investigate the SSM of proton bunches with unequal transverse aspect ratio, and its effect on the resonantly driven wakefield of the resultant micro-bunch train. Seeding the self-modulation process requires an initial wakefield with a sufficiently strong longitudinal component at the plasma wavelength,

$\lambda_p = 2\pi c \sqrt{m_e \epsilon_0 / n_p e^2}$, where n_p is the plasma density, ϵ_0 is the permittivity of free space, m_e and e are the electron mass and charge, respectively. This may be achieved by ionising the plasma by a co-propagating laser pulse, nominally placed at the midpoint of the gaussian proton beam to create a 'discontinuity' in beam longitudinal profile seen by the plasma. Such seeding is required to control the initial phase of the modulation process along the gaussian longitudinal beam profile, to ensure an efficient resultant micro-bunch arrangement upon saturation of the SSM growth [3].

SIMULATIONS

The 3D quasi-static PIC code *QuickPIC* [4] was used to perform numerical simulations of the proton-bunch-plasma interaction. We consider long proton bunches with parameters similar to the SPS bunches, with Lorentz factor $\gamma = 427$. Two sets of simulations were performed scanning over the beam transverse aspect ratio, with $n_p = 2 \times 10^{14} \text{ cm}^{-3}$, corresponding to a plasma skin-depth, $c/\omega_p = \lambda_p / 2\pi = 0.373 \text{ mm}$, where c is the speed of light and the plasma frequency, $\omega_p = 8.04 \times 10^{11} \text{ rad s}^{-1}$. In the first set, the primary interest was understanding the small-scale micro-bunch-field interaction. Bunches with flat-top longitudinal profiles and a bi-gaussian profile in x and y were used. At the seed point, there is a sharp linear rise to a maximum density of $n_{b0} = 0.04 a^{-1} n_p$ ($8.0 \times 10^{12} a^{-1} \text{ cm}^{-3}$) with the factor of a^{-1} included so that the bunches have equal total charge per xy slice. The simulations were performed with 3D cartesian grids $x \times y \times z$ of $256 \times 256 \times 4096$ cells, spanning physical dimensions $6 \times 6 \times 130 c/\omega_p$, corresponding to a grid density of $21 \times 21 \times 31$ cells per c/ω_p . Here, $\xi = ct - z$ is a coordinate backward along the bunch axis, zeroed at the seed point $2 c/\omega_p$ from the front edge of the simulation window. The aspect ratio was varied by changing the x r.m.s. size as $\sigma_x = a\sigma_y$, where $\sigma_y = 0.536 c/\omega_p$ (0.2 mm) was kept fixed, and a varied from 1.0 to 0.4 (1.0 and 0.8 for long windows). We take equal normalised emittance in x and y , $\epsilon_n = 3.5 \text{ mm mrad}$. For simplicity, only bunches with zero longitudinal momentum spread (0.035% for the SPS beam) [5] and uncorrelated transverse momentum spread $\sigma_{xx} = \sigma_{yy}$, $\sigma_{xy} = 0$ were considered. This means $\sigma_{x'} = a^{-1}\sigma_{y'}$, where $\sigma_{y'} = 0.041 \text{ mrad}$ corresponding to a doubling of $\sigma_{y'}^2$ after approx. $\beta^* \approx 5 \text{ m}$ from the beam waist in vacuum. However, this length is similar to the saturation length of the SSM at these parameters [6], which means that if, for example, the unequal aspect ratio is due to deviation of the beam waist in one or both axes from the plasma entrance, it is likely that the effect of correlated spread is important in the SSM process but is not considered in these results.

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Seed Fields

Following linear plasma wakefield theory, the absence of cylindrical symmetry of the proton beam's initial charge distribution excites a seed wakefield which contains fields with higher-order azimuthal Fourier modes [7]. For an elliptical profile, these are even-numbered modes, and predominantly those with wavenumbers $m = 2$ and $m = 4$. For an ultrarelativistic, rigid beam as in the quasi-static limit, where it is assumed to be evolving slowly on the time scale of its length, the beam self-fields vanish and the plasma wakefield is electrostatic making it possible to define an effective pseudopotential as seen by the beam particles [8]. In this section, we discuss its effect on the micro-bunch formation process. Figure 1 shows the instantaneous pseudopotential over the transverse (x - y) plane, encountered by an $a = 0.6$ beam, after the beam head has propagated $250 c/\omega_p$ into the plasma, near the position along the beam that the 20th micro-bunch would later form ($\xi \approx 120 c/\omega_p$). In the defocusing region, (left), the field has a relatively strong component in $m = 4$, leading to outward acceleration preferentially along the ellipse axes. Note, however, the difference in scale between plots. In the focusing region (right), all contours are closed around the propagation axis, but protons along the ellipse minor axis (x) see a significantly steeper potential well than along the major axis (y).

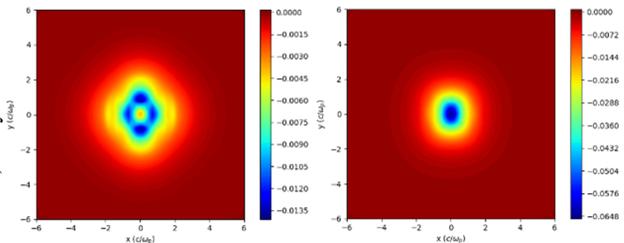


Figure 1: ‘Seed’ pseudopotential field, in units of $m_e c^2 / e$, experienced by protons at typical defocusing ($\xi = 120.0 c/\omega_p$) (left), and focusing ($\xi = 123.0 c/\omega_p$) (right) transverse cross-sections, after $250 c/\omega_p$ of propagation.

Over time, the phase of the wakefield and thus of the focusing field shifts, backwards along the beam [1], and hence the micro-bunch forms slightly behind the focusing point of the initial seed field. Figure 2 shows the defocused (left) and focused (right) protons in this period, after propagating 5.8 m in plasma. The defocused protons are completely evacuated from the central region of the wake with a nearly circular profile. The focused micro-bunch is typical of those that survive, after forming far from the head of the beam after saturation of the SSM, with a strongly azimuthally dependent profile. After the SSM saturates, the micro-bunches continue to evolve in asymmetric shapes, with pronounced features along the initial ellipse axes.

The wakefield at a given point is determined by contributions from the entire train of bunches ahead of that point, and therefore it can be expected that over time, the asymmetry in the micro-bunches' 2D profile grows at a ξ -dependent rate. Figure 3 shows the pseudopotential field experienced by the typical micro-bunch in Fig. 2.

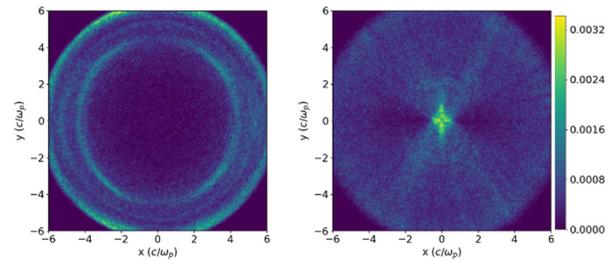


Figure 2: Proton transverse profiles at typical defocusing ($\xi = 125.0 c/\omega_p$) (left), and focusing ($\xi = 128.0 c/\omega_p$) (right) transverse cross-sections, after $15,750 c/\omega_p$ (5.8m) of propagation.

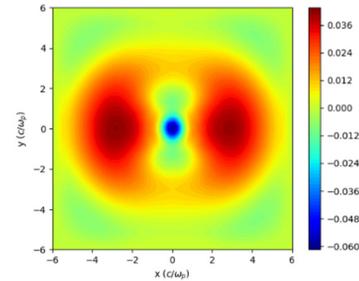


Figure 3: Pseudopotential field, in units of $m_e c^2 / e$, experienced by protons at the micro-bunches in Fig. 2 ($\xi = 125.0 c/\omega_p$) (left), and focusing ($\xi = 128.0 c/\omega_p$) (right) transverse cross-sections, after $15,750 c/\omega_p$ (5.8m) of propagation.

Phase Stability

The longitudinal electric wakefield is a useful measure of excitation effectiveness. Figure 4 shows the evolution with propagation distance of the on-axis E_z over the beam upto $\xi = 128.0 c/\omega_p$ ($\approx \sigma_z/3$ of the SPS beam) for $a = 1.0$ and $a = 0.4$.

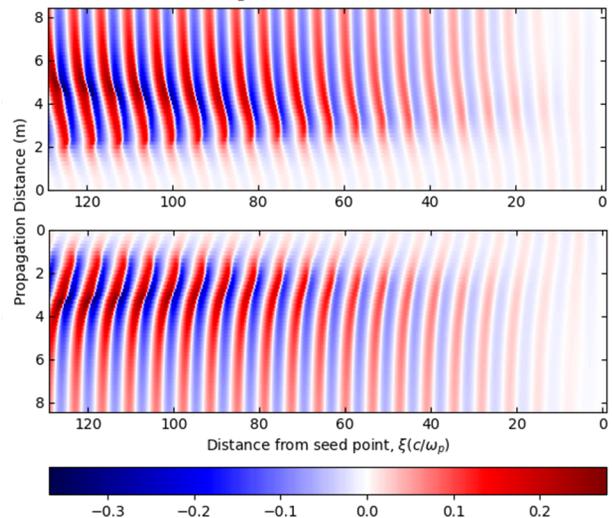


Figure 4: Variation of E_z in units of $E_0 = m_e c \omega_p / e$ with propagation distance for bunches with (top) $a = 1.0$ and (bottom) $a = 0.4$. The vertical axes are mirrored for ease of comparison by eye.

It is interesting to note that the development and phase evolution is qualitatively similar, despite the shapes of the

micro-bunches seen in Fig. 2. In fact, Fig. 5 shows variation of the maximum E_z for five values of aspect ratio a just before the 21st micro-bunch. It can be seen that for lower a , the growth of E_z due to the SSM is faster and saturates sooner than for more round bunches. Nonetheless, the maximum E_z reached is virtually independent of a .

After saturation (~ 5 m), the field begins to drop earlier for smaller a leading to lower wakefield amplitudes at a given propagation distance. It would seem that the function of the SSM, at least in this region, is robust to even $a = 0.4$.

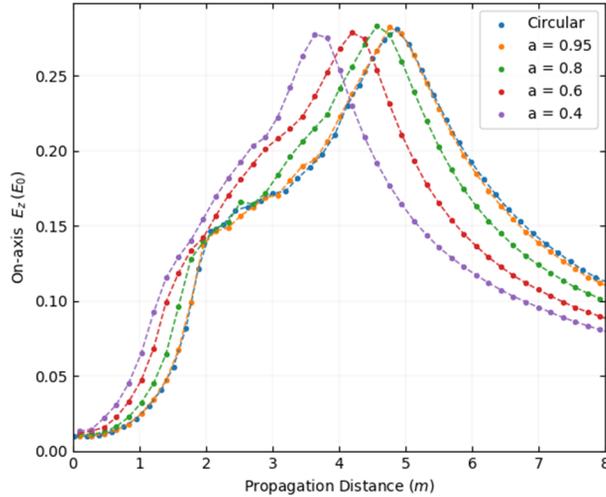


Figure 5: Evolution of the maximum longitudinal electric field, E_z for $\xi < 128.0$ c/ω_p (20th micro-bunch) in units of $E_0 = m_e c \omega_p / e$ for bunches of varying aspect ratio, as it propagates. Dashed lines are as a visual guide.

Gaussian Bunches

In this section we present preliminary results of two simulations performed using long bunches with a longitudinal half-Gaussian profile of $\sigma_z = 322$ c/ω_p (12 cm) behind the seed point, with a simulation window up to $\sim 2\sigma_z$ in order to capture the global maximum of the resonantly-driven wakefield of an SPS-like bunch. Since *QuickPIC* is not natively able to initialise half-Gaussian bunches, the continuous part of the profile was approximated using a piecewise linear function with 20 segments up to $2\sigma_z$ which yields a maximum deviation of 0.3% from a true gaussian profile within the simulated window. We use 3D cartesian grids $x \times y \times \zeta$ of $256 \times 256 \times 16384$ cells, spanning physical dimensions $6 \times 6 \times 650$ c/ω_p , corresponding to a grid density of $21 \times 21 \times 25$ cells per c/ω_p .

Figures 6 and 7 show the early evolution of a circular bunch compared to one with $a = 0.6$ with such a long window. The E_z is very similar between the two cases, even for a Gaussian bunch. Curiously, the $a = 0.6$ beam achieves a slightly higher field. However, as seen in Fig. 7, the position of the maximum field is consistently several tens of plasma periods closer to the bunch head, and spread over a smaller region in ζ than that of the circular bunch at a given propagation distance.

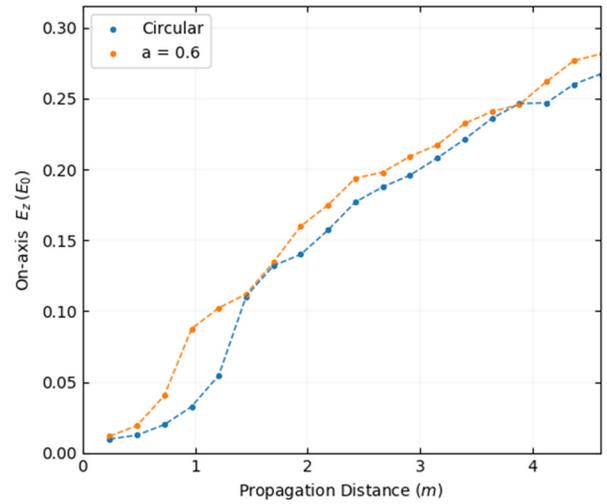


Figure 6: Evolution of the maximum longitudinal electric field, E_z for $\xi < 650.0$ c/ω_p in units of $E_0 = m_e c \omega_p / e$ for beams of two aspect ratios, as they propagate. Dashed lines are as a visual guide.

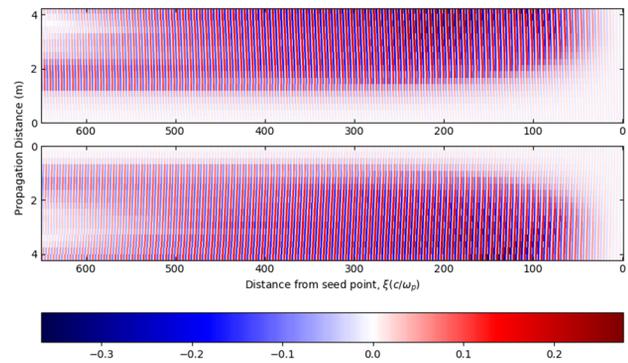


Figure 7: Variation of E_z in units of $E_0 = m_e c \omega_p / e$ with propagation distance for long bunches with $2\sigma_z = 644$ c/ω_p with (top) $a = 1.0$ and (bottom) $a = 0.6$. The vertical axes are mirrored for ease of comparison by eye.

CONCLUSIONS

We have presented preliminary results using 3D particle-in-cell simulations of the wakefield and seeded self-modulation of a long proton bunch with unequal transverse aspect ratio propagating in plasma. We have shown that the asymmetry in the transverse profile leads to the excitation of azimuthal modes in the plasma wakefield, which in turn lead to the growth of azimuthally asymmetric charge distributions in the resultant micro-bunches. While the near-axis accelerating fields and wakefield excitation effectiveness are found to be somewhat robust to variation of the aspect ratio, external injection of a witness bunch into such transverse fields may prove challenging. Further analysis of the field structure, boundary effects and particle kinetics is required to better understand the behaviour of the micro-bunches.

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