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BEAM BASED MEASUREMENTS OF RELATIVE RF PHASE

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Abstract

The ferrite loaded RF cavities of the CERN Proton Synchrotron Booster will be replaced with Finemet™ loaded cavities during Long Shutdown 2 (2019-2020). To fully realise the potential of the new cavities, the relative RF phases must be aligned along the acceleration ramp, where the revolution frequency changes by nearly a factor of 2. A beam based method of measuring the relative phase between the cavities is desired to give the best possible compensation for the frequency dependent phase shift. In this paper we present an operationally viable method to measure the phase shift as a function of RF frequency. The relative phase of the RF cavities can be aligned to within a few degrees, giving an error on the voltage seen by the beam of less than 1%.

INTRODUCTION

Prior to Long Shutdown 2 the RF cavities of the Proton Synchrotron Booster (PSB) at CERN were tuned ferrite cavities operating at $h = 1$, $h = 2$, and $6 \leq h \leq 20$. The PSB revolution frequency changes significantly during the ramp, from $f_{\mathrm{rev}} = 0.99$ MHz at injection to $f_{\mathrm{rev}} = 1.81$ MHz at extraction. During Long Shutdown 2, the ferrite cavities are being replaced with broadband Finemet™ cavities able to operate at all desired harmonics across the full frequency span simultaneously [1].

One benefit of the Finemet™ cavities will be the ability to use a distributed cavity concept, where a particular harmonic is shared across multiple cavities [2]. Correctly sharing the voltage between cavities requires the relative phases to be well aligned so that the voltage seen by the beam is as close as possible to the programmed value.

There are two contributions to the phase error; a fixed azimuthal offset ($\Delta\Phi$), caused by frequency independent effects, such as cavity locations in the ring; and a dispersive component ($\omega_{RF} D$), where ω_{RF} is the RF angular frequency and D is the dispersion of the cables and other parts of the RF system. Therefore, the RF phase in a cavity at any ω_{RF} can be expressed as $\varphi = \varphi_{\mathrm{prog}} + \Delta\Phi + \omega_{RF} D$, where φ_{prog} is the programmed phase offset. To align the phase between two cavities, a delay (dt) and azimuth (A) compensation are used such that $\omega_{RF} dt = -\omega_{RF} D$ and $A = -\Delta\Phi$.

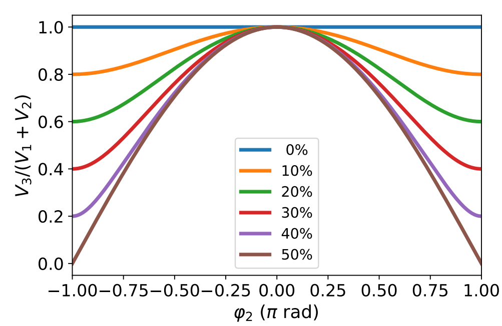
In this paper we present a beam based method to determine the phase offset between two RF cavities operating at the same harmonic, which can be readily modified to work at different harmonics. By measuring the relative phase at different points in the accelerating cycle, corresponding to different RF frequencies, both $\Delta\Phi$ and $\omega_{RF} D$ can be determined. Therefore, the correct dt and A can be programmed in case a misalignment is measured.

When splitting the voltage between two cavities, the voltage seen by the beam can be expressed as

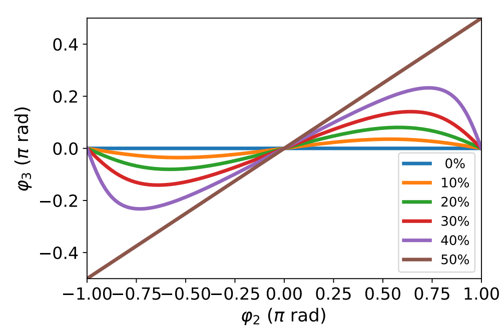
$$\begin{aligned} V(t) &= V_3 \sin(\omega_{RF} t + \varphi_3) = \\ & V_1 \sin(\omega_{RF} t + \varphi_1) + V_2 \sin(\omega_{RF} t + \varphi_2), \end{aligned} \tag{1}$$

Where $V(t)$ is the waveform seen by the beam, V_3, φ_3 are the amplitude and phase of the resultant voltage, and $V_{1,2}$ and $\varphi_{1,2}$ are the voltage and phase of the 1st and 2nd cavities. Defining cavity 1 as the master cavity, to which cavity 2 must be aligned, allows setting $\varphi_1 = 0$; φ_3 and V_3 can then be expressed as

$$\begin{aligned} V_3 &= [V_1^2 + V_2^2 + 2V_1 V_2 \cos\varphi_2]^{\frac{1}{2}}, \\ \tan\varphi_3 &= \frac{V_2 \sin\varphi_2}{V_1 + V_2 \cos\varphi_2}. \end{aligned} \tag{2}$$



(a) Voltage ratio



(b) Resultant phase

Figure 1: The ratio $V_3/(V_2 + V_1)$ (top) and the resultant phase (bottom) for different $V_2/(V_1 + V_2)$ voltage ratios in % (colour) and values of φ_2 (x-axis).

Figure 1 shows how V_3 (1a) and φ_3 (1b) vary at different ratios of V_2 to V_1 and as φ_2 varies from $-\pi$ to $+\pi$. To

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minimise the impact of a misalignment, it is clearly preferable to maximise the share of the voltage in a single cavity. However, to minimise the demand on the High Level RF, it is preferable to make the split as even as possible. In the case where voltage is shared equally, the phase misalignment must be less than 0.28 rad if the resultant voltage reduction is to be less than 1%.

METHOD

In a double RF system the synchronous phase is given by:

$$\sin(\varphi_{s,0}) = \sin(\varphi_s) + \alpha \sin(n\varphi_s + \varphi_2), \qquad (3)$$

where $\varphi_{s,0}$ is the synchronous phase with a single RF system, φ_s is the synchronous phase in the two cavity system, α is the voltage ratio (V_2/V_1), n is the ratio of the harmonic numbers (h_2/h_1) and φ_2 is the phase of the second system with respect to the first [3]. For this paper we set $n = 1$, however the method presented can be modified to $n > 1$.

To guarantee φ_s exists for all φ_2, a limit can be imposed on α:

$$\alpha < 1 - |\sin(\varphi_{s,0})|. \qquad (4)$$

We define the phases $\varphi_{2,\mathrm{max/min}}$ and $\varphi_{2,\alpha\pm}$ as the stationary points of $\partial\varphi_s/\partial\varphi_2 = 0$ and $\partial\varphi_s/\partial\alpha = 0$ respectively. It can be shown that they satisfy the following relations:

$$\sin(\varphi_{s,0}) = -\cos(\varphi_{2,\mathrm{max}}) - \alpha \qquad (5)$$
$$\sin(\varphi_{s,0}) = \cos(\varphi_{2,\mathrm{min}}) + \alpha \qquad (6)$$
$$\sin(\varphi_{s,0}) = \sin(\varphi_{2,\alpha+}) \qquad (7)$$
$$\sin(\varphi_{s,0}) = -\sin(\varphi_{2,\alpha-}). \qquad (8)$$

Combining Eq. (5) and Eq. (7) gives

$$\cos(\varphi_{2,\mathrm{max}}) + \sin(\varphi_{2,\alpha+}) + \alpha = 0, \qquad (9)$$

eliminating $\varphi_{s,0}$.

The value of φ_2 cannot be known accurately if D and $\Delta\Phi$ are unknown. Instead only the programmed phase offset φ_{prog} can be known, which is given by

$$\varphi_{\mathrm{prog}} = \varphi_2 - \phi, \qquad (10)$$

where $\phi = \omega_{RF} D + \Delta\Phi$. Therefore, by measuring ϕ at different values of ω_{RF} the values of D and $\Delta\Phi$ can be determined.

Replacing φ_2 from Eq. (9) using Eq. (10) gives

$$\cos\left(\varphi_{\mathrm{prog,max}} + \phi\right) + \sin\left(\varphi_{\mathrm{prog},\alpha+} + \phi\right) + \alpha = 0, \qquad (11)$$

An equivalent result can also be obtained using Eq. (6) and (8). Identifying $\varphi_{\mathrm{prog,max/min}}$ and $\varphi_{\mathrm{prog},\alpha\pm}$ therefore identifies ϕ and the necessary compensations to correctly align the phases along the ramp.

NUMERICAL MODEL

The method described here relies on correctly identifying the stationary points of φ_s as a function of φ_{prog} and α. The efficacy can be demonstrated by numerically calculating φ_s at different points in the accelerating cycle for various values of D, $\Delta\Phi$. The versions used in the calculations to confirm suitability across a range of conditions were:

- Version 1: $D = 200$ ns, $\Delta\Phi = \pi/3$
- Version 2: $D = 136$ ns, $\Delta\Phi = 1.3\pi$
- Version 3: $D = -250$ ns, $\Delta\Phi = 0.3\pi$

For each version the values of α, φ_{prog} and ω_{RF} were sampled from the following ranges:

- $\alpha = [0.25, 0.625]$
- φ_{prog} (rad) $= [0, 2\pi]$
- ω_{RF} (MHz) $= [6.2, 9.4]$

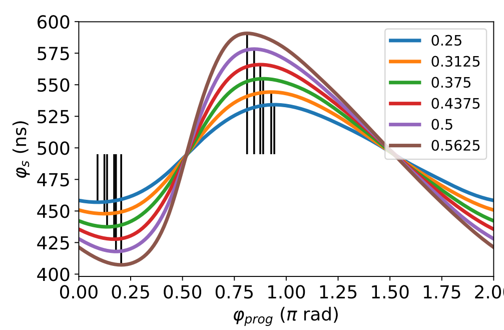

Figure 2: Phase φ_s as a function of α (coloured lines) and φ_{prog} (x-axis) with $D = 200$ ns, $\Delta\Phi = \pi/3$, $\omega_{RF} = 6.61$ MHz and $dp/dt = 1.9$ GeV c_0^{-1} s^{-1}. The vertical black lines indicate $\varphi_{\mathrm{prog,max/min}}$ for each α.

Figure 2 shows the synchronous phase in ns during a phase scan at $\omega_{RF} = 6.61$ MHz and an acceleration of $dp/dt = 1.9$ GeV c_0^{-1} s^{-1}. This example used $D = 200$ ns and $\Delta\Phi = \pi/3$. The coloured lines show a third order cubic spline interpolation of the measured data. Phases $\varphi_{2_{\mathrm{max}}}$, $\varphi_{2_{\mathrm{min}}}$ can be identified from the turning points in the interpolation.

Next, $\varphi_{\mathrm{prog},\alpha+}$ should be identified from the function

$$\delta(\varphi_{\mathrm{prog}}) = \sum_{i>j} \left[\varphi_s(\alpha_i, \varphi_{\mathrm{prog}}) - \varphi_s(\alpha_j, \varphi_{\mathrm{prog}})\right]^2, \qquad (12)$$

which satisfies $\delta(\varphi_{\mathrm{prog},\alpha\pm}) = 0$. Figure 3 shows the identified minima, with the first minimum corresponding to $\varphi_{\mathrm{prog},\alpha+}$, and the second minimum to $\varphi_{\mathrm{prog},\alpha-}$.

Since both $\varphi_{\mathrm{prog},\alpha+}$ and $\varphi_{\mathrm{prog,max}}(\alpha_i)$ were known, ϕ could be evaluated along the acceleration cycle. For a measurement with fixed α_i, it is possible to calculate ϕ by solving

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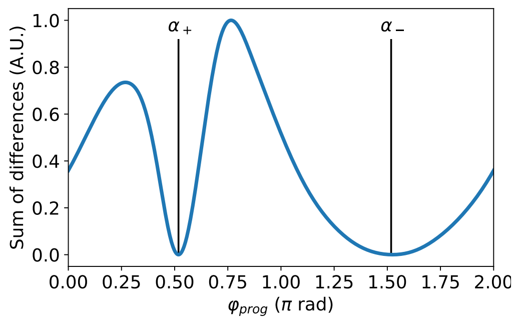

Figure 3: Sum of squared differences, used in Eq. (12), the points at $\alpha\pm$ identify $\varphi_{\text{prog}} = \varphi_{\text{prog},\alpha\pm}$.

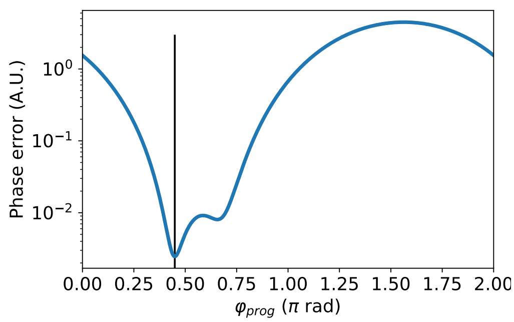

Figure 4: Plot of Eq. (15), the point indicated with the black line is where the two cavities are most closely aligned at this ω_{RF}.

Eq. (11):

$$\phi = k2\pi + \frac{-\varphi_{\text{prog},\alpha+} - \varphi_{\text{prog,max}}(\alpha_i) + \frac{\pi}{2}}{2} + \\ + \arccos\left[\frac{\alpha_i}{2}\csc\left(\frac{\varphi_{\text{prog},\alpha+} - \varphi_{\text{prog,max}}(\alpha_i) - \frac{3\pi}{2}}{2}\right)\right], \quad (13)$$

where $k \in \mathbb{Z}$. However, the exact solution is not practical as small measurement errors in $\varphi_{\text{prog,max}}(\alpha_i)$ and $\varphi_{\text{prog},\alpha+}$ lead to large errors in ϕ. To compensate for that, measurements over different α_i can be combined. The function

$$f(\alpha_i, \lambda) = \cos\left(\varphi_{\text{prog,max}}(\alpha_i) + \lambda\right) + \\ + \sin\left(\varphi_{\text{prog},\alpha+} + \lambda\right) + \alpha_i, \quad (14)$$

defined for $\lambda \in (0, 2\pi)$ satisfies $f(\alpha_i, \phi) = 0$ for all α_i, as seen from Eq. (11). The value of λ, such that $f(\alpha_i, \lambda)$ is as close to 0 as possible for all α_i, must then be identified. Defining

$$\mathcal{E}(\lambda) = \sum_i f(\alpha_i, \lambda)^2, \quad (15)$$

it follows that $\lambda = \phi$ implies $\mathcal{E}(\lambda) = 0$. This can noticed by the fact that $f(\alpha_i, \lambda)^2 \geq 0$, therefore $\mathcal{E}(\lambda) \geq 0$ and $\mathcal{E}(\phi) = 0$. Phase ϕ was found by minimising $\mathcal{E}(\lambda)$. The solution is given by the global minimum, as shown in Fig. 4. Repeating this process at different times in the ramp, corresponding to different ω_{RF}, allows ϕ as a function of ω_{RF} to be identified, as shown in Fig. 5. The phase measured at each ω_{RF} is given by the stars, with a linear regression through them used to identify D and $\Delta\Phi$.

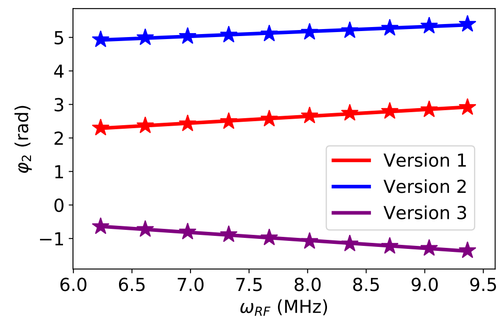

Figure 5: Multiple measures of ϕ at different ω_{RF} (stars), showing φ_2 when $\varphi_{\text{prog}} = 0$. The gradient and y-intercept of the linear regressions gives D and $\Delta\Phi$ respectively.

For the three versions shown here, the programmed and calculated D and $\Delta\Phi$ are shown in Table 1. As can be seen the errors are very small and in the worst case would result in approximately 15° misalignment between cavities, which would reduce the voltage seen by the beam by 0.9 %.

Table 1: Programmed and Calculated D and $\Delta\Phi$

	Programmed		Calculated	
	D	$\Delta\Phi$	D	$\Delta\Phi$
Version 1	200 ns	$\pi/3$ rad	203 ns	0.32π rad
Version 2	136 ns	1.3π rad	144 ns	1.28π rad
Version 3	−250 ns	0.3π rad	−235 ns	0.26π rad

CONCLUSION

In this paper we demonstrated that by identifying fixed points in φ_s as a function of φ_{prog} and α, the relative RF phase between two cavities can be measured. Measuring the phase offset at different points in the acceleration cycle, corresponding to different ω_{RF}, allows D and $\Delta\Phi$ to be calculated.

In a numerical study it was shown that two cavities could be aligned with sufficient precision to limit the voltage reduction seen by the beam to better than 1 %. This method will be applied in experiment after Long Shutdown 2 to minimise the misalignment between the new Finemet™ cavities of the CERN Proton Synchrotron Booster.

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