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VECTOR SUM & DIFFERENCE CONTROL OF SRF CAVITIES*

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Abstract

We consider the ponderomotive instability of multiple superconducting RF cavities driven from a single RF source. We add vector difference control to the usual technique of vector sum control, in order to increase the accelerating gradient threshold for ponderomotive instability.

INTRODUCTION

High power RF sources are expensive, and it is more economical for one source to power several cavities. Vector -sum control, introduced [1,2,3,4] in the 1990's, is used to control their combined amplitude and phase against disturbances. The electro-magnetic (EM) fields within a super-conducting radio frequency (SRF) cavity can be sufficiently strong to deform the cavity shape, which may lead to a ponderomotive instability at high accelerating gradient. This must be stabilized by the RF controls.

This paper was initially motivated by the discovery[5] of a ponderomotive instability in the TRIUMF ARIEL E-linac Accelerator Cryomodule, which has two SRF cavities powered by a single klystron. The electron beam is accelerated on crest of the RF wave. The instability has several features: (i) it occurs at relatively modest gradient (8-9 MV/m); (ii) the individual cavity amplitudes oscillate in anti-phase; (iii) growth takes several seconds; (iv) detuning the cavity resonance frequency above the RF is not sufficient to stabilize; and (v) as the gradient is raised, and the instability approached, the range of stable tuning angles tends to zero.

The instability is believed to derive from two features: (a) the heavily loaded cavity quality factor extends the cavity bandwidth to include a longitudinal mechanical mode at roughly 160 Hz; and (b) the linac, which employs vector sum control, operates in continuous wave (c.w.) mode, giving ample time for the development of an instability with slow growth rate. Companion papers address the first feature (a), while this investigates the latter circumstance (b). Based on the analysis, it is believed the instability can be tamed by adding vector difference control.

Although originally motivated by the 2-cavity case, difference control is extensible to many cavities; and we shall present the 2-cavity and 3-cavity cases as examples.

TWO CAVITY SYSTEM

Consider two RF cavities, generator driven (GR), with vector sum control. Take the basis vector

$$\boldsymbol{u} = \{\mathrm{av}_1, \mathrm{pv}_1, \tau\delta\omega_1, \mathrm{ag}, \mathrm{pg}, \mathrm{av}_2, \mathrm{pv}_2, \tau\delta\omega_2\}$$

Here a,p are amplitude and phase modulation indices, and the subscript 1,2 is the cavity identifier; and v and g denote "voltage" (response) and "generator" (drive), respectively.

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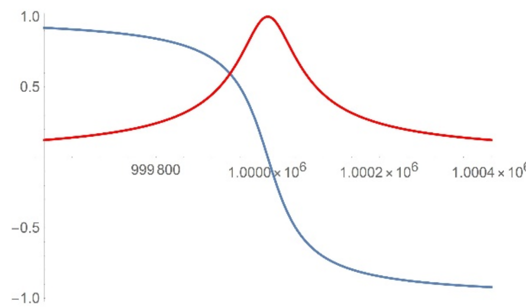

Figure 1: Cavity amplitude (red) and phase Ψ (blue).

$\delta\omega_{1,2}$ are deviations in the cavity resonance frequency. The cavity loaded time constant and detuning angle are τ and Ψ, respectively. Phase convention shown in Fig. 1. Let s be the Laplace transform variable.

The system matrix is **P** =

$$\begin{pmatrix}
1+s\,\tau_c & \mathrm{Tan}[\Psi] & 0 & -1 & -\mathrm{Tan}[\Psi] & 0 & 0 & 0 \\
-\mathrm{Tan}[\Psi] & 1+s\,\tau_c & -1 & \mathrm{Tan}[\Psi] & -1 & 0 & 0 & 0 \\
0 & \frac{K_t[s]}{2} & \frac{1}{2} & 0 & -\frac{1}{2}K_t[s] & 0 & 0 & 0 \\
\frac{K_a[s]}{2} & 0 & 0 & 1 & 0 & \frac{K_a[s]}{2} & 0 & 0 \\
0 & \frac{K_p[s]}{2} & 0 & 0 & 1 & 0 & \frac{K_p[s]}{2} & 0 \\
0 & 0 & 0 & -1 & -\mathrm{Tan}[\Psi] & 1+s\,\tau_c & \mathrm{Tan}[\Psi] & 0 \\
0 & 0 & 0 & \mathrm{Tan}[\Psi] & -1 & -\mathrm{Tan}[\Psi] & 1+s\,\tau_c & -1 \\
0 & 0 & 0 & 0 & -\frac{1}{2}K_t[s] & 0 & \frac{K_t[s]}{2} & \frac{1}{2}
\end{pmatrix}$$

There are additional equations, one for each cavity, for the mechanical mode that participated in the ponderomotive instability. The product **P.u** is a set of equations arranged as a column vector that is equal to the **0**.

$$\begin{pmatrix}
-\mathrm{ag} + (1+s\,\tau_c)\,\mathrm{av}_1 - \mathrm{pg}\,\mathrm{Tan}[\Psi] + \mathrm{pv}_1\,\mathrm{Tan}[\Psi] \\
-\mathrm{pg} + (1+s\,\tau_c)\,\mathrm{pv}_1 - \tau\,\delta\omega_1 + \mathrm{ag}\,\mathrm{Tan}[\Psi] - \mathrm{av}_1\,\mathrm{Tan}[\Psi] \\
\frac{\tau\,\delta\omega_1}{2} - \frac{1}{2}\mathrm{pg}\,K_t[s] + \frac{1}{2}\mathrm{pv}_1\,K_t[s] \\
\mathrm{ag} + \frac{1}{2}\mathrm{av}_1\,K_a[s] + \frac{1}{2}\mathrm{av}_2\,K_a[s] \\
\mathrm{pg} + \frac{1}{2}\mathrm{pv}_1\,K_p[s] + \frac{1}{2}\mathrm{pv}_2\,K_p[s] \\
-\mathrm{ag} + (1+s\,\tau_c)\,\mathrm{av}_2 - \mathrm{pg}\,\mathrm{Tan}[\Psi] + \mathrm{pv}_2\,\mathrm{Tan}[\Psi] \\
-\mathrm{pg} + (1+s\,\tau_c)\,\mathrm{pv}_2 - \tau\,\delta\omega_2 + \mathrm{ag}\,\mathrm{Tan}[\Psi] - \mathrm{av}_2\,\mathrm{Tan}[\Psi] \\
\frac{\tau\,\delta\omega_2}{2} - \frac{1}{2}\mathrm{pg}\,K_t[s] + \frac{1}{2}\mathrm{pv}_2\,K_t[s]
\end{pmatrix}$$

Despite the apparent couplings between variables, the characteristic equation factors into two polynomials.

Change of Basis Vector

From the theory of linear (matrix) algebra, we know that the characteristic equation is independent of the vector basis. However, the structure of the matrix depends on the choice of basis vector. Thus, underlying symmetry can be made manifest by suitable choice of basis. For example, block-diagonal form may result if artificial cross-couplings are eliminated. But the characteristic polynomial will factor into product form if there is an underlying symmetry, whether or not the matrix was actually block-diagonalized.

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Any linear superposition of the old bases, is also a basis. We transform to vector sum and difference coordinates:

$$\boldsymbol{v_s} = \{\mathrm{av} = (\mathrm{av}_1 + \mathrm{av}_2)/2, \mathrm{pv} = (\mathrm{pv}_1 + \mathrm{pv}_2)/2, \delta\omega\tau = (\tau\delta\omega_1 + \tau\delta\omega_2)/2\}$$

$$\boldsymbol{v_d} = \{\Delta\mathrm{av} = \mathrm{av}_1 - \mathrm{av}_2, \Delta\mathrm{pv} = \mathrm{pv}_1 - \mathrm{pv}_2, \tau\Delta\omega = \tau\delta\omega_1 - \tau\delta\omega_2\}$$

The new basis vector is $\boldsymbol{v} = \{\boldsymbol{v_s}, ag, pg, \boldsymbol{v_d}\}$.
This transformation is generated by a matrix **T.**
The new system matrix is **P'=T P T^{-1}** equal to

$$\begin{pmatrix} 1+s\tau_c & \mathrm{Tan}[\Psi] & 0 & -1 & -\mathrm{Tan}[\Psi] & 0 & 0 & 0 \\ -\mathrm{Tan}[\Psi] & 1+s\tau_c & -1 & \mathrm{Tan}[\Psi] & -1 & 0 & 0 & 0 \\ 0 & \frac{K_t[s]}{2} & \frac{1}{2} & 0 & -\frac{1}{2}K_t[s] & 0 & 0 & 0 \\ K_a[s] & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & K_p[s] & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+s\tau_c & \mathrm{Tan}[\Psi] & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\tau_c & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_t[s]}{2} & \frac{1}{2} \end{pmatrix}$$

P' is in block diagonal form, and the vector **P'.v** is partitioned {5,3} and uncoupled:

$$\begin{pmatrix} -ag + av + av\,s\,\tau_c + (-pg + pv)\,\mathrm{Tan}[\Psi] \\ -pg + pv - \delta\omega\,\tau + pv\,s\,\tau_c + (ag - av)\,\mathrm{Tan}[\Psi] \\ \frac{1}{2}(\delta\omega\,\tau + (-pg + pv)\,K_t[s]) \\ ag + av\,K_a[s] \\ pg + pv\,K_p[s] \\ \Delta av + s\,\Delta av\,\tau_c + \Delta pv\,\mathrm{Tan}[\Psi] \\ \Delta pv - \tau\Delta\omega + s\,\Delta pv\,\tau_c - \Delta av\,\mathrm{Tan}[\Psi] \\ \frac{1}{2}(\tau\Delta\omega + \Delta pv\,K_t[s]) \end{pmatrix}$$

This partition remains possible even when unwanted but identical cross-couplings are introduced: for example, poor isolation between ports of the power divider or inter-cavity mechanics.

If we supplement the system equations with those of two mechanical modes (one for each cavity, and assumed identical), we find that they enter into **P'.v** in the same way: one mechanical resonator coupled to the sum av, and the other coupled to the difference Δav.

The first 5 equations are those of a virtual cavity with all control loops present, and which can benefit from the raised thresholds of the ponderomotive instability.

The last 3 equations are those of a virtual cavity with no control except the tuning loop; this "cavity" will encounter the ponderomotive instabilities at lower threshold accelerating gradient; indeed if the tuner is slow (or absent) the threshold is that for no control loops. In principle, this is the circumstance at the ARIEL e-linac; except that the cavities are run in self-excited (SE) loop.

N-CAVITY SYSTEM

The system of transformations described above can be applied to multiple cavities driven from a single RF source; and the result is the same: 1 virtual cavity with all loops present and N-1 cavities with no control except the tuner. This is a crucial point, because it means we can employ the ponderomotive stability criteria [6] for a single cavity.
We have to generalize the concepts: there is 1 sum variable and N-1 difference variables. The differences are formed pairwise and cyclically permuted. The 3 cavity system will demonstrate the principle. The "old" base vector is $\boldsymbol{u} = \{\mathrm{av}_1, \mathrm{pv}_1, \tau\delta\omega_1, \mathrm{av}_2, \mathrm{pv}_2, \tau\delta\omega_2, \mathrm{av}_3, \mathrm{pv}_3, \tau\delta\omega_3, \mathrm{ag}, \mathrm{pg}\}$

The system matrix **P** in this basis:

$$\begin{pmatrix} 1+s\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\mathrm{Tan}[\Psi] \\ -\mathrm{Tan}[\Psi] & 1+s\tau_c & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{Tan}[\Psi] & -1 \\ 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3}K_t[s] \\ 0 & 0 & 0 & 1+s\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 & 0 & -1 & -\mathrm{Tan}[\Psi] \\ 0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\tau_c & -1 & 0 & 0 & 0 & \mathrm{Tan}[\Psi] & -1 \\ 0 & 0 & 0 & 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & -\frac{1}{3}K_t[s] \\ 0 & 0 & 0 & 0 & 0 & 0 & 1+s\tau_c & \mathrm{Tan}[\Psi] & 0 & -1 & -\mathrm{Tan}[\Psi] \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\tau_c & -1 & \mathrm{Tan}[\Psi] & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & -\frac{1}{3}K_t[s] \\ \frac{K_a[s]}{3} & 0 & 0 & \frac{K_a[s]}{3} & 0 & 0 & \frac{K_a[s]}{3} & 0 & 0 & 1 & 0 \\ 0 & \frac{K_p[s]}{3} & 0 & 0 & \frac{K_p[s]}{3} & 0 & 0 & \frac{K_p[s]}{3} & 0 & 0 & 1 \end{pmatrix}$$

The relation between old ($\mathrm{av_i}$, $\mathrm{pv_i}$, $\delta\omega_i$) and new ($\mathrm{Av_i}$, $\mathrm{Pv_i}$, $\mathrm{O_i}$) coordinates:

$$\boldsymbol{v_s} = \{\mathrm{Av}_1 = (\mathrm{av}_1 + \mathrm{av}_2 + \mathrm{av}_3)/3, \mathrm{Pv}_1 = (\mathrm{pv}_1 + \mathrm{pv}_2 + \mathrm{pv}_3)/3, \tau O_1 = (\tau\delta\omega_1 + \tau\delta\omega_2 + \tau\delta\omega_3)/3\}$$

$$\boldsymbol{v_d} = \{\mathrm{Av}_2 = \mathrm{av}_1 - \mathrm{av}_3, \mathrm{Pv}_2 = \mathrm{pv}_1 - \mathrm{pv}_3, \tau O_2 = \tau\delta\omega_1 - \tau\delta\omega_3, \mathrm{Av}_3 = \mathrm{av}_1 - \mathrm{av}_2, \mathrm{Pv}_3 = \mathrm{pv}_1 - \mathrm{pv}_2, \tau O_3 = \tau\delta\omega_1 - \tau\delta\omega_2\}$$

$$\boldsymbol{v} = \{\boldsymbol{v_s}, \boldsymbol{v_d}, ag, pg\}$$

The new system matrix is **P'=T P T^{-1}**:

$$\begin{pmatrix} 1+s\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\mathrm{Tan}[\Psi] \\ -\mathrm{Tan}[\Psi] & 1+s\tau_c & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{Tan}[\Psi] & -1 \\ 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3}K_t[s] \\ 0 & 0 & 0 & 1+s\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\tau_c & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1+s\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\tau_c & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 \\ K_a[s] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & K_p[s] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

And vector **P'.v** is partitioned {5,3,3} and uncoupled. The upper part represents a virtual cavity with all loops present:

$$\begin{pmatrix} (1 + s\,\tau_c)\,Av_2 + Pv_2\,\mathrm{Tan}[\Psi] \\ -\tau\,O_2 + (1 + s\,\tau_c)\,Pv_2 - Av_2\,\mathrm{Tan}[\Psi] \\ \frac{1}{3}(\tau\,O_2 + Pv_2\,K_t[s]) \\ (1 + s\,\tau_c)\,Av_3 + Pv_3\,\mathrm{Tan}[\Psi] \\ -\tau\,O_3 + (1 + s\,\tau_c)\,Pv_3 - Av_3\,\mathrm{Tan}[\Psi] \\ \frac{1}{3}(\tau\,O_3 + Pv_3\,K_t[s]) \end{pmatrix}$$

And the lower part represents two virtual cavities with no loops except the tuner.

$$\begin{pmatrix} (1 + s\,\tau_c)\,Av_2 + Pv_2\,\mathrm{Tan}[\Psi] \\ -\tau\,O_2 + (1 + s\,\tau_c)\,Pv_2 - Av_2\,\mathrm{Tan}[\Psi] \\ \frac{1}{3}(\tau\,O_2 + Pv_2\,K_t[s]) \\ (1 + s\,\tau_c)\,Av_3 + Pv_3\,\mathrm{Tan}[\Psi] \\ -\tau\,O_3 + (1 + s\,\tau_c)\,Pv_3 - Av_3\,\mathrm{Tan}[\Psi] \\ \frac{1}{3}(\tau\,O_3 + Pv_3\,K_t[s]) \end{pmatrix}$$

GENERAL CONSIDERATIONS

We have assumed identical cavities with identical (partially shared) control loops, & identical mechanical modes. This is not a far departure from reality: accelerator builders go to great lengths to make all components (as near as possible) identical (particularly the EM resonance frequencies and drive/SEL frequencies).

Small splitting of the EM or mechanical resonance frequencies, does not change the picture – although it does break the degeneracy of the eigenfrequencies.

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A General Problem

There is a generic and underlying problem with all "vector sum of many cavities" type of control: $V_1 + V_2 + V_3 + \ldots V_N = 0$ is a valid solution with all V_i nonzero. For short pulse operation, the initial conditions at (t=0) prevail $V_1 = V_2 = V_3 \ldots = V_N$. But for long-pulse and c.w. operation [7], any slow-growth rate instabilities which can cause the individual V_i to become independently modulated will become manifest.

A Simple Solution

The loops are ineffective for the difference variables (V_1-V_2, etc). Restoring control over individual V_i, implies introducing control that is different between the cavities, without destroying "vector sum". Form of the equations implies that the only place to do this is at the cavity tuners.

The first action is to make the tuning loop "fast", i.e. tuner time constant T much less than cavity time τ, and push the tuner gain towards its limit (see below); this will raise the monotonic instability threshold.

Exotic/Speculative Solutions

If simple means is not enough to secure desired gradient, then we could investigate new/unusual couplings and/or symmetry breaking. For example, with 2 cavities:

- Cross-coupling: control of $\Delta\omega = \delta\omega_1 - \delta\omega_2$ based on feedback of $\Delta\text{av} = \text{av}_1 - \text{av}_2$
- Symmetry breaking: tuning control $\delta\omega_1$ propto pv_1 and $\delta\omega_2$ propto av_2; Or AC couplings $\delta\omega_1$ propto ($\text{pv}_1 - \text{pv}_2$) and $\delta\omega_2$ propto ($\text{av}_1 - \text{av}_2$).

The latter are speculative, and perhaps too asymmetric. We pursue the cross-coupling a little further. The basis vector is $\{\text{av}_1, \text{pv}_1, \tau\delta\omega_1, \text{av}_2, \text{pv}_2, \tau\delta\omega_2, \text{av}_3, \text{pv}_3, \tau\delta\omega_3, \text{ag}, \text{pg}\}$. The coupling from amplitude to resonance frequency is $K_{ta}[s]$. The column vector **P.u** is:

$$\begin{pmatrix} -\text{ag} + (1 + s\,\tau_c)\,\text{av}_1 - \text{pg}\,\text{Tan}[\Psi] + \text{pv}_1\,\text{Tan}[\Psi] \\ -\text{pg} - \delta\omega_1\,\tau_c + (1 + s\,\tau_c)\,\text{pv}_1 + \text{ag}\,\text{Tan}[\Psi] - \text{av}_1\,\text{Tan}[\Psi] \\ \frac{\delta\omega_1\,\tau_c}{2} - \frac{1}{2}\text{pg}\,K_t[s] + \frac{1}{2}\text{pv}_1\,K_t[s] + \frac{1}{2}\text{av}_1\,K_{ta}[s] - \frac{1}{2}\text{av}_2\,K_{ta}[s] \\ \text{ag} + \frac{1}{2}\text{av}_1\,K_a[s] + \frac{1}{2}\text{av}_2\,K_a[s] \\ \text{pg} + \frac{1}{2}\text{pv}_1\,K_p[s] + \frac{1}{2}\text{pv}_2\,K_p[s] \\ -\text{ag} + (1 + s\,\tau_c)\,\text{av}_2 - \text{pg}\,\text{Tan}[\Psi] + \text{pv}_2\,\text{Tan}[\Psi] \\ -\text{pg} - \delta\omega_2\,\tau_c + (1 + s\,\tau_c)\,\text{pv}_2 + \text{ag}\,\text{Tan}[\Psi] - \text{av}_2\,\text{Tan}[\Psi] \\ \frac{\delta\omega_2\,\tau_c}{2} - \frac{1}{2}\text{pg}\,K_t[s] + \frac{1}{2}\text{pv}_2\,K_t[s] - \frac{1}{2}\text{av}_1\,K_{ta}[s] + \frac{1}{2}\text{av}_2\,K_{ta}[s] \end{pmatrix}$$

After transforming to the sum and difference basis, the column vector **P'.v** is partitioned {5,3} and uncoupled:

$$\begin{pmatrix} -a_g + a_v + s\,\tau_c\,a_v + (-p_g + p_v)\,\text{Tan}[\Psi] == 0 \\ -\delta\omega\,\tau_c - p_g + p_v + s\,\tau_c\,p_v - (-a_g + a_v)\,\text{Tan}[\Psi] == 0 \\ \delta\omega\,\tau_c + (-p_g + p_v)\,K_{\tau_c}[s] == 0 \\ a_g + a_v\,K_a[s] == 0 \\ p_g + p_v\,K_\rho[s] == 0 \\ \Delta a_v + s\,\tau_c\,\Delta a_v + \Delta p_v\,\text{Tan}[\Psi] == 0 \\ -\Delta\omega\,\tau_c + \Delta p_v + s\,\tau_c\,\Delta p_v - \Delta a_v\,\text{Tan}[\Psi] == 0 \\ \Delta\omega\,\tau_c + 2\,\Delta a_v\,K_{ta}[s] + \Delta p_v\,K_{\tau_c}[s] == 0 \end{pmatrix}$$

The lower portion, in the difference variables, has an additional term in the resonance frequency feedback that provides greater flexibility of control.

STABILITY OF CAVITY & TUNER

We consider now the ponderomotive instability for a cavity equipped only with resonance tuning control and no other feedback. We adopt the notation of Ref. [6].

Suppose the tuner is "fast", i.e. has constant gain for several decades. In such case, the characteristic is a quartic in *s* with coefficients.

$$a_0 = Q\Omega^2(K_t - 2K_L\text{Tan}[\Psi] + \text{Tan}[\Psi]^2)$$
$$a_1 = \Omega(2K_t + Q\tau\Omega(1 + K_t) + 2\text{Tan}[\Psi]^2)$$
$$a_2 = \tau\Omega(2 + Q\tau\Omega) + (Q + 2\tau\Omega)K_t + Q\text{Tan}[\Psi]^2$$
$$a_3 = \tau(Q + 2\tau\Omega + QK_t),\ a_4 = Q\tau^2$$

Here K_t stands in place of $(1+K_t)$.

The *monotonic instability* occurs for $a_0<0$ and $\text{Tan}[\Psi]>0$, leading to the threshold on Lorentz force detuning:

$$K_L < (\text{Cot}[\Psi]K_t + \text{Tan}[\Psi])/2$$

Oscillatory Instability

The fourth Routh determinant RH_4 implies an instability threshold when $\text{Tan}[\Psi]<0$, thus:

$$K_L\{Q\rho(Q + 2\rho + QK_t)^2\}/(1 + K_t) < -\{(Q^2 + 2\text{Cot}[\Psi]^2((2 - Q^2)\rho^2 + Q^2K_t + Q\rho(1 + K_t)) + (Q + \rho(2 + Q\rho))\text{Cot}[\Psi]^4 (2\rho K_t + Q(\rho^2 + K_t^2)))\}\text{Tan}[\Psi]^3$$

Here ρ = τ Ω where Ω is mechanical resonance frequency. We consider illustrative cases in the limit of large gain and mechanical quality factor Q.

Classical regime $\rho \approx Q$

$$K_L < -\frac{\text{Cot}[\Psi](Q^2 + K_t^2)}{K_t} - \frac{2(-Q^2 + 2K_t)\text{Tan}[\Psi]}{Q^2K_t} - \frac{\text{Tan}[\Psi]^3}{Q^2K_t}$$

Heavily loaded regime $\rho \approx 1$

$$K_L < -\frac{2\text{Cot}[\Psi]K_t}{Q} - \frac{2\text{Tan}[\Psi]}{Q} - \frac{\text{Tan}[\Psi]^3}{QK_t}$$

Intermediate regime $\rho = \sqrt{Q}$

$$K_L < -\frac{\text{Cot}[\Psi](Q^{3/2} + 2K_t + \sqrt{Q}K_t^2)}{QK_t} - \frac{2(-Q^{3/2} + K_t + \sqrt{Q}K_t)\text{Tan}[\Psi]}{Q^2K_t} - \frac{\text{Tan}[\Psi]^3}{Q^{3/2}K_t}$$

Slow Tuner

The slow tuner is modelled as $K_t/(1+sT)$. In such case, the characteristic is a quintic. Ideally, in the absence of Lorentz force effects, the system response is damped and nonoscillatory. Such case leads to a limit on the gain: $K_t \le \frac{(T+\tau)^2}{4T\tau}$. The minimum occurs when T=τ. Large gain is permitted when T>>τ or T<<τ. The former/latter pushes roots toward s= -1/τ and s = -Q/τ respectively. So T<τ is preferred. Both polynomial coefficients a_0 and a_1 can lead to instability when $\text{Tan}[\Psi]>0$. The monotonic case $a_0<0$ always occurs before $a_1<0$ provided that T ≤ τ. The Routh determinants RH_4 and RH_5 can both produce instability for $\text{Tan}[\Psi]<0$.

CONCLUSION

We have provided a comprehensive analysis of the ponderomotive stability of multiple SRF cavities driven from a single source. We introduced vector-difference control in order to improve the stability.

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