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PONDEROMOTIVE INSTABILITY OF TWO SELF-EXCITED CAVITIES*

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Abstract

We consider the ponderomotive instability of two superconducting RF cavities self-driven from a single RF source with vector-sum control.

CAVITY TUNING

Slater[1] gave the relation between changes in the stored energy and the natural frequency for EM fields in a metallic cavity when the boundary is perturbed:

$$[1-(\omega_c/\omega_{c0})^2] = \frac{\int_{\Delta V}(\mu_0|\vec{H}_0|^2 - \varepsilon_0|\vec{E}_0|^2)dV}{\int_V(\mu_0|\vec{H}_0|^2 + \varepsilon_0|\vec{E}_0|^2)dV}$$

The sources of volume change (ΔV) are: (i) the cavity tuner; (ii) Lorentz Force Detuning (LFD); and (iii) microphonics. All three disturbances act through the agency of the cavity mechanical modes. Schulze[2] gives a concise introduction to mechanical eigenmodes and generalized coordinates q. The dynamical response of each mode, index μ, is governed by an equation of the form:

$$\ddot{q}_\mu + \frac{2}{\tau_\mu}\dot{q}_\mu + \Omega_\mu^2 q_\mu = \frac{\Omega_\mu^2}{c_\mu} F_\mu$$

We assume that the cavity has axial symmetry, and that the mechanical and electromagnetic modes can each be grouped into transverse and longitudinal modes.

Mechanical Tuner

Provided that the tuner does not break the symmetry, then it couples only to the longitudinal mechanical modes (MM). This is true for the TESLA-style cavity, provided there is no buckling. It would be untrue if a pill-box cavity were tuned by a piston at the radial boundary.

Lorentz Force Detuning (LFD)

There are currents and charges on the interior (metallic) surface of the cavity (true for NC and SC, but different depths). These are acted upon by the electric (**E**) and magnetic (**H**) fields via the Lorentz force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. Because the cavity normal mode determines the spatial configuration of both the charges /currents and the fields, the resultant Lorentz pressure is $P = (\mu_0|\mathbf{H}|^2-\varepsilon_0|\mathbf{E}|^2)/4$. Apart from the squaring, this radiation pressure has the same spatial pattern as the cavity fundamental EM mode being used for acceleration of the particle beam. If the fundamental mode has axial symmetry, then LFD will couple only to the longitudinal MMs; and when high radiation pressure coincides with anti-nodes of the MM, coupling is strong. The changes (ΔV) in the mechanical shape resulting from pressure are used to find the Electro-Magnetic (EM) resonance frequency change. The static response to the Lorentz pressure is the sum of the DC response of all the mechanical modes of the cavity, acting collectively. Schulze seems to imply (from fundamental principles) that in general (i.e. always) the static detuning satisfies $\omega_c(|E|>0) < \omega_c(|E|=0)$; and that the contributions from individual mechanical modes is also negative. This is confirmed by measurement on a real cavity, or computer simulations (CST, COMSOL, etc). Because H is proportional to E, it is usual to write the resonant frequency as a function of E^2: $(\omega_c-\omega_{c0})/(2\pi) = -|k|E^2$.

Noise/Vibration Sources

Noise/vibration sources may enter into the cavity RF wave in two ways: either modulating the waveform before it enters the cavity, or acting inside the cavity through so-called “microphonics”.

External RF modulations Each SRF cavity in the TRIUMF ARIEL E-linac has two coaxial-type input couplers. The inner conductor is a cantilever, and prone to vibration. Displacement of the conductor changes the impedance, leading to an impedance mismatch and modulation incident RF wave. The high-power couplers are cooled with forced air: the impulsive and turbulent flow producing noise-like vibrations from 20 to 300 Hz that are imprinted on the arriving RF wave via the mechanism of impedance modulation.

Microphonics

These are mechanical vibrations that change the cavity shape, resulting in changes in the EM resonance frequency. (Note, there are also vibrations which change the cavity shape but do not change the resonance frequency.) There are many sources:

- Acoustic noise from fluids, gases, turbulence, bubbles
- Mechanical disturbance from rotary and reciprocating equipment, passing vehicular traffic, etc.

If the vibrations have spatial/directional and temporal overlap with the mechanical modes, then coupling occurs; but not necessarily to the EM mode. The MM has also to couple to the EM mode for the vibration to have an effect.

The fundamental electrical mode typically has no azimuthal dependence. In principle, there should be exactly zero coupling to the transverse MMs. The frequency change resulting from the linear part of the mechanical displacement is indeed zero – because there are equal and opposite linear displacements on opposite walls. But there is a 2nd order change in the volume proportional to mode displacement squared which gives a non-zero contribution when multiplied by the radiation pressure distribution.

The fundamental EM mode has cylindrical symmetry – it cannot tell difference between left, right, up, down; but can tell difference between short and long. The consequence of this: for longitudinal (transverse) MMs: the RF

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frequency modulation is equal (twice) the mechanical frequency. If a transverse MM is excited at frequency Ω, then the time variation of the EM resonance is Δω.Cos(2Ω t). This frequency doubling makes it difficult for a mode to participate in an RF instability; because, for instability to occur, all parts of the system have to oscillate at the same frequency. Thus longitudinal (transverse) mechanical modes couple strongly (weakly) to the fundamental EM mode, leading to 1st order and 2nd order effects, respectively. This being so, it is the longitudinal MMs that will participate in a ponderomotive instability.

The mode spectrum of transverse MMs is denser than the longitudinal, particularly at low frequency. Thus structure vibrations tends to be dominated by the transverse, but not the RF modulation spectrum. In contrast, the longitudinal MM spectrum is sparse at low frequency and the noise sources tend to roll off at higher frequency; but these modes respond strongly. The signature of a longitudinal mode is that when you excite it, say with an external shaker, the cavity response is large, RF may unlock or trip.

Thus, to good approximation, the static and dynamic LFD is dominated by the longitudinal mode spectrum – and this facilitates predictions based on a small number of MMs. For TESLA-like structures there are modes at roughly 40 (weak) and 160 Hz and two more between 200 and 300 Hz.

PONDEROMOTIVE INSTABILITY

SRF experts (including this author) use the term ponderomotive to mean an instability mediated by dynamic LFD. The static Lorentz force detuning results in a lowering of the EM resonance frequency, and an increase/reduction of the cavity amplitude depending as the drive frequency ω is below/above resonance ωc. Let TanΨ ≈ (ωc – ω)τc where τc is the cavity EM time constant.

If the static LFD (Δωc = -|kμ|V02) is not compensated by the tuner, the self-consistent voltage is the solution of a cubic equation and is limited by the available generator voltage. But, in the limit of small coupling [Δωc×τc <<1], there is an approximate form:

$$V_0^2 = \text{Cos}[\Psi]^2 V_{g0}^2\left(1 + 2\text{Cos}[\Psi]^3\text{Sin}[\Psi]k_\mu V_{g0}^2\tau\right)$$

The Lorentz force acting on the inner surface of the RF cavity couples the EM resonator to a mechanical mode. Think of the cavity voltage V(t)× Exp[iωt] as having been demodulated to baseband, leaving V(t). The MM behaves as a driven oscillator in the coordinate Δωc with external drive proportional to V2(t)≈ V02[1+2a] where a(t) is the modulation index. In the presence of the MM, and only in that case, the EM resonator behaves as a parametric resonator in the coordinate V, with equation of motion τcdV/dt+[1- jTanΨ- jΔω(t)τc]V=Vg and Δω = -|kμ|V2. The linearized form of the equations correctly predicts the threshold for instability and the growth rate for small amplitudes. For large amplitudes, additional phenomena such as limit cycles may appear.

In the monotonic instability, the almost steady state response of the EM and mechanical mode is exploited: the resonance frequency falls, the cavity voltage changes according to the derivative of the resonance curve w.r.t. frequency: voltage falls/rises above/below resonance, respectively, leading to corresponding changes in the DC excitation of the MM. Hence perturbations above/below are self-limiting/regenerative, respectively. The EM resonator response is Cos[Ψ], the derivative is Sin[Ψ], so simplistically we expect the instability threshold to scale as 1/(Cos[Ψ] Sin[Ψ]) when Ψ>0. During the instability, the EM field performs mechanical work on the cavity, changing its shape. The instability is "slow" compared with the EM and mechanical filling times. For the large amplitude motions, the cavity resonance frequency will slide toward the drive frequency.

In the oscillatory instability, the EM resonator pumps the mechanical resonator and visa versa. For the coupling to be effective, the amplitude modulation (AM) frequency has to be within the bandwidth of the MM, and the frequency modulation of the RF cavity (induced by the MM) has to be within the bandwidth of the EM mode. During an oscillation of the MM, the cavity resonance moves up and down in frequency, leading to changes in the amplitude response. But it is equivalent and simpler to think of the resonance as fixed and the RF drive as being frequency modulated. In this picture, the RF cavity is alternately driven at upper and lower sidebands (ω±Ω), Ω being the MM frequency. The net effect depends on the filling times of the EM and MM resonator, and also on whether both side bands are on the same side of resonance or whether they straddle the EM resonance. Above/below resonance, the lower sideband excites/damps and the upper sideband damps/excites. Differencing of the sidebands leads to a net excitation which is proportional to Cos(Ψ+Ωτc)- Cos(Ψ-Ωτc) ≈ -2 |Ωτc|SinΨ for small values. Hence the oscillatory instability occurs above resonance (i.e. Ψ<0).

In the parlance of "sidebands", the monotonic instability is single sideband with frequency offset tending to zero.

It is noteworthy that the linearized theory predicts which side of the EM resonance these instabilities occur; it is simply the derivative of the resonance curve (w.r.t. frequency) which counts, and this changes sign: positive below and negative above resonance. There is no need to invoke non-linear mechanisms to introduce this behaviour.

TWO-CAVITY SYSTEM

Consider two SRF cavities each self-excited excited with loop phases Θ_1 and Θ_2, under vector sum control. The loops share a common feedback point where the quadrature signal B(t) is injected. Following from Delayen [3] and the exposition [4], the system matrix is **P**=

$$\begin{pmatrix} 1+s\tau_c & \frac{s\tau_c}{2\omega} & 0 & -1 & \text{Sin}[\Theta_1] & 0 & 0 & 0 \\ -\text{Tan}[\Psi_1] & \tau_c & -\tau_c & \text{Tan}[\Psi_1] & -\text{Cos}[\Theta_1] & 0 & 0 & 0 \\ \frac{K_L}{\tau_c} & 0 & 1+\frac{s^2}{\Omega^2}+\frac{s}{Q\Omega} & 0 & 0 & 0 & 0 & 0 \\ \frac{A}{2} & 0 & 0 & 1 & 0 & \frac{A}{2} & 0 & 0 \\ 0 & \frac{F}{2s} & \frac{F}{2s} & 0 & 1 & 0 & \frac{F}{2s} & \frac{F}{2s} \\ 0 & 0 & 0 & -1 & \text{Sin}[\Theta_2] & 1+s\tau_c & \frac{s\tau_c}{2\omega} & 0 \\ 0 & 0 & 0 & \text{Tan}[\Psi_2] & -\text{Cos}[\Theta_2] & -\text{Tan}[\Psi_2] & \tau_c & -\tau_c \\ 0 & 0 & 0 & 0 & 0 & \frac{K_L}{\tau_c} & 0 & 1+\frac{s^2}{\Omega^2}+\frac{s}{Q\Omega} \end{pmatrix}$$

Acting on the system vector

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$$\boldsymbol{u} = \{a_{v1}, \delta\omega_1, \delta\omega\mu_1, a_g, \mathrm{dB}, a_{v2}, \delta\omega_2, \delta\omega\mu_2\}$$

The product **P.u** is a set of equations arranged as a column vector that is equal to the **0**. We adopt the notation of Ref.[4]. Θ_1 and Θ_2 relate the individual cavity resonance frequency (ω_{c1}, ω_{c2}) to the self-excitation frequency ω. Ψ_1 and Ψ_2 relate the individual cavity resonance frequency to the reference frequency ω_r. When the cavities become locked, $\omega=\omega_r$.

In principle, Θ1 and Θ2 are similar but may be unequal; and likewise for Ψ_1 and Ψ_2. In the case of the E-linac EACA, the power divider introduces phase shifts depending on the steady state power split between the cavities and so Θ_1 and Θ_2 must be re-adjusted.

For simplicity, we shall take Θ1 = Θ2 =Θ and Ψ1 = Ψ2 = Ψ. The characteristic determinant factorizes into the product of two polynomials: a cubic and quartic in the Laplace frequency variable *s*. The cubic contains terms in K_L and Ψ. The quartic contains terms in K_L, Θ and Ψ. The cubic/quartic is the same as for an RF cavity with/without control loops. Both these cases were treated in [4]. In the case that the phases are all properly adjusted and near zero, the quartic (with control loops) is more stable. However, this is not necessarily trivial to arrange and may be disturbed by additional detunings from other mechanical modes (a.k.a. microphonics).

Thus, the vector sum control leads to a behavior in which there are two virtual cavities, one less stable than the other. This comes as no surprise, vector sum control is under-determined: there are more variables than control parameters. It will not change the characteristic equation, but we may make a linear transform to sum ($a_v = a_{v1} + a_{v2}$, etc) and difference ($\delta a_v = a_{v1} - a_{v2}$, etc) variables; and in terms of these the system matrix takes on block diagonal form, with each block a virtual cavity.

Our model does not contain local tuning control of the individual cavities. [This happens to be a good approximation to the ARIEL E-linac Accelerator Cryomodule, which currently has slow tuning of near-DC offsets; but is being upgraded to fast tuning with piezo crystals.] Based on experience with the generator driven (GD) case, we would expect this to lead to an increase of the instability thresholds provided that the tuner is fast compared with the cavity (loaded) time constant.

Routh Determinant

Formulae were provided in Ref.[4] for the stability criteria arising from the condition that polynomial coefficients and Routh determinants all be positive. Here we display results graphically, Figs.1-4, for the 4th determinant of the quartic for a variety of conditions. Left and right images in each figure are low and high Lorentz coupling, respectively; influence of control loops is evident.

CONCLUSION

The E-linac ponderomotive instability [5], can be understood as a consequence of vector sum control.

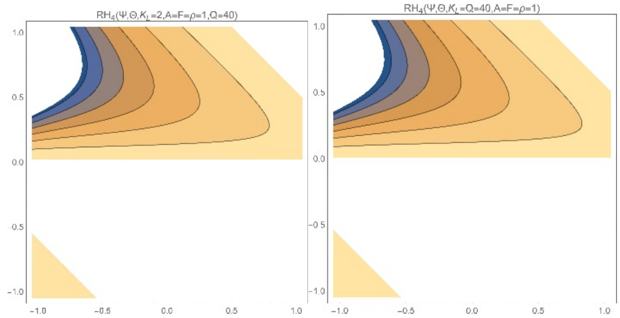

Figure 1: RH_4 in Ψ,Θ plane. Coloured areas unstable. Classic oscillatory condition is upper left quadrant: high frequency side TanΨ<0 and SinΘ>0. Heavily loaded regime $\tau_c\Omega$=1. Small control loop gains.

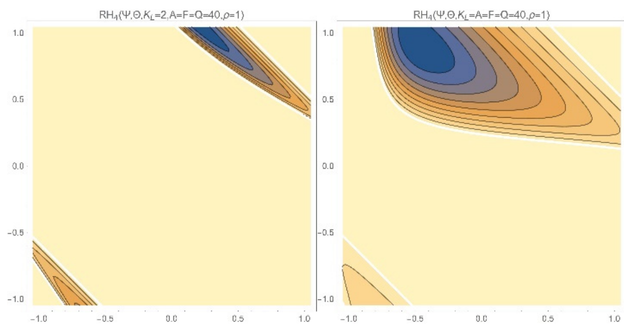

Figure 2: RH_4 in Ψ,Θ plane. Cream-coloured areas stable. Heavily loaded regime $\tau_c\Omega$=1. Large control loop gains. Stable area increased, compared with Fig. 1.

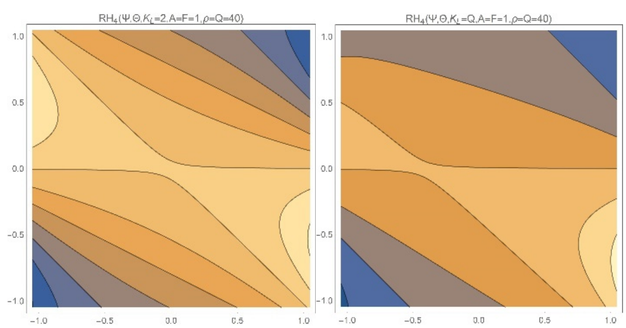

Figure 3: RH_4 in Ψ,Θ plane. Coloured areas stable. Unloaded regime $\tau_c\Omega$=Q (mechanical). Small control loop gains.

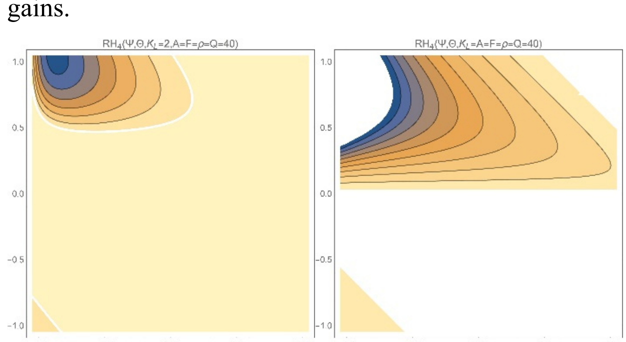

Figure 4: RH_4 in Ψ,Θ plane. Left: cream-coloured areas stable. Right: white areas stable. Unloaded regime $\tau_c\Omega$=Q (mechanical). Large control loop gains. Stable area reduced compared with Fig. 3.

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